Solution Key to Second Round of 4th IMAS
Middle Primary Division

1. What is the sum of the digits of the value of $100\times100-2015$?

   (A) 27 (B) 29 (C) 30 (D) 34 (E) 39

【Suggested Solution #1】

Since $100\times100-2015=10000-2015=7985$, then the sum of all the digits in the difference of the given expression is $7+9+8+5=29$.

【Suggested Solution #2】

We know that $100\times100=10000=(9999+1)$ while the sum of all its digits is $9+9+9+9+1=37$, and the sum of the digits in 2015 is $2+0+1+5=8$, then the sum of all the digits in the difference of the given expression is $37-8=29$.

Answer: (B)

2. If $6\otimes2=6+66=72$ and $2\otimes3=2+22+222=246$, what is the value of $5\otimes3$?

   (A) 3735 (B) 605 (C) 615 (D) 625 (E) 37035

【Suggested Solution】

$5\otimes3=5+55+555=615$.

Answer: (C)

3. In a gymnastic competition, an athlete receives a score from each of seven judges. After the highest score and the lowest score have been removed, the average of the remaining five scores is the actual score for that athlete. If the seven judges give scores of 9.2, 9.5, 9.3, 9.6, 9.1, 9.6 and 9.4 to an athlete, what is the actual score for this athlete?

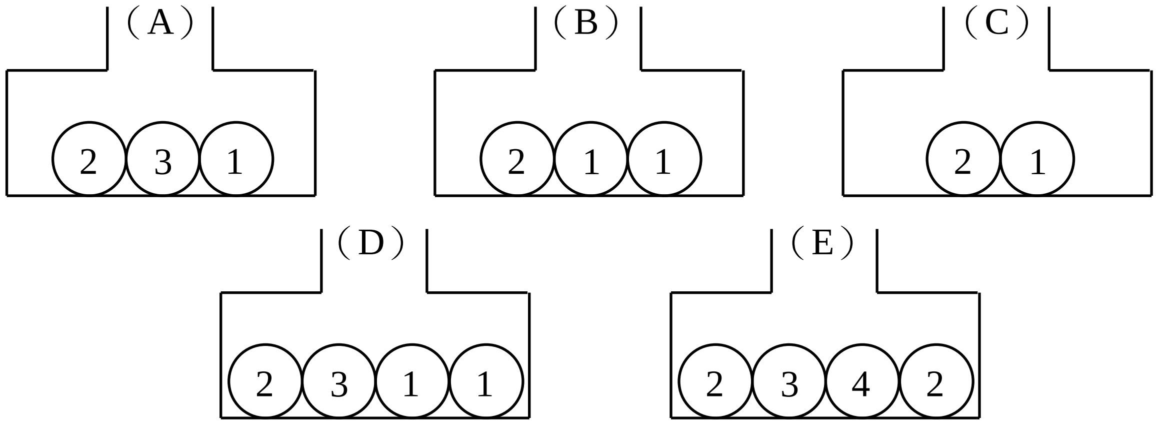
   (A) 9.3 (B) 9.38 (C) 9.4 (D) 9.42 (E) 9.5

【Suggested Solution】

After the highest score 9.6 and the lowest score 9.1 were removed, the average of the remaining scores of the 5 judges is $(9.2+9.3+9.4+9.5+9.6)\div5=9.4$.

Answer: (C)

4. In a shopping mall, a ball is drawn from one of the five boxes shown below. A door prize is won if the number of the ball is 1. From which box should a ball be drawn so that the chance of winning a door prize is as big as possible?



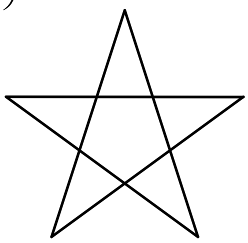
【Suggested Solution】

The chance (or probability) that the ball with label number 1 will be drawn from box (A), (B), (C) and (D)is $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{2}$ and $\frac{2}{4}=\frac{1}{2}$, respectively. Since box (E)does not have the ball with a label of number 1, so the chance is 0. Hence, the chance that the ball with a label number 1 will be drawn from box (B) is the most possible.

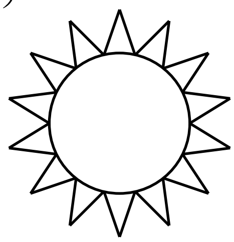
Answer: （B）

5. Which of the following five figures is not possible to trace without lifting the pencil from the paper or retracing any part of it?

