Solution to First Round of 2013 IMAS Middle Primary Division

1. In an art class, the teacher presented shaped patterns. How many circular figures are there?

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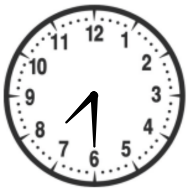
（A）1 （B）2 （C）3 （D）4 （E）5

【Suggested Solution】

There are three (3) circles shown in the above figures. Hence, we must select C.

Answer: C

2. The clock in the diagram shows the time 7:30. The hour hand is between the numbers 7 and 8 and the minute hand points at number 6. What number will the minute hand be pointing 40 minutes later?



（A）2 （B）4 （C）6 （D）8 （E）10

【Suggested Solution #1】

After 40 minutes, the time indicated in the clock is 8:10, the minute hand must be pointing at 2. Hence, we select A.

【Suggested Solution #2】

The minute hand will move from one digit to the next digit every 5 minutes and 40 minutes later it will move 8 digits, that is $6+8-12=2$, the minute hand will pointing at digit 2, so we select option A.

Answer: A

3. Which of the following is the closest length of time for one day?

（A）half day （B）2 days （C）23 hours
（D）26 hours （E）1410 minutes

【Suggested Solution #1】

Let us first convert all the time into minutes' measurement. Since 1 day = 1440 minutes, half day = 720 minutes, two days = 2880 minutes, 23 hours = 1380 minutes, 26hours = 560 minutes. Then it follows

$$1440-720=720$$

$$2880-1440=1440$$

$$1440-1380=60$$

$$1560-1440=120$$

$$1440-1410=30,$$

Hence, 1410 minutes is closest to one day. Therefore, we choose E.

【Suggested Solution #2】

Since 1 day = 24 hours= 1440 minutes and half day must be one-half of 1440 minutes, so there is a difference of 720 minutes in half day while two days is more than a whole day by 1440 minutes. 23 hours is one hour less than a whole day, there is a difference of 60 minutes. The difference of 26 hours and a whole day is 2 hours, which means they differ by 120 minutes. Lastly, the difference between 1410 minutes and a whole day is 30 minutes only. Hence, we conclude that 1410 minutes is nearer to one whole day. So, we select E.

Answer: E

4. In the amusement park, a roller-coaster ride requires 5 tokens. Each token costs 5 dollars. How many dollars does Mickey have to spend for one ride?

(A) 5 (B) 10 (C) 15 (D) 20 (E) 25

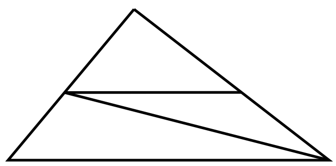
【Suggested Solution】

To be able ride a roller-coaster, he requires 5 tokens and each token costs $ 5.
Hence, Mickey must spend $\$5 \times 5 = \$25$. So, we select E.

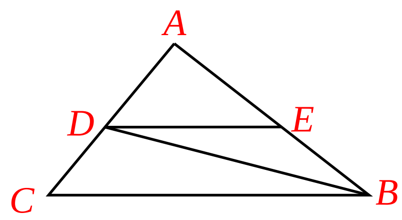
Answer: E

5. How many different triangles can be found in the diagram?

(A) 1 (B) 3 (C) 4
(D) 5 (E) 6



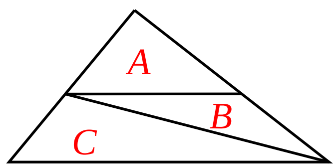
【Suggested Solution #1】



Based on the given diagram, we can name five different triangles as $\triangle ADE$, $\triangle DEB$, $\triangle DBC$, $\triangle ADB$, $\triangle ABC$. Therefore, we choose D for our answer.

【Suggested Solution #2】

From the given diagram, we easily recognize each of the regions *A*, *B* and *C* as triangles of different sizes. When we combine regions *A* and *B*, they produce another triangle of different size. When regions *A*, *B* and *C* are combined, the biggest triangle is produced. Therefore, there are 5 different triangles in the diagram.



