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**Solution to
Fourth International Mathematics Assessment for Schools
Round 1 of Middle Division**

1. What is the value of $2 + 0 + 1 + 4 + 2 \times 0 \times 1 \times 4$?
(A) 0 (B) 5 (C) 7 (D) 9 (E) 15

【Suggested Solution】

$2 + 0 + 1 + 4 + 2 \times 0 \times 1 \times 4 = 7$. Hence, we select (C).

Answer: (C)

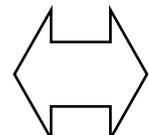
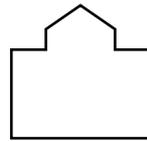
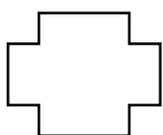
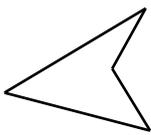
2. Which of the following numbers is the smallest?
(A) 298 (B) 312 (C) 231 (D) 357 (E) 101

【Suggested Solution】

Since $101 < 231 < 298 < 312 < 357$, then the smallest number is 101. So, we select (E).

Answer: (E)

3. Which of the following polygons has the greatest number of sides?
(A) (B) (C) (D) (E)



【Suggested Solution】

The polygon in option (A) has 4 sides, the polygon in option (B) has 12 sides, the polygon in option (C) has 9 sides, the polygon in option (D) has 10 sides while the polygon in option (E) has 10 sides. Hence, since the polygon in option (B) has the greatest number of sides, so we select (B).

Answer: (B)

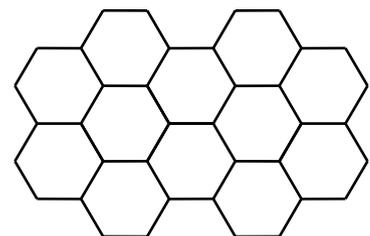
4. The word “2014IMAS” appears on the screen. After each minute, the leftmost character moves over to become the rightmost character. How many minutes will elapse before the word “2014IMAS” appears on the screen once again?
(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

【Suggested Solution】

There are eight characters in this word. It takes 8 minutes for the word “2014IMAS” to reappear. So, we select (D).

Answer: (D)

5. The side length of each hexagon in the diagram is 1 cm. What is the perimeter, in cm, of the figure formed from these hexagons?
(A) 18 (B) 20 (C) 22
(D) 24 (E) 26



【Suggested Solution】

The given diagram has 26 line segment of 1 cm long, so its perimeter is 26 cm. Hence, we select (E).

Answer: (E)

【Note】 By symmetry property of a diagram, We just need to count the number of sides of the upper half and then be multiplied by 2.

6. Thirty students numbered from 1 to 30 stand in a row. The teacher announces, “Will those numbered from 1 to 10 inclusive take one step forward, and those numbers 20 to 30 inclusive take one step backward.” How many students remain in place?
(A) 9 (B) 10 (C) 11 (D) 20 (E) 21

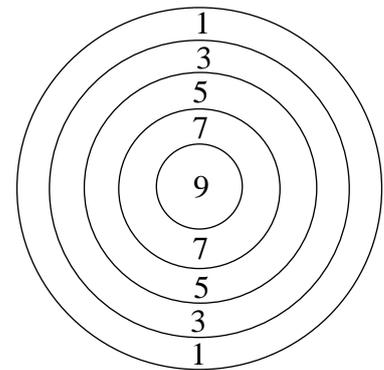
【Suggested Solution】

We know that students with numbered 11 to 19 will not move one step forward or take one step backward, so there are a total of $19 - 11 + 1 = 9$ students, not moving, remain in their original position. We select (A).

Answer: (A)

7. Max throws four darts at the target shown in the diagram. All four darts hit the target, each scoring a different number of points. What is the minimum number of points Max has scored?

- (A) 4 (B) 10 (C) 16
(D) 20 (E) 24



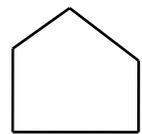
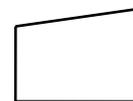
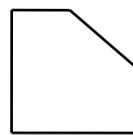
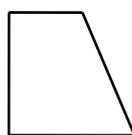
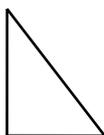
【Suggested Solution】

From the given information, there is one target that Max did not hit, so the minimum number of points Max has scored is $1 + 3 + 5 + 7 = 16$ points. Hence, the answer is (C).

