2012 UPPER PRIMARY FIRST ROUND SOLUTION

1. What is the value of 101010－10101?

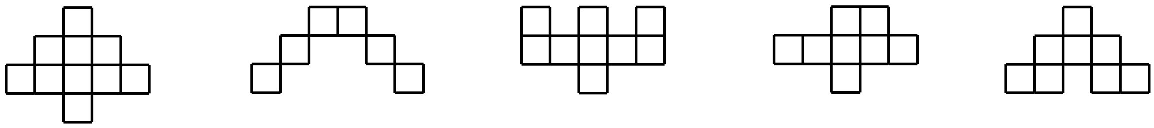
（A）90909　（B）10101　（C）100000　（D）900000　（E）10000

【Suggested Solution】

101010－10101=90909.　Answer：(A)

2. The following figures are formed by identical squares. Which figure has the longest perimeter?

（A）　（B）　（C）　（D）　（E）



【Suggested solution】

Let the side lengths of squares are 1.The perimeter of figure A is 18; the perimeter of figure B is 22; the perimeter of figure C is 20; the perimeter of figure D is 16; the perimeter of figure E is 18. Thus the figure with largest perimeter is B.

Answer：(B)

3. Which of the following sets of fractions has sum greater than 1?

(A) $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$　(B) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$　(C) $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$

(D) $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$　(E) $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}$

【Suggested solution】

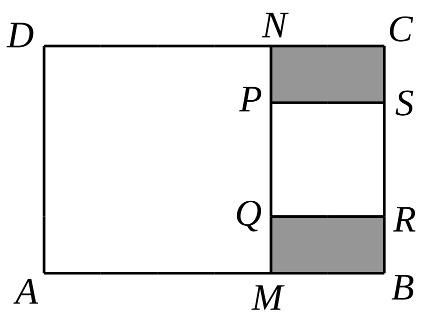
The sum of fractions in A is $\frac{47}{60}$ (which is smaller than 1), the sum of fractions in B is $\frac{11}{12}$ (which is smaller than 1), the sum of fractions in C equal to 1, the sum of fractions in D is $\frac{19}{20}$(which is smaller than 1), the sum of fractions in E is greater than that of C and is $\frac{31}{30}$ which is greater than 1, so answer is E.

Answer：(E)

【Remark】

We can compare the sums with 1 with almost no calculations. Choice A is eliminated since $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}<\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$. Choice B is eliminated since $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}<\frac{1}{2}+\frac{1}{4}+\frac{1}{4}=1$. Choice D is eliminated since $\frac{1}{2}+\frac{1}{4}+\frac{1}{5}<\frac{1}{2}+\frac{1}{4}+\frac{1}{4}=1$. Choice E is the correct one since $\frac{1}{2}+\frac{1}{3}+\frac{1}{5}>\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$.

4. The diagram to the right shows two squares *AMND* and *PQRS* inside a rectangle *ABCD*. The areas of the two squares are 16 cm^2 and 4 cm^2 respectively. What is the sum of area of the shaded regions in cm^2?



（A）3 （B）4 （C）5

（D）6 （E）7

【Suggested solution】

The side lengths of two squares *AMND* and *PQRS* are 4cm and 2cm respectively. Hence MN = 4 cm，$PS = PQ = RQ$ = 2 cm, and the sum of areas of the shaded regions in the figure is

$NP\cdot PS + QM\cdot RQ = PS\cdot(NP+QM) = PS\cdot(MN-PQ) = 2\times(4-2) = 4\,\text{cm}^2$.

Answer：(B)

5. Mike jogs at constant speed along a road with lamp posts which are evenly spaced. It takes him 2 minutes to go from the first lamp post to the fifth lamp post. He continues jogging to the n-th lamp post before turning back. If he has been jogging for 12 minutes when he returns to the first lamp post, what is the value of n?

（A）6 （B）9 （C）11 （D）12 （E）13

【Suggested solution】

Mike spends half the time going away from the first lamp post. He reaches the fifth lamp post in 2 minutes, the ninth lamp post in 4 minutes and the thirteenth lamp post in 6 minutes. Hence n=13.

Answer：(E)

6. The smallest interior angle of a triangle is 50°. Which of the following statements about this triangle is correct?

（A）It must be isosceles. （B）It must be right angled.

（C）It must be acute angled. （D）It must be obtuse angled.

（E）None of these is correct.