Answer: D

6. Walter has two options in going to school. He can (a) walk 8 minutes to the bus stop, and rides the bus for 15 minutes to the school, or (b) walks 10 minutes to the LRT station, rides the train for 10 minutes to school. If he does not have to wait for the bus at the bus stop, nor the train on the subway station, what is the minimum number of minutes required for him to get to school?

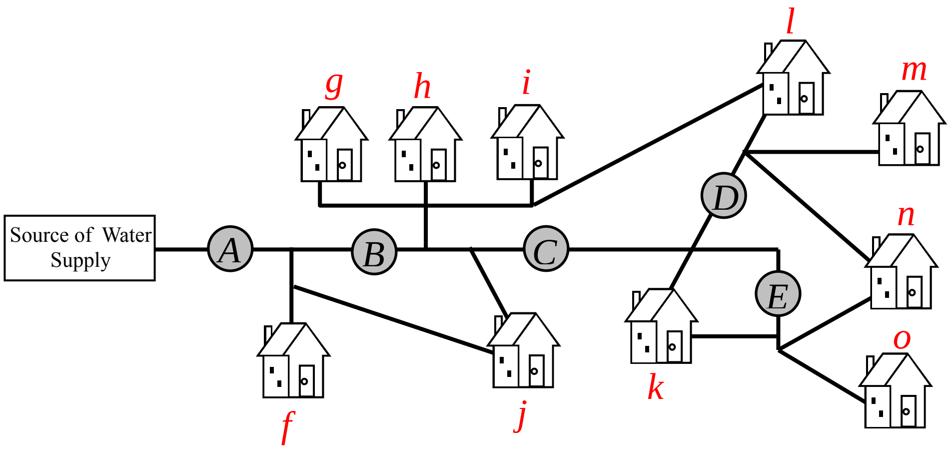
(A) 18 (B) 20 (C) 23 (D) 25 (E) 33

【Suggested Solution 】

When Walter selects option (a) for going to school from his house, he needs to travel 8 + 15 =23 minutes; when he selects option (b), then he needs to travel 10 + 10 =20 minutes. Therefore, Walter needs at least 20minutes for him to get to school from his house.

Answer: B

7. The diagram shows the water distribution system in the neighbourhood. There are five valves indicated by capital letters. When water flows into a house, it will not flow out to another houses. Which valve must be closed in order to shut off the water to exactly four houses?



(A) A (B) B (C) C (D) D (E) E

【Suggested Solution】

Let us name the 10 houses as *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n* and *o* and analyze the switch (on/off) positions of the valve to determine our target :

When only valve *A* is shut off, then all the users will have no water, that is; a total of 10 houses will be waterless.

When only valve *B* is shut off, since the water of house *j* can be supplied by valve *A*, so there are two houses *f* and *j* will still have water, that is; 8 houses will have no water.

When only valve *C* is shut off, since the water of house *l* can be supplied by valve *B*, so there are four houses *k*, *m*, *n* and *o* will have no water.

When only valve *D* is shut off, since the water of house *l* can be supplied by valve *B* while *n* can be supplied by valve *E*, so only house *m* is affected with water interruption. When only valve *E* is shut off, since the water of house *n* can be supplied by valve *E* while *k* can be supplied by valve *C*, so there will be only one house *o* be affected with no water.

Thus, we choose C.

Answer: C

8. A giraffe invites 28 small animals to a Plain Peach Party. In a group photo, the giraffe is in the middle. Counting from the left, which position does the giraffe occupy?

(A) 1 2 (B) 13 (C) 14 (D) 15 (E) 16

【Suggested Solution】

If the giraffe's position is at the middle, then animals at the left and right sides are equal in number. Hence, there are $28 \div 2 = 14$ animals on either side; therefore the giraffe is on the 15^{th} position.

Answer: D

9. A kangaroo jumps 6 meters forward, 4 meters backward, 7 meters forward, 8 meters backward, and then it rests. How many meters apart are the current position and the initial position of the kangaroo?

(A) 1 (B) 3 (C) 4 (D) 6 (E) 8

【Suggested Solution #1】

Since the kangaroo jumps 6 m forward, then the kangaroo is 6 m from the initial position, then he turns back and jumps 4 m, so he is now $6-4=2$ m from the initial position, and this time he turns forward and jumps 7 m again, which is $2 + 7 = 9$ m from the initial position, and lastly turns back to jump 8 m, which is $9-8=1$ m from the initial position.