Answer: (C)

8. Which of the polygonal board below cannot be obtained from a rectangular board after one straight cut?

- (A) (B) (C) (D) (E)



【Suggested Solution】

If a saw was able to cut a rectangular board thru the two non-consecutive vertices, then one piece that was cut will be in the shape same as option (A). If a saw was able to cut a rectangular board thru anyone of its vertices, then the shape in one piece may either be same as that figure of option (A), option (B) or option (D).

If a saw will not cut a rectangular board thru anyone of its vertices, then one piece that was cut will either be in the shape the same as that of option (A), option (B), option (C) or option (D).

Hence, only the shape same as option (E) will never occur. So we select option (E).

Answer: (E)

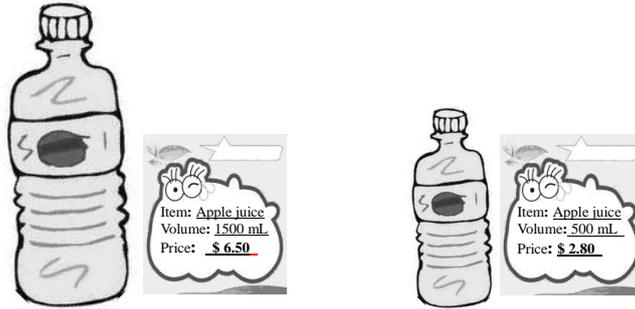
9. Benches are provided for children watching a movie. Each bench can seat 3 children. What is the minimum number of benches required to seat 25 children?
 (A) 7 (B) 8 (C) 9 (D) 10 (E) 11

【Suggested Solution】

We know that $25 = 8 \times 3 + 1$, hence if there are 8 available benches, then at most 24 children will sit and 1 child cannot sit on any bench. Thus, we need at least 9 benches. So, we select option (C).

Answer: (C)

10. A large bottle of apple juice costs 6.5 dollars while a small bottle of apple juice costs 2.8 dollars. How many dollars less is the cost of a large bottle compared to the total cost of three small bottles?



- (A) 1.9 (B) 2.1 (C) 2.3 (D) 2.8 (E) 3.7

【Suggested Solution】

The cost of 3 small bottles of apple juice is $\$2.8 \times 3 = \8.4 . Hence buying 1 large bottle of apple juice will be less than buying 3 small bottle of apple juices by $\$8.4 - \$6.5 = \$1.9$. Therefore, we select option (A).

Answer: (A)

11. A shell may be traded in for two baskets of fruit or three baskets of vegetable. Which of the following may not be obtained by trading in at most two shells?



- (A) two baskets of fruit and three baskets of vegetable
 (B) six baskets of vegetable
 (C) two baskets of fruit
 (D) three baskets of fruit and two baskets of vegetable
 (E) three baskets of vegetable

【Suggested Solution】

We can use 2 shells to trade in the foods of option (A) or option (B), we can also use 1 shell to trade in the foods of option (C) or option (E), because it is not possible to trade in 3 baskets of fruits, hence option (D) is unobtainable.

Answer: (D)

12. Lana is a student in Grade 4. Which of the following is the closest approximation to her age?

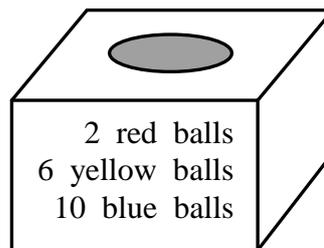
- (A) 120 hours (B) 120 days (C) 120 weeks
(D) 120 months (E) 120 years

【Suggested Solution】

Since $120 \text{ hours} < 120 \text{ days} < 120 \text{ weeks} < 3 \text{ years}$, the time in the first three expression each less than 3 years, which is not practical to be the age of a grade 4 student. Option (E) is obviously cannot be the age for a grade 4 pupil. But $120 \text{ months} = 10 \text{ years}$, which is more realistic. So, we select option (D).

Answer: (D)

13. There are 2 red balls, 6 yellow balls and 10 blue balls in a box. One ball is drawn at random. Which of the following statement is correct?



