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## Junior Division Round 2

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### Questions 1 to 5, 4 marks each

1. When the digits 0, 1, 2, 5, 6, 8 and 9 are rotated  $180^\circ$ , they become 0, 1, 2, 5, 9, 8 and 6 respectively. What does 9105 become when the four-digit number is rotated  $180^\circ$ ?
- (A) 6150      (B) 6102      (C) 5016      (D) 2019      (E) 2016

Answer : \_\_\_\_\_

2. What is the value of the expression  $(a-b)(a+b-c) + (b-c)(b+c-a) + (c-a)(c+a-b)$
- (A) 0      (B)  $a^2 + b^2 + c^2$       (C)  $ab + bc + ca$   
(D)  $a^2 + b^2 + c^2 - ab - bc - ca$       (E)  $a^2 + b^2 + c^2 + ab + bc + ca$

Answer : \_\_\_\_\_

3. We wish to dissect a square into  $n$  squares which need not be of the same size. For which of the following values of  $n$  is this impossible?
- (A) 5      (B) 6      (C) 7      (D) 8      (E) 9

Answer : \_\_\_\_\_

4. The total number of players on three badminton teams is 29. No two players on the same team play against each other, while every two players on different teams play each other exactly once. What is the maximum number of games played?
- (A) 265      (B) 270      (C) 276      (D) 280      (E) 282

Answer : \_\_\_\_\_

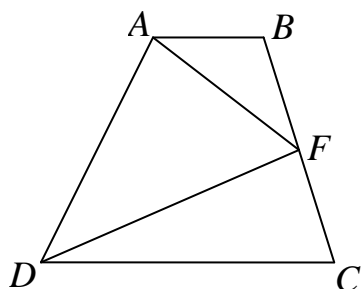
5. Two distinct quadratic polynomials  $f(x)$  and  $g(x)$  with leading coefficients equal to 1 satisfy  $f(1) + f(3) + f(5) = g(1) + g(3) + g(5)$ . Find all solutions of  $f(x) = g(x)$ .
- (A)  $x \leq 0$       (B)  $-2 \leq x \leq 0$       (C)  $0 \leq x \leq 1$       (D)  $2 \leq x \leq 2$       (E) 3

Answer : \_\_\_\_\_

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**Questions 6 to 13, 5 marks each**

6. The diagram shows a quadrilateral  $ABCD$  with  $AB$  parallel to  $DC$ .  $F$  is the midpoint of  $BC$ . If the area of triangle  $AFD$  is  $10 \text{ cm}^2$ , what is the area, in  $\text{cm}^2$ , of  $ABCD$ ?



Answer : \_\_\_\_\_  $\text{cm}^2$

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7. Lily has 2014 chocolates. She eats one on the first day. Each day after, she eats twice as many as the day before, until all the chocolates have been eaten. How many chocolates did she eat on the last day?

Answer : \_\_\_\_\_ chocolates

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8. Leon is putting 99 apples into boxes of two different sizes. A large box can hold 12 apples while a small box can hold 5 apples. All boxes must be full. How many boxes will he need if this number must be greater than 10?

Answer : \_\_\_\_\_ boxes

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9. In a convex quadrilateral  $ABCD$ ,  $AB = 3$ ,  $BC = 5$ ,  $CD = 6$ ,  $DA = 10$ , and the length of the diagonal  $AC$  is a positive integer. How many different possible shapes can  $ABCD$  take?

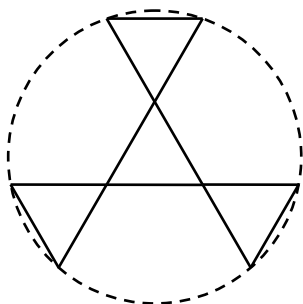
Answer : \_\_\_\_\_ shapes

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10. Divide the ten positive integers from 1 to 10 into two groups so that when the product of the numbers in the first group is divided by the product of the numbers in the second group, the quotient is a positive integer. What is the minimum value of this quotient?

Answer : \_\_\_\_\_

11. A cardboard windmill with three blades is made from four equilateral triangles of side length 6 cm. Two triangles sharing a common vertex have the corresponding sides lying on the same straight line, as shown in the diagram. The area of the circle swept out by the blades of the windmill is  $x \text{ cm}^2$ . What is the greatest integer less than or equal to  $x$ ?



Answer : \_\_\_\_\_

12. Let the real numbers  $a_1, a_2, a_3, a_4$  and  $a_5$  be such that  $a_{n+1} = |a_n| - |a_n - 1|$  for  $1 \leq n \leq 4$ . If  $a_5 = \frac{1}{2}$  and  $a_1 = \frac{p}{q}$ , where  $p$  and  $q$  are relatively prime positive integers, what is the value of  $p + q$ ?

Answer : \_\_\_\_\_

13. What is the minimum perimeter of a parallelogram which may be partitioned into 462 equilateral triangles of side length 1 cm?

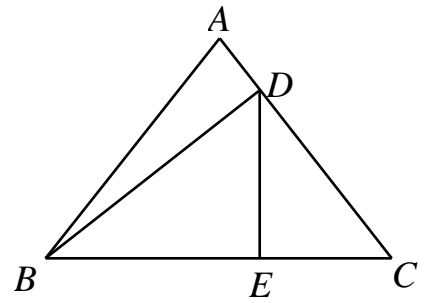
Answer : \_\_\_\_\_ cm

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**Questions 14 to 15, 20 marks each**

**Detailed solutions are needed for these two problems**

14. In an acute triangle  $ABC$ ,  $AB = AC$ .  $D$  is the foot of the perpendicular from  $B$  to  $CA$ , and  $E$  is the foot of the perpendicular from  $D$  to  $BC$ . If  $BC = AB + AD$ , prove that  $BE = CD$ .



15. A positive integer  $x$  with  $n \geq 2$  digits is written down twice in a row and the  $2n$ -digit number so obtained is divisible by  $x^2$ . Prove that the first two digits of  $x$  are 1 and 4 in that order.
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