【Suggested Solution】

Since the smallest interior angle in a triangle is 50°, the maximum interior angle will not exceed 180°－50°－50° = 80°. Therefore, this triangle must be acute angled. It does not have to be isosceles as the angles may be (50°, 60°, 70°).

Answer：(C)

7. Each of A, B, C and D has a watch.

(1) A's watch is 10 minutes faster than standard time but A believes that it is 5 minutes slower.

(2) B's watch is 5 minutes slower than standard time but B believes that it is 10 minutes faster.

(3) C's watch is 5 minutes faster than standard time but C believes that it is 3 minutes faster.

(4) D's watch is 5 minutes slower than standard time but D believes that it is 10 minutes slower.

According to their watches and their beliefs, they go to school in order to be just in time. Who is late?

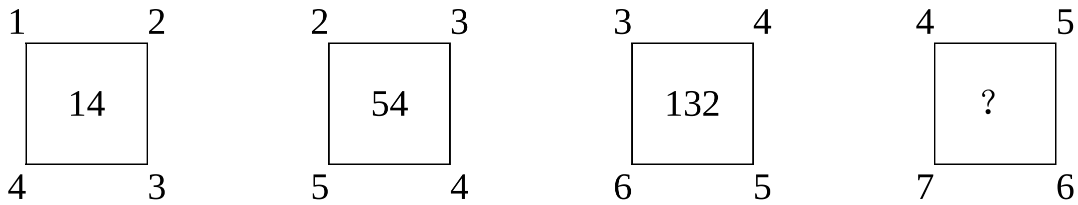
(A) A (B) B (C) C (D) D (E) No one

【Suggested solution】

B is the only one who is late, by 10－(-5)=15 minutes. A is early by 5－(-10)=15 minutes, C is early by 5－3=2 minutes and D is early by (-5)－(-10)=5 minutes.

Answer：(B)

8. The diagram below shows four squares with numbers which exhibit a certain pattern. What number should be inside the fourth box?



(A) 210 (B) 260 (C) 288 (D) 308 (E) 330

【Suggested solution】

A possible pattern is that the number inside the box is equal to the sum of the two numbers at the bottom corners times the product of the two numbers at the top corners. Hence the number inside the fourth box should be $4\times5\times(7+6)=260$.

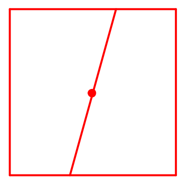
Answer：(B)

9. A piece of square paper is folded along a straight line, dividing it into two regions with the same area and the same shape. How many such straight lines are there?

(A) 1 (B) 2 (C) 3

(D) 4 (E) infinitely many

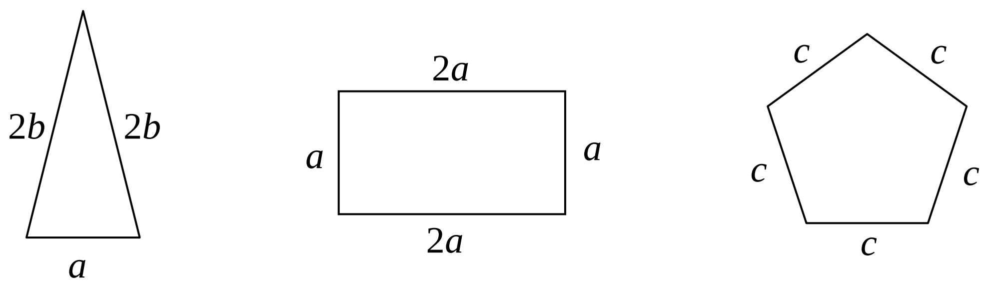
【Suggested solution】



Any straight line passing through the center of the square cuts the square into two congruent regions as shown in the diagram to the right. The two halves are identical by a 180° rotation. There are infinitely many such straight lines.

Answer：(E)

10. The three figures in the diagram below have equal perimeters. What is $a:b:c$?



(A) 1：1：1 (B) 2：4：5 (C) 4：6：5

(D) 16：20：25 (E) 20：25：24

【Suggested solution】

We have $4b+a=6a=5c$ so that $4b=5a$ and $6a=5c$. Thus $a:b=4:5$ and $a:c=5:6$, so that $a:b:c=20:25:24$.

