Solution to First Round of Third IMAS Junior Division

1. What is the value of the expression $|-2013|+2^0+1^3$?

   (A) 2014 (B) 2015 (C) 2016 (D) -2010 (E) -2011

【Suggested Solution】

Since $|-2013|=2013$, $2^0=1^3=1$. Thus, the value of the given expression is 2013+1+1=2015.

Answer: B

2. Which of the following real numbers has the greatest absolute value?

   (A) $-\pi$ (B) $\sqrt{7}$ (C) 3.1 (D) -2 (E) $\frac{23}{8}$

【Suggested Solution】

The absolute value of each of the five expressions is $\pi \approx 3.1416$, $\sqrt{7} \approx 2.645$, 3.1, 2 and $\frac{23}{8}=2.875$. Hence the expression with the greatest absolute value is $-\pi$.

Answer: A

3. Which of the following five numbers is divisible by 6?

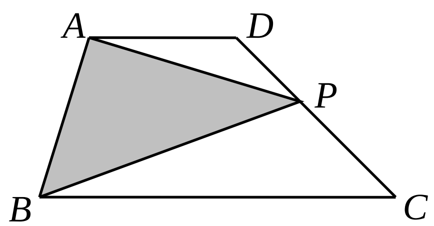
   (A) 332 (B) 363 (C) 494 (D) 522 (E) 586

【Suggested Solution】

A number is divisible by 6 if it is divisible both by 2 and 3. While a number is divisible by 2, its last digit must be an even number and for a number that is divisible by 3 when the sum of the digits of the number is divisible by 3. After doing the divisibility test, sum up all the digits of 332, 494, 522 and 586 are 8, 17, 9 and 19; respectively. None of them is divisible by 6. While the sum of all the digits of 522 is 9, which is divisible by 9 and the last digit is even.

Answer: D

4. In the diagram, *AD* is parallel to *BC*. A point *P* moves from *C* to *D* along the side *CD*. Which of the following is the accurate description of the change in the area of $\triangle ABP$ during the motion?



   (A) becomes larger (B) becomes smaller

   (C) becomes larger then reduce to smaller

   (D) becomes smaller then enlarge to bigger (E) no changes

【Suggested Solution】

Let us assume AB as the base of $\triangle ABP$ and observe the changes of point P whose first location is at point C then moving sliding to point D, we are able to discover the distance from P to side AB keeps on decreasing and becomes smaller, then the area of $\triangle ABP$ also decreases and becomes smaller, hence we choose (B).

Answer: B

5. If x is a real number, which of the following is an accurate description of the expression $|x|-x$?

（A）must be positive （B）may be positive or zero （C）must be negative
（D）may be negative or zero （E）may be any number

【Suggested Solution】

If $x \geq 0$, then $|x|-x=x-x=0$. If $x<0$, then $|x|-x=-x-x=-2x>0$. Thus the value of the given expression is either positive number or 0.

Answer: B

6. In a promotional sale, a store reduces the prices of all merchandises by 40%. If payment is made using a membership card, then there is a further reduction of 10%. What is the combined reduction in using a membership card?

（A）40 % （B）46 % （C）50 % （D）54 % （E）60 %

【Suggested Solution】

The actual payment amount is $60\% \times 90\% = 54\%$ of the original price, so it is cheaper by $1-54\% = 46\%$.

Answer: B

7. The length of each side of a triangle is a different odd positive integer. What is the minimum perimeter of this triangle?

（A）9 （B）11 （C）13 （D）15 （E）21

【Suggested Solution】

Since the length of two longer sides differ by at least 2 units. Hence, by the triangle inequality theorem, we know the length of the shortest side is least 3 units. Thus, the smallest possible length of three sides of the triangle is 3, 5, 7 units. Therefore, the perimeter is 3 +5 +7 = 15 units.

Answer: D

8. The coordinates of a point in the plane are $(w,\ 1-w)$, where w is a real number. Which of the following is an accurate description of the position in this point?

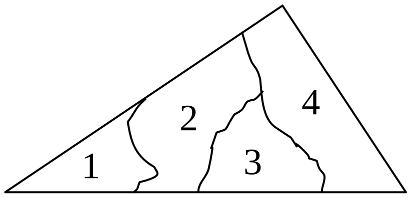
（A）cannot be in the fourth quadrant （B） cannot be in the third quadrant
（C） cannot be in the second quadrant （D） cannot be in the first quadrant
（E） can be anywhere

【Suggested Solution】

The sum of x-coordinate and y-coordinate of point A is 1, so it is not possible that both x- coordinate and y- coordinate are negative, hence, it is certainly not in the third quadrant.

