2012 Upper Primary Division Second Round Solution

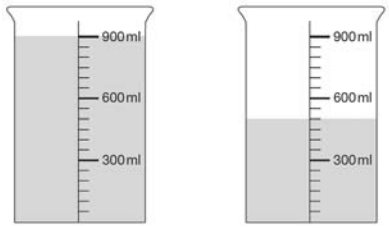
1. According the know exchange rate, Andrew converted $\frac{15700}{6.28} = 2500$ US dollars before the trip. After the trip, he has 2500－1612 = 888 US dollars, so he can converted his remained money to 6.25×888 = 5550 Yuan.

Answer：(A)

2. Because the sum of two odds is an even, so (A) is wrong. Since 1 + 2 = 3, (B) is wrong. 2 and 3 are both primes, and their sum is an odd, so (C) is wrong. Also, 1+1=2, so (D) is wrong. Among three consecutive positive integers, if the first one is not divided by 3, its remainder is either 1 or 2. If it is 1, the third integer is divided by 3; if it is 2, the second integer is divided by 3. Hence, (E) is right.

Answer：(E)

3. From the picture we know that six graduates are equal to 300 ml, so each graduate is equal to 50 ml. Observe that the water in the left cylinder is 400 ml more than that of the right one. So we need to pour $\frac{400}{2} = 200$ ml into the right cylinder.

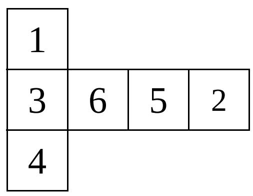


Answer：(D)

4. According to the property of numbers divided by 3 (the sum of every digit is divided by 3), we can exclude (B) and (D). Also, according to the property of numbers divided by 4 (the number last two digit is divided by 4), we can exclude (E). Since 201312 is not divided by 7, we choose (C).
In fact, 201348 = 3 × 4 × 7 × 2397.

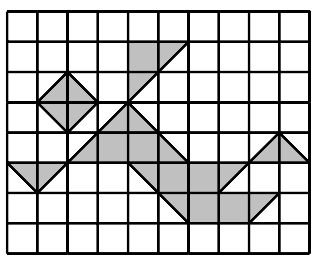
Answer：(C)

5. Among five choices, 3 appears most frequently. So we focus on 3. If (A) and (B) are the same, (A) should be (B)'s upside-down. So 1 and 4 are on opposite face, which contradicts (C). Hence, one of (A), (B) and (C) is different from the others. Also, if (C) and (D) are the same, (D) should be (C)'s counterclockwise. That is, 1 and 2 are on opposite face, which contradicts (E). So one of (C), (D) and (E) is different from the others. Therefore, (C) must be different from the other dices. The other four dices should be as this figure.

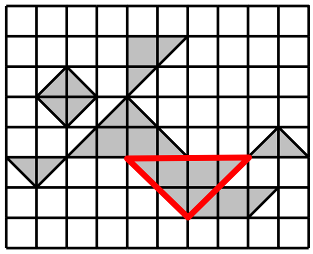


Answer：(C)

6. 【**Solution 1**】 There are 8 × 10 =80 squares in the grid. The shaded part consists of 18 isosceles right triangles, which are equal to 9 squares, and 7 squares. Hence, the shaded part is $\frac{16}{80} \times 100\% = 20\%$ of the figure.



【**Solution 2**】The area of the grid is 8 × 10 =80. Observe that the shaded part is composed by Tangram. So the area of shaded part is four times as large as the red triangle, that is, 16. So the shaded part is $\frac{16}{80}\times100\%=20\%$ of the figure.



Answer：20%

7. 【**Solution 1**】Ben was (6－4)x2=4 km behind Andy when he rode on the bike, so it took him 4÷(10－6)= 1 to catch up with Andy. At the same time, they arrived at town B together. Therefore, it took Andy 3 hours to arrive town B from town A. Then, the distance between two towns is 6x3=18 km.