Answer：(E)

11. A triangle is formed with 10 matchsticks of equal length connected end to end. No matchsticks are bent or broken. How many different triangles can be formed?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

【Suggested solution】

By the Triangle Inequality, the sum of two sides of a triangle is greater than the third side. The only possible triangles have side lengths (2, 4, 4) and (3, 3, 4).

Answer：(A)

12. The total weight of 5 apples is equal to that of 6 bananas, and the total weight of 3 bananas is equal to that of 4 oranges. How many apples have the same total weight as 16 oranges?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

【Suggested solution】

From the question, we can deduce that 6 bananas have the same weight as 8 oranges. Thus 5 apples will have the same weight as 8 oranges. Hence, 10apples will have the same weight as 16 oranges.

Answer：(C)

13. Jane begins observing a hyacinth plant on last Friday, when some of its flowers bloom. Thereafter, the number of flowers which bloom on any day is equal to the number of flowers which have already bloomed. No flowers wither. All the flowers have bloomed by the following Thursday. On which day of the week have exactly half of the flowers bloomed?

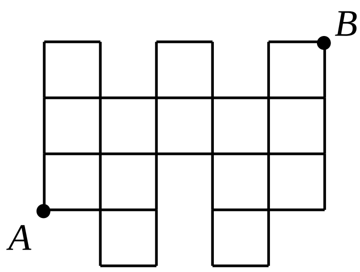
(A) Saturday (B) Sunday (C) Monday
(D) Tuesday (E) Wednesday

【Suggested solution】

Since the number of flowers which bloom on any day is equal to number of flowers which have already bloomed, the number of flowers which bloom on the last day is equal to one half of the total number of flowers. This means that the day for exactly half of the flowers to have bloomed is Wednesday, the day before the last day.

Answer：(E)

14. The side length of each square in the diagram below is 1 unit. An ant needs 5 seconds to crawl 1 unit horizontally, and 6 seconds to crawl 1 unit vertically. It requires 1 second to change direction. What is the minimum number of seconds for the ant to crawl from point A to point B?



(A) 43 (B) 45 (C) 46 (D) 47 (E) 48

【Suggested solution】

In order to minimize the time for the ant to crawl from point A to point B, it should take as few changes of direction and crawl vertically as few times as possible. The ant can crawl 2 units upwards from point A, change direction to right, then crawl 5 units horizontally, then change to upwards direction, finally crawling 1 unit and reach the point B. So the minimum time requires $2\times6+1+5\times5+1+6=45$ seconds。

Answer：(B)

15. During the holidays, Dick worked part-time washing bowls in a restaurant. He got paid 3 dollars for washing one bowl. If he broke a bowl, he got no pay for washing it, and must pay 9 dollars to the owner. In one week, Dick washed 500 bowls and earned 1368 dollars. How many bowls did he break?

（A） 7　　（B） 8　　（C） 9　　（D） 10　　（E） 11

【Suggested solution】

After washing 500 bowls, Dick should receive $3\times500=1500$ dollars, but he only got 1368 dollars, which was 132 dollars less. For each bowl Dick broke, he lost 3 dollars of income and 9 dollars in compensation, for a total of 12 dollars. Thus Dick had broken $132\div12=11$ bowls.

Answer：(E)

16. Four coloured lamps are arranged in a 2×2 array, where the letters **R**, **Y**, **B** and **W** represent the colours red, yellow, blue and white respectively. Each lamp changes its colour every minute. In the first minute, the colours of the lamps in the two rows are interchanged. In the second minute, colours of the lamps in the two columns are interchanged. See the diagram below. This cycle is then repeated. What are the colours of the four lamps at the 60th minute?

R	**Y**
B	**W**

(start)

→

B	**W**
R	**Y**

(1st minute)

→

W	**B**
Y	**R**

(2nd minutes)

……

（A）

R	**Y**
B	**W**

（B）

B	**W**
R	**Y**

（C）

W	**B**
Y	**R**

（D）

Y	**R**
W	**B**

（E）

Y	**R**
B	**W**

【Suggested solution】

R	**Y**
B	**W**

(initial)

→

B	**W**
R	**Y**

(1st minute)

→

W	**B**
Y	**R**

(2nd minutes)

→

Y	**R**
W	**B**

(3rd minutes)

→

R	**Y**
B	**W**

(4th minutes)

……

According to the pattern of change in colour of lamps, the colour of lamp changed back to its original colour every 4 minutes, 60 (1 hour equal to 60 minutes) is a multiple of 4, and the colour of lamp is equal to the status at the start after an hour.

