4th International Mathematics Assessments for Schools (2014-2015)

Junior Division Round 2

Time: 120 minutes

Printed Name ____________________ *Code* ______________ *Score* ____________

Instructions:

- Do not open the contest booklet until you are told to do so.
- Be sure that your name and code are written on the space provided above.
- Round 2 of IMAS is composed of three parts; the total score is 100 marks.
- Questions 1 to 5 are given as a multiple-choice test. Each question has five possible options marked as A, B, C, D and E. Only one of these options is correct. After making your choice, fill in the appropriate letter in the space provided. Each correct answer is worth 4 marks. There is no penalty for an incorrect answer.
- Questions 6 to 13 are a short answer test. Only Arabic numerals are accepted; using other written text will not be honored or credited. Some questions have more than one answer, as such all answers are required to be written down in the space provided to obtain full marks. Each correct answer is worth 5 marks. There is no penalty for incorrect answers.
- Questions 14 and 15 require a detailed solution or process in which 20 marks are to be awarded to a completely written solution. Partial marks may be given to an incomplete presentation. There is no penalty for an incorrect answer.
- Use of electronic computing devices is not allowed.
- Only pencil, blue or black ball-pens may be used to write your solution or answer.
- Diagrams are not drawn to scale. They are intended as aids only.
- After the contest the invigilator will collect the contest paper.

The following area is to be filled in by the judges;
the contestants are not supposed to mark anything here.

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total Score	Signature
Score																	
Score																	

Junior Division Round 2

Questions 1 to 5, 4 marks each

1. Which of the following expressions is equal to

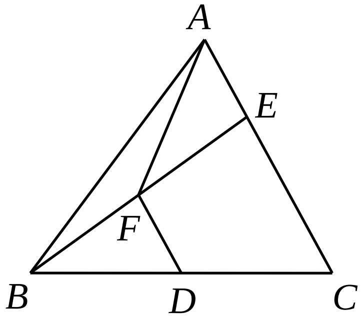
$$(a+1)(b-1)+(b+1)(c-1)+(c+1)(a-1)?$$

（A） $ab+bc+ca-3$ （B） $ab+bc+ca$
（C） $ab+bc+ca+2a+2b+2c+3$ （D） $ab+bc+ca-2a-2b-2c-3$
（E） $ab+bc+ca-2a-2b-2c+3$

Answer：________________

2. In triangle *ABC*, *D* is the midpoint of *BC*. *E* is an arbitrary point on *CA*, and *F* is the midpoint of *BE*. If the area of triangle *ABC* is 120 cm^2 and the area of the quadrilateral *AFDC* is 80 cm^2, what is the area, in cm^2, of triangle *BDF*?

（A） 10 cm^2 （B） 15 cm^2 （C） 17.5 cm^2 （D） 20 cm^2 （E）25 cm^2



Answer：________________

3. Let *m* be a positive integer such that m^3 can be expressed as a sum of *m* consecutive odd integers. For instance, $2^3=3+5$, $3^3=7+9+11$ and $4^3=13+15+17+19$. If 999 is one of the consecutive odd integers in the expression for m^3, what is the value of *m*?

（A） 30 （B） 31 （C） 32 （D） 33 （E） 34

Answer：________________

4. The perimeter of an equilateral triangle is a cm while the perimeter of a square is b cm. If the area of the square is half the area of the triangle, what is the value of $\frac{a^2}{b^2}$?

（A） $\frac{3\sqrt{3}}{8}$ （B） $\frac{3\sqrt{3}}{4}$ （C） $\frac{3\sqrt{3}}{2}$ （D） $\frac{3\sqrt{3}}{3}$ （E） $6\sqrt{3}$

Answer：________________

5. Mindy has two boxes, containing 0 and n pieces of candy respectively, where n is a positive integer. She adds 4, 3 and 2 pieces of candy to one of the boxes in that order, always adding to the box containing fewer pieces of candies. If the two boxes have the same number of pieces of candy, then she adds to either of them. In the end, there is 1 more piece of candy in one box than in the other. How many possible values of n are there?

（A） 2 （B） 3 （C） 4 （D） 5 （E） 6

Answer：________________

Questions 6 to 13, 5 marks each

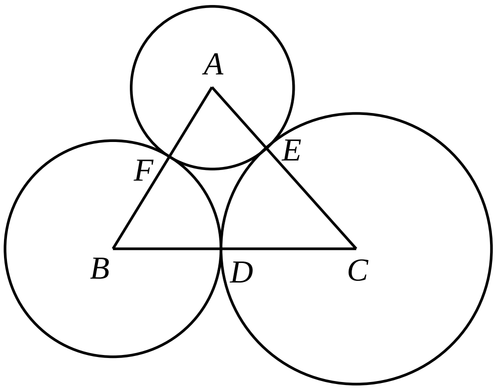
6. Let a, b and c be real numbers such that x^2+5x-3 is one of the factors of the polynomial x^3+ax^2+bx+c. What is the numerical value of $a+b+2c$?