【**Solution 2**】Assume that it took Ben t hours to arrive town B after starting to ride the bike, so

$$2\times4+10t=2\times6+6t,\ t=1.$$

Therefore, the distance between town A and town B is $2\times4+10\times1=18$ km.

Answer：18 km

8. 【**Solution 1**】Since we can get a second cup of juice by paying 1 more dollar while buying a cup at the regular price of 7 dollars, we will get 8 cups by buying 4 cups and paying 4 more dollars. There are 9 persons, so we need 1 more cup. They totally cost 4 × 7 + 4 + 7 = 39 dollars.

【**Solution 2**】We know that we can get 1 more cup of juice by paying 1 more dollar while buying a cup at the price of 7 dollars. That is, 2 cups of juice cost at least 8 dollars. Now we need 9 cups. Since 9 = 2 × 4 + 1, it cost us at least 4 × 8 + 1 × 7 = 39 dollars to buy the drink.

Answer：39 dollars

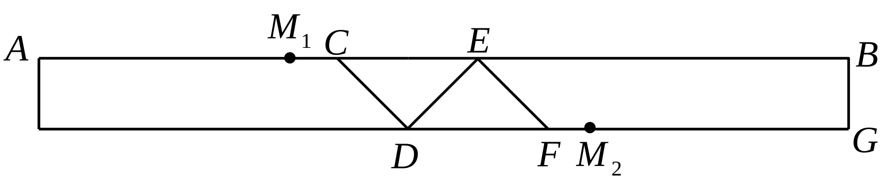
9. Assume that there are x men and 35－x women in the party. Because the number of times men handshake with women is equal to that of women handshake to men,

$$4x=6(35-x),\ x=21.$$

So there are 21 men in the party.

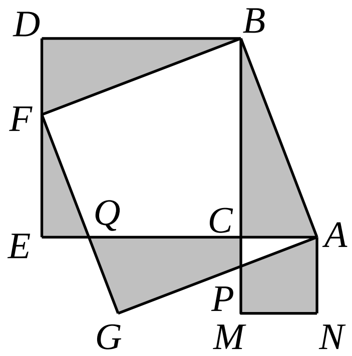
Answer：21 men

10. Unfold the strip, the crease should show as the followed picture. By condition, $AM_1=GM_2$. Since $CM_1=FM_2=3$, we know that AC = FG. Hence, the picture is centrally symmetric. So $AC=\frac{30}{2}-3-\frac{3}{2}=10.5$ cm.



Answer：10.5 cm

11. Assume that AG meets CM at P, EC meets FG at Q. By Pythagorean theory, we know that the area of $BFGA$ is equal to the sum of the area of $BDEC$ and that of $ACMN$. From the figure we know the area of shaded part is $2(S_{\Delta ABC}+S_{CQGP})$. Observe that $\triangle ABP$ is as large as $\triangle AQG$, and $\triangle ACP$ is overlapping. Hence, $S_{\Delta ABC}=S_{CQGP}$. So the area of shaded part is $4S_{\Delta ABC}=48$ cm^2. Therefore, the area of $\triangle ABC$ is 12 cm^2。



【**Note**】In the solution, we use the condition of $\triangle ABP=\triangle AQG$. we are not testing students if they have the knowledge about congruence of triangles. Instead, we are testing if they know $\triangle ABP$ and $\triangle AQG$ are the same through observation.

Answer：12 cm^2

12. 【**Solution 1**】We can get 2 strands of rope when folding for one time; 4 strands of rope when folding for two times; 8 strands of rope when folding for three times; 16 strands of rope when folding for four times; 32 strands of rope when folding for five times;64 strands of rope when folding for six times. If we cut the rope in halves after folding six times, the rope will be cut at 64 different places. That is, it will separate into 65 pieces.