Answer: B

9. Mickey accidentally drops a triangular sheet of glass, breaking it into four pieces as shown in the diagram. He wishes to take only one of the pieces to a repair shop so that he can reproduce a triangular sheet of glass. How many different choices of this piece does he have?



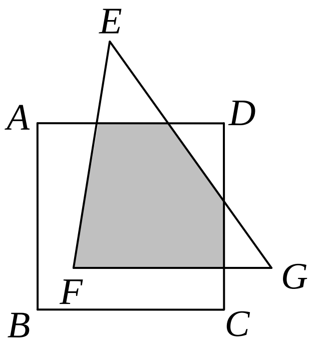
（A） 4　　（B） 3　　（C） 2　　（D） 1　　（E） 0

【Suggested Solution】

Bring the 4th piece of glass to the glass shop, the glass shop can measure out the length of one side of the triangle glass and two of the angles. Apply ASA Theorem, this triangle is uniquely determined. If we bring the other pieces, the information cannot be sufficiently determine the shape and size of the triangular glass, Hence, one possible way only.

Answer: D

10. In the diagram, *ABCD* is a square. The common part of *ABCD* and triangle *EFG* is shaded. Its area is $\frac{4}{5}$ of that of *EFG* and $\frac{1}{2}$ of that of *ABCD*. If the area of triangle *EFG* is 40 cm^2, what is the length of a side of *ABCD*, in cm?



（A） 4　　（B） 5　　（C） 8　　（D） 10　　（E） $\sqrt{2}$

【Suggested Solution】

From the diagram, we observe that the shaded region is an overlap between the square and triangle, its area is $40\times\frac{4}{5}=32\text{ cm}^2$. Since it was given that the area of the shaded region is half that of the square *ABCD*, then the area of square is $32\times 2=64\text{ cm}^2$, this implies the length of each side is 8 cm.

Answer: C

11. What is the simplified value of $\dfrac{3^{2013}-3^{2011}}{3^{2013}+3^{2012}}$?

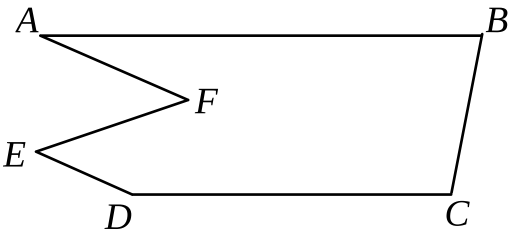
（A） $\frac{2}{3}$　　（B） $\frac{4}{5}$　　（C） $\frac{3}{2}$　　（D） $\frac{1}{2}$　　（E） $\frac{3}{4}$

【Suggested Solution】

Dividing both numerator and denominator by 3^{2011}, the given expression can be simplified as $\dfrac{3^2-1}{3^2+3}=\dfrac{8}{12}=\dfrac{2}{3}$.

Answer: A

12. Linda cuts out a shape as shown in the diagram. AB is parallel to CD and the measure of angle AFE is 40°. What, in degrees, is the total measure of angles BAF, FED and EDC?

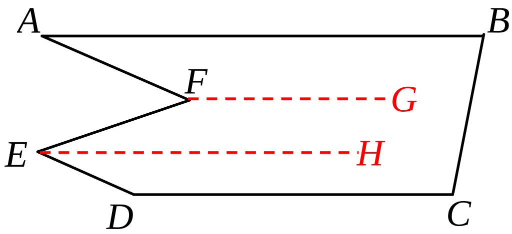


（A） 200 （B） 220 （C） 300

（D） 320 （E） not uniquely determined

【Suggested Solution #1】

Construct two auxiliary line segments $\overline{FG}$ and $\overline{EH}$ parallel to $\overline{AB}$ as in the diagram.



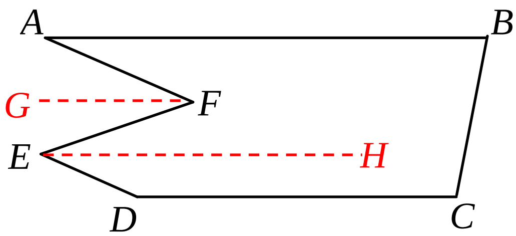
Then $\angle FAB+\angle AFG=180°$, $\angle GFE+\angle FEH=180°$, $\angle HED+\angle EDC=180°$.