- (A) The probability of drawing a blue ball is the lowest.
(B) It is equally likely to draw a ball of any colour.
(C) The probability of drawing a yellow ball is the highest.
(D) It is less likely to draw a yellow ball than a blue ball.
(E) The probability of drawing a yellow ball is the lowest.

【Suggested Solution】

Since the number of red balls inside the box is the least, then the probability of drawing red ball is the smallest. We also note that the number of blue balls inside is more than the number of red or number yellow balls, so the probability of drawing blue ball is higher than any other ball. Because the number of yellow balls inside the box is less than the number of blue balls, then the probability of drawing yellow ball is lower than blue ball. Hence, we select option (D).

Answer: (D)

14. Whenever the hour hand and the minute hand of a clock coincide, the number of germs in a dish increases by 10. Between 1:30 pm and 6:30 pm on the same day, by how many has the number of germs increased?

- (A) 20 (B) 30 (C) 40 (D) 50 (E) 60

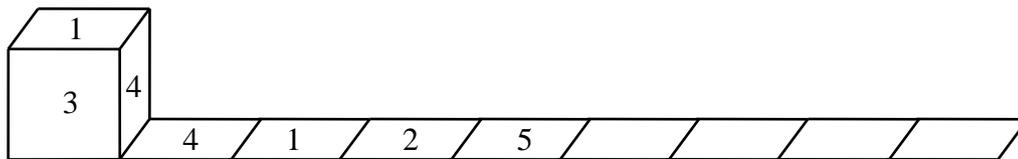


【Suggested Solution】

We know that the minute hand and the hour hand coincide once in every hour from 2 pm to 6 pm. The next time the two hands will be coincide must be after 6:30 pm. Thus, from 1:30 pm to 6:30 pm, the two hands coincide 4 times. Hence the number of germs increase by 40. So, we select option (C).

Answer: (C)

15. The diagram shows a cubical die moving on a 1 by 8 board by tilting over an edge. The number on the face touching the board is imprinted on that square of the board. The numbers in the first four squares are 4, 1, 2 and 5. What is the total of all eight numbers?



- (A) 21 (B) 22 (C) 23 (D) 24 (E) 25

【Suggested Solution】

When rolling the cubical die on the tilted position, the following four numbers have been seen keep repeating: 4, 1, 2 and 5, then total of all eight numbers is $4 + 1 + 2 + 5 + 4 + 1 + 2 + 5 = 24$. Hence, we select option (D).

Answer: (D)

16. The two stars in the diagram represent the same number. The sum of the three numbers in the second row is equal to twice the sum of the three numbers in the first row. What number does each star represent?

| | | | | |
|---|---|---|----|----|
| 5 | 6 | ☆ | | |
| | | ☆ | 19 | 20 |

- (A) 7 (B) 8 (C) 13 (D) 17 (E) 18

【Suggested Solution】

From the given information, it shows the sum of the three numbers in the second row is equal to twice the sum of the three numbers in the first row, it follows that the difference of the sum of three numbers in the second row and the sum of three numbers in the first row equal the sum of three numbers in the first row, that is; the difference of these two rows is $19 + 20 - 5 - 6 = 28$, then we have $☆ = 28 - 5 - 6 = 17$. Thus, we select option (D).

Answer: (D)

17. A workman is moving 40 panes of glass. He gets 2 dollars for each pane. However, if he breaks one pane, he will have to pay 8 dollars instead of getting 2 dollars. If his total pay is 60 dollars, how many panes has he broken?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

【Suggested Solution】

Assume the workman has not broken any glasses, then he gets $\$2 \times 40 = \80 , but he receive only $\$60$, so it means he broke $(80 - 60) \div (2 + 8) = 2$ pieces of glasses. Thus, we select (B).

Answer: (B)

18. Some children stand in a line and call out the numbers 1, 2 and 3 in cyclic order, starting with 1. If the last child calls out 2, which of the following number can be the number of children in the line?

- (A) 24 (B) 25 (C) 26 (D) 27 (E) 28

【Suggested Solution】

From the given information, we are searching the number from the 5 options so that when the total number of children divided by 3 will give a remainder of 2. It is only option (C) from all options meets the requirement of the problem. Hence, we select option (C).

Answer: (C)

19. Max has a red box which contains 6 blue boxes. Each blue box contains 4 green boxes. How many boxes does Max have in total?