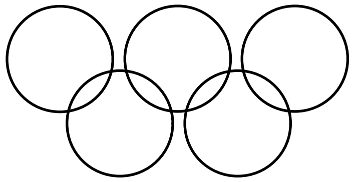
（A）



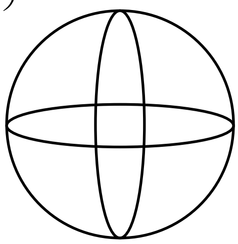
（B）



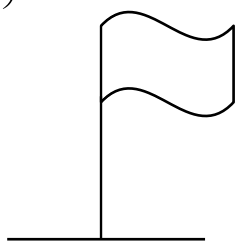
（C）



（D）



（E）



【Suggested Solution】

Solving a problem such as searching for a path in a figure that visits each edge (or arc) exactly once, we must consider the vertices associated with the each edge (or arc), that is; the figure is possible to produce a path travelling each edge exactly once depends on the degree (number of connection of each edge or arc to a particular vertex) of each vertex, then the degree of each vertex is either odd number or even number. Thus, a necessary condition for a certain figure to be traced without lifting the pencil from the paper and retracing any of the edges (arcs) more than once, then it must have exactly 0 or 2 vertices of odd degree.

In diagram (A), there are 10 vertices of even degree and no vertices of odd degree, diagram (B) contains 28 vertices of even degree and 0 vertices of odd degree, diagram (C) has 8 vertices of even degree and no vertices of odd degree, diagram (D) has 8 vertices of even degree and 0 vertices of odd degree, diagram (E) has 3 vertices of even degree and 4 vertices of odd degree, so it is not possible to draw diagram (E) without lifting a pencil from the paper and without tracing any of the edges (arcs) more than once.

Answer: （E）

6. At the bookstore, Lily spends half of her money buying mathematics books and two-thirds of the remaining amount on Chinese literature books. She has just enough money left to buy an English literature book which costs $18. How much money does Lily have initially?

【Suggested Solution】

From the given information, we know that Lily spent one-third of the remaining half of the money to buy the English book , that is equivalent to one-sixth of the money or $18 to buy the English book, hence she has $\$18\times6$ or $\$108$ originally.

Answer: $108

7. There are three kinds of objects, spheres, cylinders and cubes. Three spheres have the same total weight as two cylinders, and five spheres have the same total weight as six cubes. How many cubes will have the same total weight as five cylinders?

【Suggested Solution】

From the given information, the total weight of 3 spheres has the same total weight as 2 cylinders, and the total weight of 5 spheres has the same total weight as 6 cubes, then we know the total weight of 15 spheres and the total weight of 10 cylinders are the same, likewise the total weight of 15 spheres is the same as the total weight of 18 cubes, it follows the total weight of 10 cylinders is the same as the total weight of 18 cubes. Therefore, the total weight of 5 cylinders has the same total weight as the weight of 9 cubes.

Answer: 9 cubes

8. In a video game, 1 point is awarded for eating the first apple, 2 points for eating the second apple, and so on, with 1 additional point is awarded for the next apple to be eaten. What is the total number of points awarded for eating ten apples?

【Suggested Solution】

From the given information, we know the total number of points when reaching eating the 10th apple will be rewarded $1+2+3+4+5+6+7+8+9+10=11\times5=55$ points.

Answer: 55 points

9. In the month of May of a certain year, there are five Sundays and four Mondays. On which day of the week does May 1 fall in that year?

【Suggested Solution】

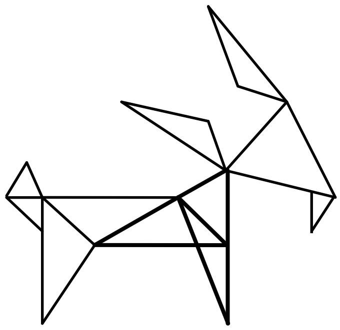
Refer the calendar below：

Mon	Tue	Wed	Thu	Fri	Sat	Sun
				※	※	×
×	×	×	×	×	×	×
×	×	×	×	×	×	×
×	×	×	×	×	×	×
×	×	×	×	×	×	×

From the above calendar, there are 5 Sundays and 4 Mondays, so from the first Sunday until the fifth Sunday, there will be 29 days. Since there are 31 days in the month of May, then the remaining two days must fall before the first Sunday and not after the fifth Sunday, which we label it using ※ in the calendar above. Therefore, May 1 will fall on a Friday of that year.

Answer: Friday or 5

10. In the following diagram, how many different triangles are there, including overlapping triangles?



【Suggested Solution】

In order to ensure the counting of triangles will not be repeated, we will do the counting of triangles by classification.

Case 1 There are no other triangles inside the triangle that we consider, so a total of 12 triangles.

Case 2 There are two smaller triangles inside the big triangle that we consider, so there are 2 such kinds of triangles.

Case 3 There are three smaller triangles inside the big triangle that we consider, there are also 2 such kinds of triangles.