Adding the above three equations and subtract to $\angle AFG+\angle GFE=360°-40°=320°$, we have

$$\angle BAF+\angle FED+\angle EDC=220°$$ 。

【Suggested Solution #2】

Construct two auxiliary line segments $\overline{FG}$ and $\overline{EH}$ parallel to $\overline{AB}$ as in the diagram.



We have $\angle FAB=\angle AFG$, $\angle GFE=\angle FEH$,

$\angle HED+\angle EDC=180°$ and $\angle AFG+\angle GFE=\angle AFE=40°$.

Then

$$\begin{aligned}&\angle BAF+\angle FED+\angle EDC\\&=\angle BAF+(\angle FEH+\angle HED)+\angle EDC\\&=\angle AFG+\angle GFE+\angle HED+\angle EDC\\&=\angle AFE+180°\\&=220°\end{aligned}$$

Answer: B

13. May and Cherry bought the same kind of colored pens from a stationery store. Such a pen costs more than \$10. May's total bill has reached \$182 while Cherry's total bill is \$221. What is the total number of pens which May and Cherry bought?

(A) 28 (B) 29 (C) 30 (D) 31 (E) 32

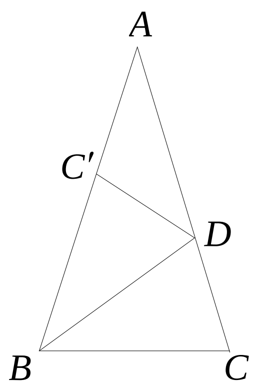
【Suggested Solution 】

Since $182=2\times7\times13$ and $221=13\times17$, the amount both May and Cherry spend in buying coloured pens is the same and is more than \$10, so the price of the coloured must be \$13. Therefore, both bought a total of $2\times7+17=31$ colored pens.

Answer: D

14. The diagram shows the outcome of a folded piece of triangular paper such that the vertex *C* becomes the point C' on the side *AB*. If $AB=AC$ and $C'A=C'D$, what is the measure, in degrees, of angle *A*?

(A) 18 (B) 20 (C) 24 (D) 30 (E) 36



【Suggested Solution #1】

From the given information, the base of isosceles $\triangle ABC$ is *BC*. Hence $\angle DCB<90^\circ$. Since $\triangle BC'D$ is the folded portion of $\triangle$ *BCD*, so the two triangles are congruent, then $\angle BC'D=\angle BCD<90^\circ$. In other words, $\angle AC'D>90^\circ$ and $\triangle AC'D$ is an isosceles triangle with $\angle AC'D$ as the vertex angle. Now, assume $\angle A=\angle ADC'=x^\circ$ and $\angle BC'D$ is an exterior angle of $\triangle AC'D$, then $\angle BC'D=2x^\circ$. But $\triangle ABC$ is an isosceles triangle. Hence $\angle ABC=\angle ACB=\angle BC'D=2x^\circ$. By sum of interior angles of a triangle, we have

$$\angle ABC+\angle ACB+\angle A=180^\circ$$
$$2x^\circ+2x^\circ+x^\circ=180^\circ$$
$$x=36$$

【Suggested Solution #2】

From the given information, *BC* is the base of isosceles $\triangle ABC$, then $\angle DCB<90^\circ$, since $\triangle BDC'$ is the folded portion of $\triangle BCD$, so that the two triangles are congruent, it follows that $\angle BC'D=\angle DCB<90^\circ$, or $\angle AC'D>90^\circ$, thens $\angle AC'D$ is the vertex angle of isosceles $\triangle AC'D$. We may assume $\angle A=\angle ADC'=x^\circ$, that is;

$\angle BC'D=\angle C=\dfrac{180^\circ-x^\circ}{2}=90^\circ-\dfrac{x^\circ}{2}$, but $\angle BC'D$ is the exterior angles of $\triangle AC'D$, then $90^\circ-\dfrac{x^\circ}{2}=x^\circ+x^\circ$, which obtain $x=36$.

Answer: E

15. Mickey is asked to multiply four positive integers, but he adds them instead. Amazingly, his correct answer is equal to the correct answer for the multiplication problem. What is the sum of these four numbers?

(A) 6 (B) 8 (C) 9 (D) 10 (E) 12

【Suggested Solution】

Let a, b, c, d represent the four positive integers where d is the largest. When $a = b = c = d$, then we have $a^4 = a$, that is; $a^3 = 4$, which is not possible! Hence, these four positive integers are not all equal. This implies the sum of these four numbers should be greater than d and less than $4d$, so we have $d < abcd < 4d$. Therefore, $abc = 2$ or $abc = 3$.