Answer：________________

7. How many triples of integers (x, y, z) are such that $|xyz|=6$?

Answer：________________

8. In triangle ABC, $AB = 7$ cm, $AC = 8$ cm and $BC = 9$ cm. A circle with centre A intersects AB at F and AC at E. The circles with centres B and C and radii BF and CE, respectively, are tangent to each other at a point D on BC. What is the total area, in cm^2, of these three circles? (Taking $\pi = 3.14$)



Answer： ____________ cm^2

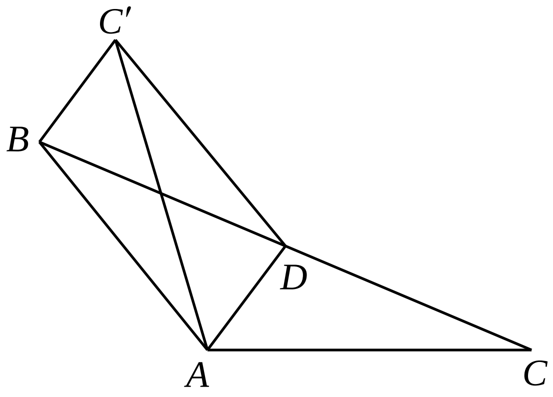
9. There are 2 counters in the first row, and each subsequent row has one more counter than the preceding row. If there are 2015 counters altogether, how many rows of counters are there?

Answer： ____________ rows

10. In a book fair, the organizers give a book to each participant. Each male participant gives every other male participant a book, and each female participant gives every other female participant a book. If the total number of books received by the male participants is 31 more than the total number of books received by the female participants, how many participants are there altogether?

Answer： ____________ participants

11. D is a point on the side BC of triangle ABC such that $\angle BAD = 76°$. When the point C is reflected across AD to the point C', $ABC'D$ is a parallelogram. What is the measure, in degrees, of $\angle ADC$?



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Answer：

12. Let k be a non-zero integer such that the equation $x+\dfrac{9k^2-81}{x}=10k$ has two distinct integer roots. What is the difference when the smaller root is subtracted from the larger one?

Answer：

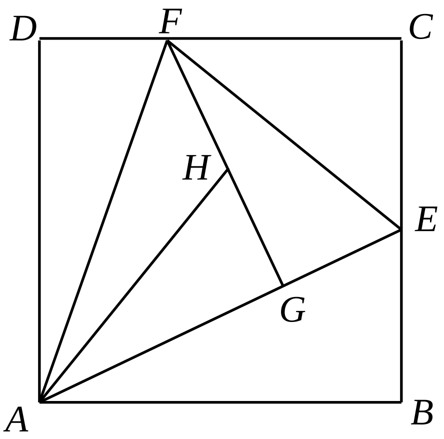
13. The integers 1, 2, 3, …, 20 are divided into two groups. The sum of all the numbers in one group is n, while the product of all the numbers in the other group is also n. What is the maximum value of n?

Answer：

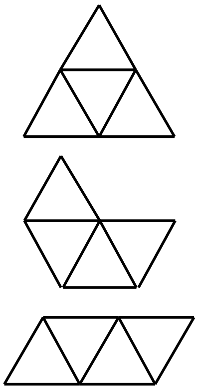
Questions 14 to 15, 20 marks each

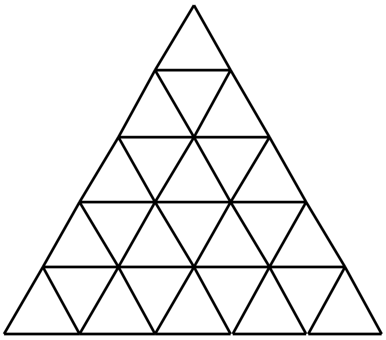
Detailed solutions are needed for these two problems

14. E is a point on the side BC and F is a point on the side CD of a square $ABCD$ such that the perimeter of triangle CEF is equal to half the perimeter of $ABCD$. G is the point on AE such that FG is perpendicular to AE, and H is the point on FG such that $AH = EF$. Prove that AH is perpendicular to EF.

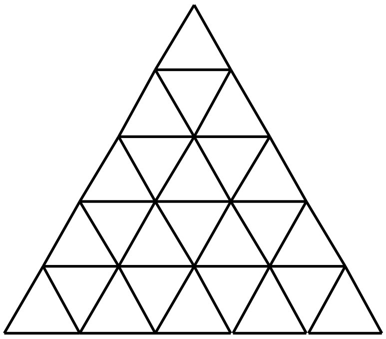


15. Each side of an equilateral triangle is divided into 5 equal parts by 4 points, and these points are joined by lines parallel to the sides of the triangle, dividing into 25 small equilateral triangles. A tetriamond is a shape formed of 4 small equilateral triangles joined edge to edge. There are three tetriamonds as shown in the diagram below on the left.





(a) Show that if 7 of the small triangles are painted, then it may be impossible to fit any tetriamond inside the large triangle without covering up any part of the painted small triangles. (4 marks)



(b) Prove that if 6 of the small triangles are painted, then it is always possible to fit a tetriamond inside the large triangle without covering up any part of the painted small triangles. (16 marks)