【Suggested Solution #2】

From the given information, we know the kangaroo jumps to a forward direction in a total distance of $6 + 7 = 13$ m while he jumps to a backward direction in a total distance of $4 + 8 = 12$ m. Therefore, the distance that the kangaroo jumps from the initial position to the current position is $13-12=1$ m.

Answer: A

10. A round table can seat 5 guests and a square table can seat 4 guests. Which of the following combinations of tables can seat 36 guests without leaving any empty seat?

(A) 1 round table and 2 square tables (B) 2 round tables and 4 square tables

(C) 3 round tables and 5 square tables (D) 4 round tables and 4 square tables

(E) 5 round tables and 3 square tables

【Suggested Solution】

If there is 1 round table and 2 square tables, then only $1\times5+2\times4=13$ guests can be seated. The number of tables available is insufficient for the guests. Hence, it can't be the solution!

If there are 2 round tables and 4 square tables, then only $2\times5+4\times4=26$ guests can be seated. The number of tables available is insufficient for the guests. Hence, it can't be the solution!

If there are 3 round tables and 5 square tables, it can accommodate $3\times5+6\times4=39$ guests. There are several empty seats. Hence, it can't be the solution!

If there are 4 round tables and 4 square tables, it can accommodate $4\times5+4\times4=36$ guests, number of seats is exactly the same as the number guests. This kind of arrangement can be the solution.

If there are 5 round tables and 3 square tables, it can accommodate $6\times5+2\times4=38$ guests, there are several empty seats. Hence, it can't be the solution!

Answer: D

11. In a supermarket, apples are sold at 150 dollars for 6 pieces, and pears are sold at 30 dollars for 2 pieces. How much more expensive is the cost of each apple than a cost of each pear?

(A) 1 (B) 3 (C) 5 (D) 10 (E) 12

【Suggested Solution】

We can easily determine that each apple costs 150 ÷ 6 = \$25, each pear cost 30 ÷ 2 = \$15, then the price of an apple is expensive than each pear by $25-15=\$10$.

Answer: D

12. Altogether, there are 240 books owned by 4 children. If Ace gives Bea 3 books, Bea gives Cec 4 books, Cec gives Dee 5 books and Dee gives Ace 6 books. Then each has the same number of books. Initially, how many books belong to the child with the least number of books?

(A) 57 (B) 58 (C) 59 (D) 60 (E) 61

【Suggested Solution】

From the given information, we know that 4 children own 240 books, so when those books are equally divided, each of them are supposed to have 240 ÷ 4 = 60 books. But from the problem, we know that Ace has $6-3=3$ more books, Bea has $4-3=1$ book less, Cec has $5-4=1$ book less and Dee has $6-5=1$ book less. Thus, we know that originally Ace must have 57 books while each of Bea, Cec and Dee has 61 books.

Answer: A

13. Zachary has a computer program which accepts an input and produces an output. Some of the data are shown in the table below.

Input	1	2	3	4	5	6	7
Output	4	7	10	13	16	?	22

What is the output when the input is 6?

(A) 17 (B) 18 (C) 19 (D) 20 (E) 21

【Suggested Solution #1】

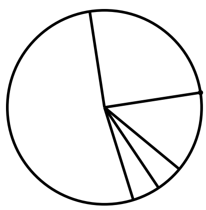
From the given information, it shows that the output data is 1 more than three times than the input data. Thus, when we enter, the output becomes 3 × 6 +1 = 19.

【Suggested Solution #2】

From the given information, we discover that the output data is an arithmetic sequence composed of 4, 7, 10, 13, 16, ..., with a common difference of 3. Hence, when the input data is 6, the output data is 16 +3 = 19.

Answer: C

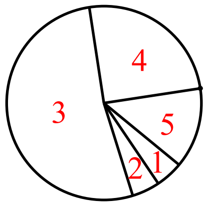
14. Hanna divides a circular piece of paper into 5 regions as shown in the diagram. She wants to paint each region using a color so that two regions sharing a common side with different colors. What is the smallest number of colors she needs?