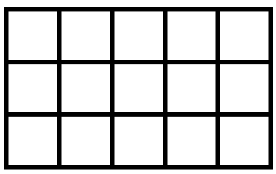
【**Solution 2**】the pieces we get when cutting in halves after folding for several times have the rule as follow:

Cut the rope in halves after folding for 1 time, we will get $2^1+1=3$ pieces;
Cut the rope in halves after folding for 2 time, we will get $2^2+1=5$ pieces;
Cut the rope in halves after folding for 3 time, we will get $2^3+1=9$ pieces;
Cut the rope in halves after folding for 4 time, we will get $2^4+1=17$ pieces;
Cut the rope in halves after folding for 5 time, we will get $2^5+1=33$ pieces;
Cut the rope in halves after folding for 6 time, we will get $2^6+1=65$ pieces.

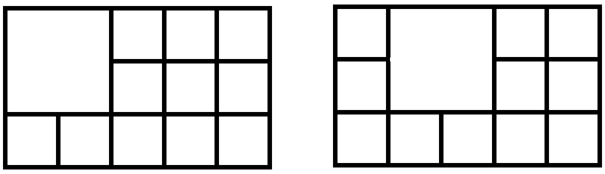
Answer：65 pieces

13. Because we will use at most two pieces of square of 20 cm cm when making paper which length is 50 cm and width is 30 cm, so we turn the problem into several cases:

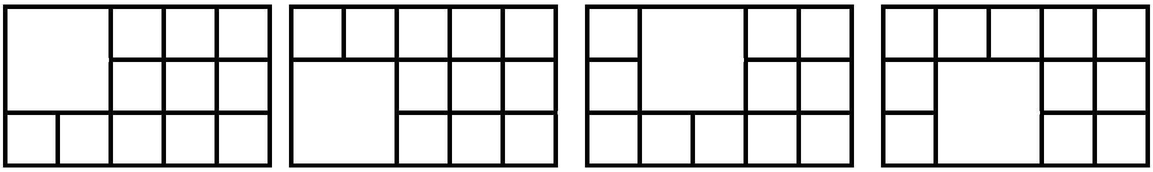
（Case 1）There is only 1 way if we use only squares of 10 cm.



（Case 2） We use one piece of square of 20 . There are 8 ways, as followed there are two kinds of them.



（Case 3） We use two pieces of squares of 20 cm. Because the width is 30 cm, the two pieces should be one on the left side, the other one on the right side. we turn them into several cases:



(a) (b) (c) (d)

Case (a)：There are 4 different places to put the other piece.

Case (b)：There are 4 different places to put the other piece.

Case (c)：There are 2 different places to put the other piece.

Case (d)：There are 2 different places to put the other piece.

To sum up, there are 1+8+4+4+2+2=21 different ways.

Answer：21 ways

14. Assume that the perfect squares between n and n+100 are

$$m^2,\ (m+1)^2,(m+2)^2,(m+3)^2,(m+4)^2,\ (m+5)^2.$$

While $m \geq 8$, $(m+5)^2 - m^2 = 10m + 25 > 100$. Hence, $m \leq 7$. (5 marks)

From the problem we know that $(m+6)^2 > n+100$ and $(m-1)^2 < n$, so

$(m+6)^2 - (m-1)^2 = 14m + 37 > 100$. That is, $m \geq 5$.

Hence, $5 \leq m \leq 7$. (5 marks)

（Case 1） If m=5, six perfect numbers are 25, 36, 49, 64, 81, 100. Because the largest perfect number smaller than 25 is 16, the smallest perfect number larger than 100 is 121, the range of n are $17 \leq n \leq 20$. Hence, there are 4 possible values.

（Case 2） If m=6, six perfect numbers are 36, 49, 64, 81, 100, 121. Because the largest perfect number smaller than 36 is 25, the smallest perfect number larger than 121 is 144, the range of n are $26 \leq n \leq 36$. Hence, there are 11 possible values.

（Case 3） If m=7, six perfect numbers are 49, 64, 81, 100, 121, 144. Because the largest perfect number smaller than 49 is 36, the smallest perfect number larger than 144 is 169, the range of n are $44 \leq n \leq 49$. Hence, there are 6 possible values. (10 marks)

To sum up, there are 4+11+6=21 possible values.