When $abc = 2$, then $a + b + c = 1 + 1 + 2 = 4$, it follows $4 + d = 2d$, then $d = 4$ and the product of these four number is 8.

When $abc = 3$, then $a + b + c = 1 + 1 + 3 = 5$, it follows$5 + d = 3d$, then d is not an integer, which did not satisfy the given condition.

Thus, the sum of these four positive integers is 8.

Answer: B

16. Mickey starts working with his report at 7:30 am. By 10:10, he has finished $\frac{2}{3}$ of his report. He takes one-hour break and then continues to work at the same rate. At what time will he finish his report?

(A) 10：50 (B) 11：20 (C) 11：40 (D) 12：30 (E) 12：50

【Suggested Solution】

Mickey finishes $\frac{2}{3}$ of his report; that is, he works for 2 hours and 40 minutes which starts at 7:30 am till 10:10 am. Since he still needs to complete $\frac{1}{3}$ of his report or he must work one-half of the time of 2 hours and 40 minutes which is equivalent of 1 hour and 20 minutes plus an hour of break. Thus, he should complete his report at 12:30.

Answer: D

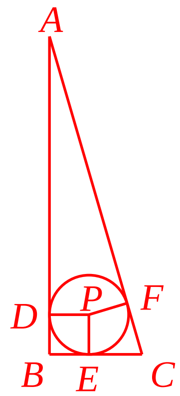
17. P is a point inside a triangle whose side lengths are 7 cm, 24 dm and 25 cm. If P is at the same distance from all three sides of the triangle, what is this distance, in cm?

(A) 1 (B) 1.5 (C) 2 (D) 2.5 (E) 3

【Suggested Solution】

Let us name the triangle as $\triangle ABC$. Since $7^2+24^2=25^2$, so $\triangle ABC$ is a right triangle, then its area is $\frac{1}{2}\times 7\times 24=84$ cm^2. Let the distance from point P to three sides of $\triangle ABC$ represent as x cm, and area of $\triangle ABC$ is the sum of $\triangle APB$, $\triangle BPC$, $\triangle CPA$, so we have

$84=S_{APB}+S_{BPC}+S_{CPA}=(7+24+25)\times\frac{x}{2}$. Therefore, $x=3$.

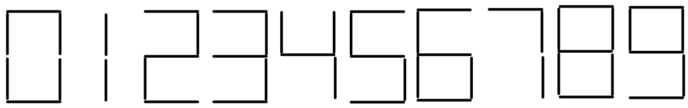


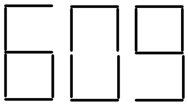
【Suggested Solution #2】

Let the given triangle as $\triangle ABC$. Since point P is equidistant to three sides of $\triangle ABC$, then P must be the center of inscribed circle of $\triangle ABC$, or called it as in-center, that is; the distance from P to three sides of $\triangle ABC$ is the radius. As shown in the diagram, the tangent segments $AD = AF$, $BD = BE$, $CF = CE$ and so we have $PD = PE = DB = BE = \dfrac{AB + BC - AC}{2} = \dfrac{7 + 24 - 25}{2} = 3$ cm。

Answer: E

18. The diagram shows how each of the digits 0 to 9 can be made from matchsticks. In this representation, the number 609 reads the same way upside down as right side up. How many such three-digit numbers can be formed if the first digit may not be 0?





（A）30 （B）36 （C）42 （D）49 （E）245

【Suggested Solution】

For all those matchstick numbers when Upside Down will still be a readable number are 0, 1, 2, 5, 6, 8 and 9. Among them, the digit 0, 1, 2, 5 and 8 still show the same number while digit 6 becomes number 9 and digit 9 becomes number 6. Hence, the possible hundreds' digit are 1, 2, 5, 6, 8 and 9; that is 6 ways, while the tens' digit are 0, 1, 2, 5 and 8; there are 5 ways. Therefore, there is a total of 6×5=30 ways of forming three-digit number.

Answer: A

19. The alien clock divides the earth day into 10 of their hours, each of which is divided into 100 of their minutes. If they plan to attack the earth at 6：36 am our time, what is the time indicated on their clock?