(A) 1 (B) 2 (C) 3

(D) 4 (E) 5

【Suggested Solution #1】



Let us assign the five different regions as regions 1 to 5 as in the diagram. If region 1 is painted with color *A*, since we are required to use the least number of colors, so we may paint color *B* in region 2 and 5, now region 4 must be painted with a color different from region 5, that is; region 4 may use color *A* which is the same as region 1 and finally region 3 will be painted with a color different from that of region 2 (color *B*) and region 4 (color *A*). So, we know region 3 must be painted with a third color. Thus, Hanna must use at least 3 colors to paint the circular piece of paper.

【Suggested Solution # 2】

If there are only two color pens available, then by Pigeonhole Principle we know that at least one color must be painted in three regions, but the diagram proves it not possible because every three regions there are at least two regions which are adjacent to one another, so there must be at least 3 color pens in order to paint the regions as shown in the diagram at the right.

Answer: C

15. Three rabbits are digging for radishes in a field. The White Rabbit and the Spotted Rabbit dig up 13 radishes between them. The Spotted Rabbit and the Black Rabbit dig up 11 radishes between them. The Black Rabbit and the White Rabbit dig up 16 radishes between them. What is the total number of radishes dug up by the three rabbits?

(A) 10 (B) 11 (C) 15 (D) 16 (E) 20

【Suggested Solution】

Since $13 + 11 + 16 = 40$, is exactly two times the number of radishes pulled out by the three rabbits, so the three rabbits pulled out a total of $40 \div 2 = 20$ radishes.

Answer: E

16. Three travellers are crossing a desert together. When Mickey has finished his water supply, Don still has 5 bottles of mineral water and Jan has 4 bottles. They share the water equally among them. Mickey pays the others 36 dollars for the water he has received. How many dollars should go to Don?

(A) 8 (B) 12 (C) 16 (D) 20 (E) 24

【Suggested Solution】

From the information, each person shares $(5+4)\div 3=3$ bottles of mineral water, so that Mickey must pay $36\div 3=\$12$ for each bottle of mineral water, that is; Don gives Mickey $5-3=2$ bottles of mineral water while Jan gives $4-3=1$ bottle of mineral water to Mickey. Therefore, Mickey should pay $2\times 12=\$24$ to Don as the cost of mineral water.

Answer: E

17. From a box of chocolate, Mickey takes out half the number of pieces and put one piece back. Then he takes out half of the remaining number of pieces of chocolates and puts one piece back. After he does this for a total of 5 times, there are only three pieces of chocolates are left in the box. How many pieces of chocolates are in the box initially?

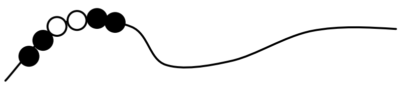
(A) 158 (B) 78 (C) 38 (D) 34 (E) 18

【Suggested Solution】

Working backward, we know that there are $(3-1)\times 2=4$ pieces of chocolates in the box before the 5th operation, it follows that there are $(4-1)\times 2=6$ pieces of chocolates in the box before the 4th operation , $(6-1)\times 2=10$ pieces of chocolates in the box before the 3rd operation, there are $(10-1)\times 2=18$ pieces of chocolates in the box before the 2nd operation and there are $(18-1)\times 2=34$ pieces of chocolates in the box before the 1st operation starts. Hence, there are 34 pieces of chocolate in the box originally.

Answer: D

18. A necklace has 27 beads. When part of the necklace becomes visible, it appears that the first two beads are black, the next two are white, the next two are black, the next two are white, as shown in the diagram. If this pattern continues, what is the total number of black beads in the necklace?



(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

【Suggested Solution】

Let us consider two black beads and two white beads as a group, then 27 beads have six groups and there remain 3 beads, and by rules of the pattern in the string of beads, there must be 2 black beads in the remaining 3 beads. Since in each group, there are 2 black beads, so a total of $2\times 6+2=14$ black beads must be in the necklace.