Answer：21

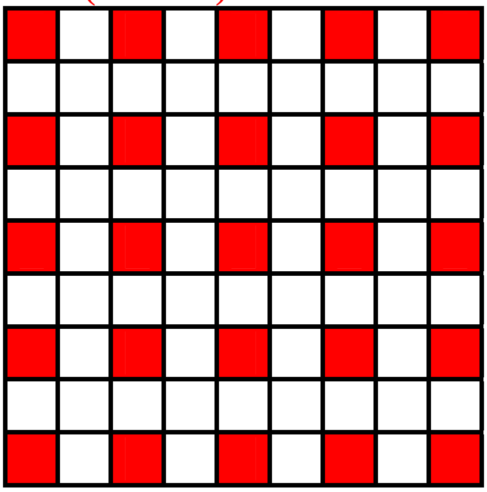
【Marking Scheme】

Find $m \leq 7$, 5 marks;

Find $m \geq 5$, 5 marks;

Discuss the three situations exactly, 10 marks. If only one or two situations are exact, 5 marks.
Only exact solution without the solving process, 5 marks.

15. As shown, we color the squares lie on both odd row and odd column. There are totally 5×5=25 red squares. (5 marks)



Generally call these shapes "polyomino", specifically called the first shape "tromino".

It's clear that a polyomino cover at most a red square. Hence, we need at least 25 polyominoes to cover the chessboard. Assume that *m* trominoes are used, *n* copies of other two shapes are used, then

$$m+n\geq 25。 \quad (5\ \text{marks})$$

Also, each tromino will cover 3 squares, each copy of the other two shapes will cover 4 squares, then

$$3m+4n=9\times 9=81。$$

Hence,

$$4n=81-3m.$$

Also,

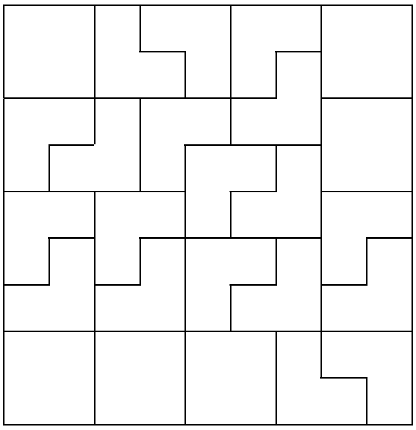
$$4m+4n\geq 100.$$

Therefore,

$$4m+(81-3m)\geq 100,$$

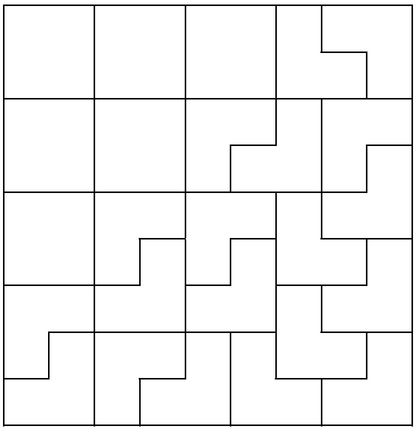
$$m\geq 19.\ (5\ \text{marks})$$

So we need at least 19 trominoes. As followed there is a case satisfied. (5 marks)



Answer：19 copies

【**Note**】Because 25 polyominoes are used, including 19rominoes, there are only 6 pieces of the other two shapes. Generally speaking, we only need to arrange these 6 pieces properly, the rest of squares can be covered by 19 trominoes. For example, here is another way to cover the rectangle,



【Marking Scheme】

Giving the exactly painting way , 5 marks;

Knowing $m+n \geq 25$, 5 marks;

Find $m \geq 19$, 5 marks;

Construct a covering way satisfied the conditions, 5 marks;

Only numerical solution without constructing a covering way, 0 marks.