（A）1：75 （B）2：25 （C）2：75

（D）3：15 （E）3：25

【Suggested Solution】

From the given information, we know that the "clock" of an alien is different with our mother earth. Every one hour of the alien is 10% of one day in our earth, every one minute of the alien is 0.1% of one day in our earth. Hence, from 0:00 until 06:36 on the earth, which is equivalent as 6 hours and 36 minutes or 396 minutes of the earth, and we know that one whole day on our earth is equivalent to $60 \times 24 = 1440$ minutes. Hence, it is $\dfrac{396}{1440} = 27.5\%$ of the alien time. Therefore, at the time when aliens are going to attack our mother earth, it must be displayed on their clock as 2:75 .

Answer: C

20. Fanny, Lily and Sherry all shop at regular intervals, Fanny shops once every 3 days, Lily once every 4 days and Sherry once every 5 days. Yesterday, all three went shopping. How many in the next 100 days, starting from today (today is the first day), will at least two of them be shopping together?

(A) 16 (B) 17 (C) 18 (D) 19 (E) 20

【Suggested Solution #1】

From the information, Fanny and Lily go shopping together every $3 \times 4 = 12^{th}$ day, while $100 = 12 \times 8 + 4$, that is; in 100 days, both of them go shopping 8 times together. Similarly, Fanny and Sherry go shopping 6 times together. Lily and Sherry go shopping 5 times together. But three of them go shopping together on every $3 \times 4 \times 5 = 60^{th}$ day, then in 100 days three of them go shopping only 1 time. Thus, in the next 100 days starting today, there are $(8-1)+(6-1)+(5-1)+1=17$ days at least two of them go shopping together.

【Suggested Solution #2】

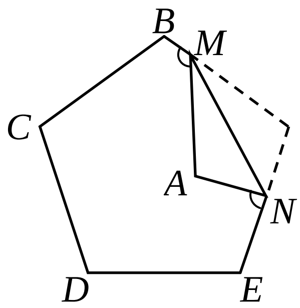
Since the least common multiple of 3, 4 and 5 is 60, hence for every 60 days, three of them will go to shopping together, let us observe the situation before the 60^{th} day as below:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Fanny			◎			◎			◎			◎			◎
Lily				◎				◎				◎			
Sherry					◎					◎					◎
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Fanny			◎			◎			◎			◎			◎
Lily	◎				◎				◎				◎		
Sherry					◎					◎					◎
	31	32	33	34	34	36	37	38	39	40	41	42	43	44	45
Fanny			◎			◎			◎			◎			◎
Lily		◎				◎				◎				◎	
Sherry					◎					◎					◎
	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Fanny			◎			◎			◎			◎			◎
Lily			◎				◎				◎				◎
Sherry					◎					◎					◎

From the above chart, we know that for the first 60 days, at least two of them go shopping together on the following 10 days: they are the 12^{th}, 15^{th}, 20^{th}, 24^{th}, 30^{th}, 36^{th}, 40^{th}, 45^{th}, 48^{th}, 60^{th} day. From the 61^{st} day to the 100^{th} day, it will be a repetition of the first 40 days. Hence, there are 7 days for at least two of them go shopping together and they are: the 72^{nd}, 75^{th}, 80^{th}, 84^{th}, 90^{th}, 96^{th} and 100^{th} day. Therefore, in the first 100 days, there are $10 + 7 = 17$days that at least two of them go shopping together.

Answer: B

21. The diagram shows a regular pentagon *ABCDE* with a point *M* on *AB* and a point *N* on *AE*. The pentagon is folded along the segment *MN* so that the vertex *A* is now inside the original pentagon. What, in degrees, is the total measure of the angles *AMB* and *ANE*?



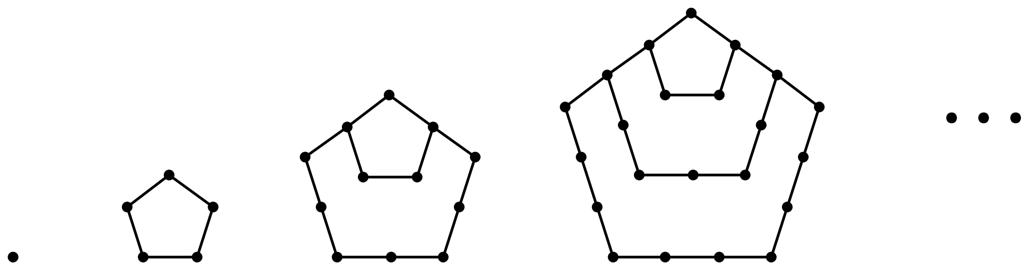
【Suggested Solution】

Each interior angle of a regular pentagon is $108°$, , so we have $\angle AMN + \angle ANM = 72°$. Therefore,

$$\angle AMB + \angle ANE = (180° - 2\angle AMN) + (180° - 2\angle ANM)$$
$$= 360° - 2 \times 72° = 216°$$

Answer: 216

22. Anne arranges some pebbles in the sand forming a pattern of interesting configurations as shown in the diagram. The numbers of pebbles used in the first four configurations are 1, 5, 12 and 22 respectively. What is the number of pebbles used in the tenth configuration of this pattern?