Answer: B

19. The digits 1 to 9 are placed inside the squares in the diagram, with a different digit in each of the boxes. Only the digit 2 is shown. If the equations are correct, what is the two-digit number formed by the digits in the first two boxes from the left?

$$\square\,\square \div \square\,\square = \square\,\square - \square\,\square = \boxed{2}$$

(A) 98 (B) 86 (C) 78 (D) 76 (E) 68

【Suggested Solution #1】

Let us assume the above mathematics expression as $\overline{AB}\div\overline{CD}=\overline{EF}-\overline{GH}=2$.

Then from the expression $\overline{AB}\div\overline{CD}=2$, let us consider the following cases:

When $\overline{AB}=98$, then $\overline{CD}=49$, the digit 9 repeated twice. Hence option E cannot be the answer.

When $\overline{AB}=86$, we have $\overline{CD}=43$, at this instance the possible value of E, F, G and H are 1, 5, 7, and 9. Since it was given $\overline{EF}-\overline{GH}=2$, then $F=7, H=5$ or $F=9, H=7$ or $F=1, H=9$. No matter what values will be assign to E and G, no value will satisfy the mathematics sentence $\overline{EF}-\overline{GH}=2$. Hence option B is not the answer.

When $\overline{AB}=78$, it follow $\overline{CD}=39$, so the values of E, F, G and H may be the digits 1, 4, 5 and 6. But $\overline{EF}-\overline{GH}=2$, this implies $F=6, H=4$ and we know that there are no possible values can be assign to E, G to meet the condition $\overline{EF}-\overline{GH}=2$. Hence option C cannot be the answer.

When $\overline{AB} = 76$, then $\overline{CD}$=38, it follow the values of E, F, G and H can assign as 1, 4, 5 and 9. Since $\overline{EF} - \overline{GH} = 2$, so we have *F* = 1, *H* = 9, so that *E* = 5, *G* = 4 in order $\overline{EF} - \overline{GH} = 2$; that is; $76 \div 38 = 51 - 49 = 2$. This met the condition of the problem.

When $\overline{AB} = 68$, it follow $\overline{CD}$=34, so the value of digits *E*, *F*, *G* and *H* could be 5, 7, 8 and 9. But $\overline{EF} - \overline{GH} = 2$, so we have *F* = 7, *H* = 5 or *F* = 9, *H* = 7. Then for any value of E and G, we can't establish $\overline{EF} - \overline{GH} = 2$. Hence option E is not the answer.

【Suggested Solution #2】

We may assume the given mathematical expression as $\overline{AB} \div \overline{CD} = \overline{EF} - \overline{GH} = 2$.

Since $\overline{AB} \div \overline{CD} = 2$, we can rewrite it as $\overline{AB} = 2 \times \overline{CD}$ while $\overline{EF} - \overline{GH} = 2$ where *E*, *G* must be distinct digits and there is a regrouping in the subtraction, so we predict *F* = 1 and *H* = 9:

(i) If $\overline{EF} = 81$, $\overline{GH} = 79$, at this instance the values of *A*, *B*, *C* and *D* are 3, 4, 5 and 6. But $\overline{AB} = 2 \times \overline{CD}$, it follow that *B* = 6, *D* = 3 and for whatever the values of *A* and *C*, it cannot satisfy $\overline{AB} = 2 \times \overline{CD}$. Not Possible!

(ii) If $\overline{EF} = 71$, $\overline{GH} = 69$, the values of *A*, *B*, *C* and *D* can be assign by the digits 3, 4, 5 and 8. Since $\overline{AB} = 2 \times \overline{CD}$, it follow that *B* = 8, *D* = 4 and whatever the values of A and C, it cannot satisfy $\overline{AB} = 2 \times \overline{CD}$. Not Possible!

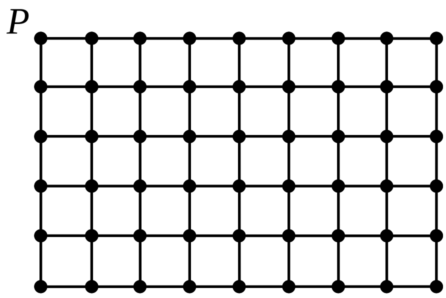
(iii)If $\overline{EF} = 61$, $\overline{GH} = 59$, then the possible values of *A*, *B*, *C* and *D* are 3, 5, 7 and 8. But $\overline{AB} = 2 \times \overline{CD}$, we conclude *B* = 8, *D* = 4 or *B* = 4, *D* = 7; and whatever the values of A and C, it cannot satisfy $\overline{AB} = 2 \times \overline{CD}$. Not Possible!