- (A) 10 (B) 11 (C) 24 (D) 25 (E) 31

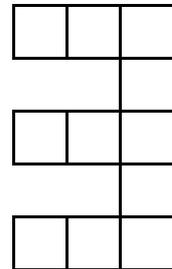
【Suggested Solution】

From the given information, there are of $4 \times 6 = 24$ green boxes, 6 blue boxes and 1 red box. Thus, Max has $24 + 6 + 1 = 31$ boxes in all. We must select (E).

Answer: (E)

20. The diagram shows a figure obtained by putting together 11 squares of the same size. If the perimeter of the figure is 48 cm, what is the area, in cm^2 , of the figure?

- (A) 11 (B) 22 (C) 33
(D) 44 (E) 48



【Suggested Solution】

From the given diagram, we know the perimeter of given polygon composed from 24 sides of the small square, using the given information, the side length of each small square must be 2 cm. It follows that the area of each small square is 4 cm^2 , there are 11 squares of the same size, hence the total area is 44 cm^2 . Hence, we select option (D).

Answer: (D)

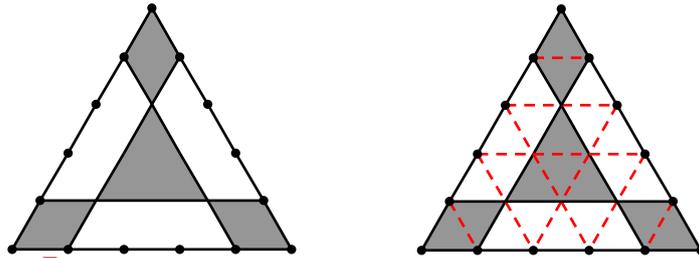
21. Some students are lined up in a rectangular array. Max is the 8th student in his column from the front, and the 13th from the back. There are 15 students to his left and 15 students to his right in the same row. How many students are in the line-up?

【Suggested Solution】

From the given information, we know the formation has $8 + 13 - 1 = 20$ rows, $15 + 15 + 1 = 31$ columns. Thus, there are $20 \times 31 = 620$ students in the line-up.

Answer: 620

22. The area of a triangle is 50 cm^2 . Each side is divided into five equal parts, and some pairs of division points are joined as shown in the diagram. What is the total area, in cm^2 of the shaded regions?

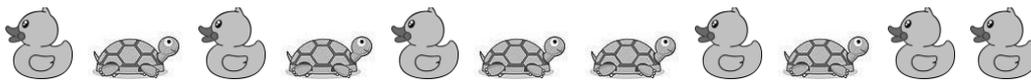


【Suggested Solution】

After connecting the division points by pairs, there will be 25 congruent equilateral triangles as shown in the diagram at the right. So the area of each small equilateral triangle is $50 \div 25 = 2 \text{ cm}^2$. There are 10 small equilateral triangles in the shaded region, then the total area of the shaded region is $2 \times 10 = 20 \text{ cm}^2$.

Answer: 020

23. Oliver arranges his toy ducks and toy turtles in a row as shown in the diagram. He wishes to have all the toy ducks on the left and all the toy turtles on the right. He may switch the position of any two adjacent toys. What is the minimum number of switches he will require?



【Suggested Solution】

We know that for each switch of two adjacent toys may make one toy duck move at most one position to the left, let us name the position of each toy from left to right as 1 to 11. Our main target is to arrange the toys in the manner that all 6 toy ducks will be on the left side. Since the initial position of 6 toy ducks are 1, 3, 5, 8, 10, 11, then we need to operate at least $(1-1) + (3-2) + (5-3) + (8-4) + (10-5) + (11-6) = 17$ times of switches in order to reach our goal. Hence, we must perform switches at least 17 times. So, we select (C).

Answer: 017

24. Every digit of a seven-digit multiple of 3 is 0, 2 or 3, and there are more 3's than 0's. If each of 0, 2 and 3 appears at least once, what is the sum of all seven digits?