Answer：(A)

17. The elder sister is 12 years old, the younger sister is 8 years old and brother is 3 years old. Their birthdays are in the same day. When the sum of their ages equal 50, what is the age of the younger sister?

(A) 14 (B) 15 (C) 16 (D) 17 (E) 18

【Suggested solution】

This year, the sum of their ages equal to 23. When sum of their ages equal to 50, it already pass through (50－23) ÷ 3 = 9 years. Therefore the age of younger sister at that time is 8 + 9 = 17。

Answer：(D)

18. Mrs. Wong worked at the airport. She had two consecutive days of rest after working for eight days. If she rests on Saturday and Sunday this week, at least how many weeks later will she rest on Sunday again?

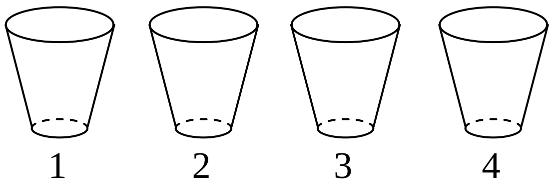
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

【Suggested solution】

The next few rest periods are Tuesday and Wednesday; Friday and Saturday; Monday and Tuesday; Thursday and Friday; Sunday and Monday, and so on. Therefore she can rest on Sunday again after 10×4+9=49 days, or 7 weeks.

Answer：(D)

19. The diagram below shows four cups in a row, labelled 1, 2, 3 and 4. Initially, there is a ball inside cup #3. In each move, the ball may be transferred to an adjacent cup. From cup #1, the ball can only go to cup #2, and from cup #4, the ball can only go to cup #3. After 2012 moves, which of the following statements about the ball is correct?



(A) It cannot be in cup #1 and cannot be in cup #2

(B) It cannot be in cup #1 and cannot be in cup #3

(C) It cannot be in cup #2 and cannot be in cup #3

(D) It cannot be in cup #2 and cannot be in cup #4

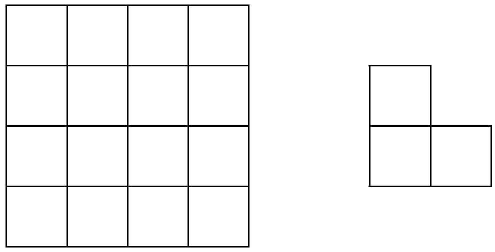
(E) It cannot be in cup #3 and cannot be in cup #4

【Suggested solution】

Since the ball will be transferred to an adjacent cup in each move, the parity of the cup number will change in each move. After 2012 moves, the parity is unchanged. Therefore the ball can only in cup #1 or cup #3 after 2012 operations and it cannot be in cup #2 or cup #4.

Answer：(D)

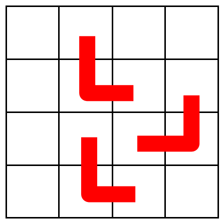
20. On a 4×4 chessboard shown in the diagram below on the left, we wish to place a minimum number of copies of the shape shown in the diagram below on the right, so that no more copies of this shape can be placed. Copies may be rotated. What is this minimum number of copies?



（A）2　　　（B）3　　　（C）4　　　（D）5　　　（E）6

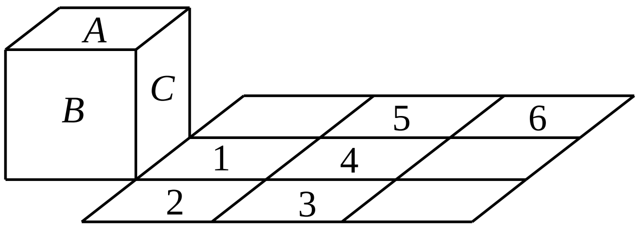
【Suggested solution】

Divide the 4×4 chessboard into four 2×2 chessboards. The copies of the shapes placed must cover at least 2 squares of each 2×2 chessboard in order to prevent another copy of the shape to be placed within the 2×2 chessboard. Thus we must cover at least 8 squares, which requires at least 3 copies. These may be placed as shown in the diagram below.



Answer：（B）

21. The diagram below shows a cube with three of its faces labelled A, B and C, and a 3×3 square with six of its squares labeled 1, 2, 3, 4, 5 and 6. The cube is tipped over so that face C lies on square 1, tipped over again so that face B lies on square 2, and so on until the cube lies on square 6. What is the sum of the numbers of the squares on which the cube has laid with face B on top?