(iv) If $\overline{EF} = 51$, $\overline{GH} = 49$, if follow that the possible values of *A*, *B*, *C* and *D* are 3, 6, 7 and 8; so that for $\overline{AB} = 2 \times \overline{CD}$ we have *B* = 6, *D* = 3 or *B* = 6, *D* = 8. When *B* = 6, *D* = 3, for whatever values of A and C to be assign, there is no possible way to establish $\overline{AB} = 2 \times \overline{CD}$. Not Possible!
When *B* = 6, *D* = 8, we know that *A* = 7, *C* = 3 so that $\overline{AB} = 2 \times \overline{CD}$, we have the correct mathematical sentence; that is; $76 \div 38 = 51 - 49 = 2$.

(v) If $\overline{EF} = 41$, $\overline{GH} = 39$, the only possible values of *A*, *B*, *C* and *D* are 5, 6, 7 and 8. But $\overline{AB} = 2 \times \overline{CD}$, it follow that *B* = 6, *D* = 8, and whatever the values of A and C, it cannot satisfy $\overline{AB} = 2 \times \overline{CD}$. Not Possible!

Answer: D

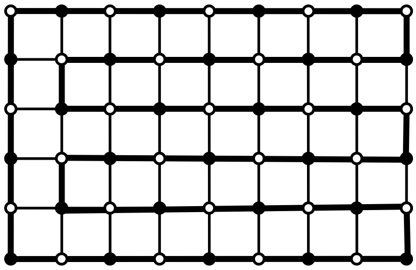
20. There are 54 grid points on a 5 by 8 grid as shown in the diagram where the side of each small square is 1 cm. Starting from point *P*, an ant crawls from point to point along the grid lines, visiting each grid point exactly once before returning to *P*. What is the maximum length of its path, in cm?



(A) 26 (B) 30 (C) 36 (D) 54 (E) 93

【Suggested Solution】

An ant starts from point P crawling along the path as shown by passing thru all the points and finally back to point A. Because the ant must pass thru each grid point once and only once, we also know the distance between two adjacent points is 1 cm, and the path has a total 54 points. So the ant must crawl through with a total distance of 54 cm, which is also the longest path.



Answer: D

21. Some cards are missing from a deck of 52 cards. If the incomplete deck is dealt to four players so that each receives the same number of cards, then 3 cards are left. If it is dealt to three players instead, with each still receiving the same number of cards, then 1 card is left. What is the maximum number of cards are there in the incomplete deck?

【Suggested Solution #1】

When the four players together play the cards, each of them gets the same number of cards, and we discover that 3 cards are left, hence the possible number of cards is 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51.

When three individuals play together, each of them gets the same number of cards. Again, 3 cards are left; hence the possible number of cards is 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49.

From above two cases, we can infer the possible the number of cards as 7, 19, 31 or 43. Thus, the maximum number of cards is 43.

【Suggested Solution #2】

Assume there are n cards in the poker, where n < 52, since we know that $n-3$ is divisible by 4 and $n-1$ is divisible by 3. So the following cases are established:

When $n-3=48$, then $n-1=50$, which is not divisible by 3!

When $n-3=44$, then $n-1=46$, which is not divisible by 3 also!

When $n-3=40$, then $n-1=42$, divisible by 3

Thus, the largest possible number of cards must be $n = 43$.

【Suggested Solution #3】

Assume there are n cards in the poker, where n < 52, then n must be a multiple of 3 when it is increased by 3, likewise n is also divisible by 3 when it is increased by 1.

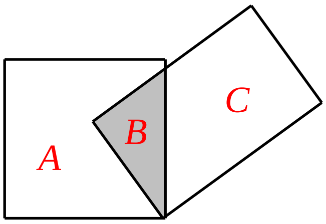
From the statement that n must be a multiple of 3 when it is increased by 3, we can also conclude that n is a multiple of 12 when increased by 3, 7 or 11.