【Suggested Solution】

We know that from the divisibility property of 3 of a certain number is: The sum of all the digits in the number must be multiple of 3. Since 0 and 3 are both multiple of 3 but digit 2 is not a multiple of 3, if these seven-digit number must be a multiple of 3, then digit 2 must appear either 3 times or 6 times in the number. If 2 will appear 6 times in the number, then only digit left must be fill in by either 0 or 3, but both digits must appear at least once. Hence, 2's must appear 3 times in the number with the remaining 4 digits as digit 3 appear 3 times and digit 0 appear 1 time. Hence, the sum of all these seven digits is $2 + 2 + 2 + 3 + 3 + 3 + 0 = 15$.

Answer: 015

25. When different buttons are pressed, a robot may move forward 1 cm, 3 cm or 5 cm. If buttons are pressed six times, how many different distances may the robot have moved?



【Suggested Solution 1】

When button 1 is pressed, the robot moves forward by 1 cm; when button 2 is pressed, the robot moves forward by 3 cm and when button 3 is pressed, the robot moves forward by 5 cm. This implies that the maximum distance a robot can move forward is 5 cm once one of the buttons is pressed, let us consider the following cases.

Case 1: When button 3 is not pressed, according to the number of button 2 pressed, there are 7 different distances the robot can move and they are: 6 cm (pressing the first button 6 times), 8 cm (pressing the first button 5 times and second button 1 time), 10 cm (pressing the first button 4 times and second button 2 times), 12 cm (pressing the first button 3 times and second button 3 times), 14 cm (pressing the first button 2 times and second button 4 times), 16 cm (when pressing first button 1 time and second button 5 times) and 18 cm (when pressing the second button 6 times and not pressing the first button at all).

Case 2: When button 3 is pressed just once, the remaining 5 presses may either be button 1 or button 2. according to the number of button 2 pressed, there are 6 different possible distances: 10 cm, 12 cm, 14 cm, 16 cm, 18 cm, 20 cm.

Case 3: When button 3 is pressed two times, the remaining 4 times may be to press button 1 or button 2. according to the number of button 2 pressed, there are 5 different possible distances: 14 cm, 16 cm, 18 cm, 20 cm, 22 cm.

Case 4: When pressing button 3 three times, similarly, the robot can move forward to any of these 4 different possible distances: 18 cm, 20 cm, 22 cm, 24 cm.

Case 5: When pressing button 3 four times, similarly, the robot can move forward to any of these 3 different possible distances: 22 cm, 24 cm, 26 cm.

Case 6: When pressing button 3 five times, similarly, the robot can move forward to any of these 2 different possible distances: 26 cm, 28 cm.

Case 7: When pressing button 3 six times. The robot can reach the distance of 30 cm. In summary, the robot can move a distance of 6 cm, 8 cm, 10 cm, 12 cm, 14 cm, 16 cm, 18 cm, 20 cm, 22 cm, 24cm, 26 cm, 28 cm and 30 cm which is a total of 13 different distances.

In summary, the robot can move a distance of 6 cm, 8 cm, 10 cm, 12 cm, 14 cm, 16 cm, 18 cm, 20 cm, 22 cm, 24cm, 26 cm, 28 cm and 30 cm which is a total of 13 different distances.

【Suggested Solution 2】

The robot may move at least $1 \times 6 = 6$ cm and can reach as far as $5 \times 6 = 30$ cm. Since it was given that the robot will always move forward in odd number of cm, so when the robot move forward 6 steps, then the distance will be an even number in cm. The possible distance that will be an even number cm between 6 cm to 30 cm are possible as follow: $6 = 1 + 1 + 1 + 1 + 1 + 1$; $8 = 1 + 1 + 1 + 1 + 1 + 3$;
 $10 = 1 + 1 + 1 + 1 + 1 + 5$; $12 = 1 + 1 + 1 + 1 + 3 + 5$; $14 = 1 + 1 + 1 + 1 + 5 + 5$;

$16 = 1 + 1 + 1 + 3 + 5 + 5;$ $18 = 1 + 1 + 1 + 5 + 5 + 5;$ $20 = 1 + 1 + 3 + 5 + 5 + 5;$
 $22 = 1 + 1 + 5 + 5 + 5 + 5;$ $24 = 1 + 3 + 5 + 5 + 5 + 5;$ $26 = 1 + 5 + 5 + 5 + 5 + 5;$
 $28 = 3 + 5 + 5 + 5 + 5 + 5;$ $30 = 5 + 5 + 5 + 5 + 5 + 5.$

Answer: 013