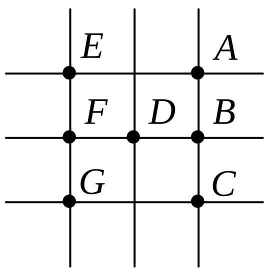


【Suggested solution】

When the cube is tipped onto square 1, face B is in front. When the cube is tipped onto square 2, face B is at the bottom. When the cube is tipped onto square 3, face B is on the left. When the cube is tipped onto squares 4 and 5, face B remains on the left. When the cube is tipped onto square 6, face B is on top. Hence the desired sum is 6.

Answer：006

22. The diagram below shows 7 points *A*, *B*, *C*, *D*, *E*, *F* and *G*.



A flea starts from A, in each move the flea jumps from a given point to another given point and must always follow any one of the two types of jumps.

(1) Jumps two steps to the west (for instance from A to E), jumps two steps to the south (for instance from A to C), or jumps one step to the southwest (for instance from A to D).

(2) Jumps one step to the east (for instance from F to D) or jumps one step to the north (for instance from F to E).

The flea must make these two types of moves alternately ((1), (2), (1), (2) and so on, or (2), (1), (2), (1) and so on). It may not land on the same point twice, and it will stop when it gets to G. How many possible routes can it follow?

【Suggested solution】

First, there are 3 choices for the first step, jump from point A to point E or point C or point D. If jump from point A to point E, next step require it to lead backwards but it is impossible to do so at point E. So this situation does not exist.

If jump from point A to point C, then the next step possible is at point B, and then jump to point F. And from F, it can jump to point E or point D. Either point E or point D will result at point G. Thus the condition has 2 routes.

If jump from point A to point D, the next step is it will arrive at point B. Then jump to point F. And from F, it can jump to point E. The point E will result at point G. Thus this condition has 1 routes.

To conclude, there are 3 possibilities for a flea to jump from point A to point G.

Answer：003

23. The number of bicycles in the school bicycles lot is a three-digit number, and the number of bicycle wheels is also a three-digit number. These six digits are 2, 3, 4, 5, 6 and 7 in some order. At most how many bicycles are there?

【Suggested solution】

Let the total number of bicycles be $\overline{abc}$ and the total number of wheels be $\overline{def}$.

From the question , we have $\overline{def}=2\overline{abc}$ ，so a can only be 2 or 3.

If a = 3，then d can only be 6 or 7。

When d = 6，b can only be 2, then c = 7, thus $\overline{abc}=327$, $\overline{def}=654$ 。

When d = 7，b can only be 6, there does not exist any suitable value for c。

If a = 2，the total number of bicycles is obviously less than 327, therefore there are at most 327 bicycles.

Answer：327

24. For any positive integers a and b, define a new operation $a \odot b$ which yields the remainder when the larger of a and b is divided by the smaller one. For example, $5\odot 12 = 12\odot 5 = 2$. Given that $(11\odot x)\odot 11 = 2$, what is the minimum value of x？

【Suggested solution】

If x=1, then $11\odot x = 0$ and $0\odot 11$ is undefined. If x=2, then $11\odot x=1$ and $1\odot 11=0$. If x=3, then $11\odot x=2$ and $2\odot 11=1$. If x=4, then $11\odot x=3$ and $3\odot 11=2$. Hence the minimum value of x is 4.

Answer：004

25. In the expression 2□3□4□5, an operation sign (addition, subtraction, multiplication or division) is placed in each □. The same operation may be repeated, and brackets may be inserted. What is the largest two-digit number that can be obtained?

【Suggested solution】

It is not hard to see that subtraction and division signs should not be used. Now a multiplication sign must be filled in the last □; otherwise the maximum value will not exceed $2 \times 3 \times (4 + 5) = 54$. If the two □s in the front are filled with multiplication signs, then the number obtained is 120, which is not a two-digit number. If both are filled with addition signs, then the largest value which may be obtained is $(2 + 3 + 4) \times 5 = 45$. Suppose one is filled with an addition sign and the other a multiplication sign. For $2 + 3 \times 4 \times 5$, the maximum value which may be obtained is $(2 + 3 \times 4) \times 5 = 70$. For $2 \times 3 + 4 \times 5$, the maximum value which may be obtained is $2 \times (3 + 4) \times 5 = 70$ also.

Answer：070