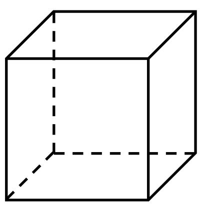
【Suggested Solution】

From the diagram, we observe the number of pebbles in the 2nd, 3rd and 4th configuration is more than 1st, 2nd and 3rd configuration by 4, 7, 10 respectively. In general, the number of pebbles in the *n*th configuration is more than the $(n-1)$th configuration by $3n-2$. Follow the same pattern, there are 13, 16, 19, 22, 25 and 28 pebbles more in the 5th, 6th, 7th, 8th, 9th and 10th configuration than the 4th, 5th, 6th, 7th, 8th, 9th configuration; respectively. Thus, the 10th configuration needs a total of $22+13+16+19+22+25+28=145$ pebbles.

Answer: 145

【Note】The n th configuration need a total of $\dfrac{n(3n-1)}{2}$ pebbles.

23. The six faces of a cubical die are labeled with six different positive integers. If the numbers on any two adjacent faces, differ by at least 2, what is the minimum value of the sum of these six numbers?



【Suggested Solution】

In order the sum of all the numbers in the six faces to be minimum, then 1 must appear in the face with the least number, otherwise the number appears in each face must be reduced by 1, then the sum will also be decreasing. Similarly, in a standard cube, the opposite side of number 1 must be the number 2, or each number on each face (except the face with number 1) must decrease by 1 also, then the sum will become smaller. We know the remaining four faces which are adjacent to number 2, we can predict the minimum sum will be 2 + 2 = 4, if the sum is more than 4, then each number on the four sides must be decreased by 1, so that the total will also be reduced. We can now enter in the opposite face of 4 by the number 5, then the remaining two faces which are adjacent with 5, so that the minimum sum of numbers in those two faces must be 5 +2 = 7, if more than 7, then each number on the two sides must each be decreased by 1, so that the total will also be reduced. Now we can enter in 8 in the opposite face of the number 7, at this time we have the minimum sum of those six faces, where these six numbers 1, 2, 4, 5, 7 and 8. Therefore, the sum of the six faces of the minimum number is 1 +2 +4 +5 +7 +8 = 27.

Answer: 027

24. The non-zero real numbers x and y satisfies

$$\left(\sqrt{x^2+2013}-x\right)\left(\sqrt{y^2+2013}-y\right)=2013.$$

What is the value of the expression $\dfrac{2013x+y}{5x+y}$?

【Suggested Solution】

We notice that

$$2013=\left(\sqrt{y^2+2013}-y\right)\left(\sqrt{y^2+2013}+y\right),$$

Divide both sides by $\sqrt{y^2+2013}-y,$

we have $\sqrt{x^2+2013}-x=\sqrt{y^2+2013}+y,$

that is; $x+y=\sqrt{y^2+2013}-\sqrt{x^2+2013}.$

Similarly, $x+y=\sqrt{x^2+2013}-\sqrt{y^2+2013},$,

then $x+y=0,$ it follow that $y=-x.$

Therefore, $\dfrac{2013x+y}{5x+y}=\dfrac{2013x-y}{5x-y}=\dfrac{2012x}{4x}=503$ 。

Answer: 503

25. Determine the least positive integer which has five three-digit divisors?

【Suggested Solution】

Let X represent the three-digit number. Assume that the largest five divisors of X are X, $\frac{X}{2}$, $\frac{X}{3}$, $\frac{X}{4}$, $\frac{X}{5}$, and each of them is a three-digit number. Then X is divisible by 60 and not less than 500, so the possible smallest value of X is 540. If one of the divisors in the first six largest divisors of X is less than or equal to $\frac{X}{6}$ and it is still a three-digit number, this implies that $\frac{X}{6}\ge 100,$ it follow that $X\ge 600.$

Thus, we conclude the least positive integer which has five-digit divisors is 540.

Answer: 540