From the statement that n is also divisible by 3 when it is increased by 1, we also conclude that n is a multiple of 12when increased by 1, 4, 7 or 10.

Hence, we know that n must be a multiple of 12when increased by 7, that is the possible value of n are 7, 12+7=19, 24+7=31 or 36+7=43. ∘ Therefore, the maximum number is $n = 43$.

Answer: 043

22. The diagram shows a 4 cm by 4 cm piece of paper overlapping a 3 cm by 5 cm piece of paper. By how many cm^2 does the area of the non-overlapped part of the square piece of paper exceeds the area of the non-overlapped part of the rectangular piece of paper?



【Suggested Solution】

The given figure shows that the square is composed of regions *A* and *B*, while the rectangle is composed of regions *B* and *C*. In order to find the difference of the two non-overlapping regions is the same as determining the difference of area of square and area of a rectangle. Hence, the area of the difference of the two non-overlapping regions is $4 \times 4 - 5 \times 3 = 1$ cm^2.

Answer: 001

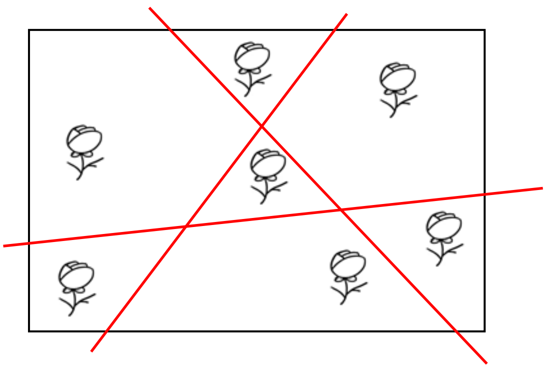
23. For the class photo of 42 students, the photo shop charges 10 dollars for the first copy and 3 dollars for each additional copy. Moreover, a bonus of 2 copies is given for an order exceeding 30 copies. If each student gets one copy, how much must they pay the photo shop altogether?

【Suggested Solution】

From the given information, we know the printing of first copy of photo costs \$10 and 2 copies are given free if printing is more than 30 copies. Hence, the class shall ask the photo shop to develop one copy plus $42-1-2 = 39$ copies, so the total payment is $10+39\times3$ or \$127.

Answer: 127

24. The diagram shows 7 flowers printed on a piece of paper. What is the smallest number of straight lines we must draw to divide the piece of paper into several regions, so that each flower is in a different region?

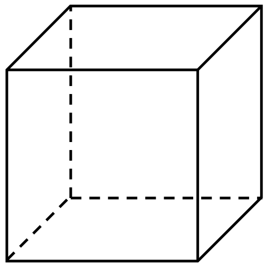


【Suggested Solution】

We may think that 2 straight lines will divide the piece of paper into the four regions, but each flower is not necessary on a separate region! By drawing 3 straight lines as in the diagram will split each flower in a separate regions.

Answer: 003

25. The six faces of a cubical die are labeled with six different positive integers. If the numbers on any two adjacent faces differ by at least 2, what is the minimum value of the sum of these six numbers?



【Suggested Solution】

In order the sum of all the numbers in the six surfaces to be a minimum, then each of the six sides must be 1, otherwise the number appear in each face must reduced by 1, then the sum will also decreasing. Similarly, in a standard cube, the opposite side of a number 1 must be the number 2, or each number on each face (except the face with number 1) must decrease by 1 also, then the sum will become smaller. We know the remaining four faces which are adjacent to number 2, we can predict the minimum sum will be $2 + 2 = 4$, if the sum is more than 4, then each number on the four sides must each decreased by 1, so that the total will also reduced. We can now entered in the opposite face of 4 by the number 5, then the remaining two faces which are adjacent with 5, so that the minimum sum of numbers in those two faces must be $5 + 2 = 7$, if more than 7, then each number on the two sides must each decreased by 1, so that the total will also reduced. Now we can entered in 8 in the opposite face of the number 7, at this time we have the minimum sum of those six faces, where these six numbers 1, 2, 4, 5, 7 and 8. Therefore, the sum of the six faces of the minimum number is $1 + 2 + 4 + 5 + 7 + 8 = 27$.

Answer: 027