There is no certain kind of big triangle containing 4 or more small triangles inside it. Thus, there are $12+2+2=16$ triangles in different position.

Answer: 16 triangles

11. There are 20 children at a party. The first girl shakes hands with 7 boys. The second girl shakes hands with 8 boys. The third girl shakes hands with 9 boys, and so on. The last girl shakes hands with all the boys. How many boys are at the party?

【Suggested Solution】

The 1^{st} girl shakes with $6+1$ boys, the 2^{nd} girl shakes with $6+2$ boys, the 3^{rd} girl shakes with $6+3$ boys, $\cdots$. Now, assuming there are *n* girls, it follows that the *n*th girl shakes hands with $6+n$ boys. From here, we know that the number of boys is 6 more than the number of girls. Hence, the total number of boys $=(20+6)\div 2=13$, while the number of girls $=(20-6)\div 2=7$.

Answer: 13 boys

12. The school has six enrichment clubs. Mickey wants to join three of them. However, there are two clubs running at the same time, he only can at most choose one of them. How many different choices does he have?

【Suggested Solution #1】

Let the six enrichment clubs be represented as A, B, C, D, E, F such that the two clubs E and F have the same running time, then let us list out the possible clubs Mickey might want to join:

A, B, C; A, B, D; A, B, E; A, B, F; A, C, D; A, C, E; A, C, F; A, D, E;
A, D, F; B, C, D; B, C, E; B, C, F; B, D, E; B, D, F; C, D, E; C, D, F.

Therefore, there are 16 different ways which Mickey may choose to participate in three different clubs.

【Suggested Solution #2】

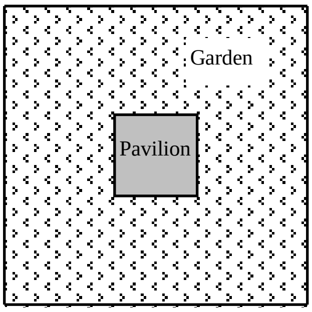
If Mickey didn't participate in two clubs scheduled at the same time, so he must select three clubs from the remaining 4 clubs, hence there will be 4 different ways of joining them. If Mickey participates in any one of the two clubs scheduled at the same time, then there are 2 possible ways of making choices, and then he will participate in the two clubs from the remaining four clubs, then there will be $\frac{4\times3}{2}=6$ different ways of joining them. Thus, there are a total of $6\times2=12$ different ways of joining them. Therefore, a total of $4+12=16$ different ways of selecting the 6 clubs.

【Suggested Solution #3】

Suppose the activities of all the six clubs are scheduled at different time, then Mickey must select three clubs from six enrichment clubs, then there will be $\frac{6\times5\times4}{3\times2\times1}=20$ different ways of selecting the clubs. Since the problem notifies us that there are two clubs conducting their activities at the same time, and we know that there are 4 different ways which Mickey will participate in both clubs having activities at same time and one of the four activities from the other four clubs. Therefore, Mickey can participate $20-4=16$ different ways of selecting the six clubs.

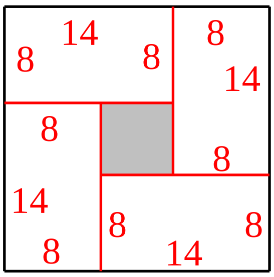
Answer: 16 different ways

13. A square garden has a square pavilion at its centre. The distance on each outer side of the pavilion to its corresponding side of the garden is 8 m. If the total area of the part of the garden outside the pavilion is 448 m^2, what is the area, in m^2, of the pavilion?



【Suggested Solution】

Refer to the given diagram. Extend each side of the pavilion to its corresponding boundary side of the garden so that it will divide the garden into four smaller rectangles of equal area. Since the distance on each outer side of the pavilion to its corresponding side of the garden is 8 m, it follows that the width of the rectangle is 8 m, but the area of the part of the garden outside the pavilion is 448 m^2, so we have the area of each small rectangle as $448\div4=112\,m^2$, so that the side length of the small square is $112\div8=14\,m$. Hence, the area of the pavilion is $(14-8)^2=6^2=36\,m^2$.



Answer: 36 m^2

14. One digit is chosen from each of the three groups {1, 4, 7}, {2, 5, 8} and {3, 6, 9}. The chosen digits are arranged in any order to form a three-digit number. How many such three-digit numbers are divisible by 6?

【Suggested Solution】

By observation, we discover that each of the number in the first set when divided by 3 will give a remainder of 1, each number in the second set when divided by 3 will give a remainder of 2 and each number in the third set when divided by 3 or when each of them is divided by 3 will give a remainder of 0. Hence, according to the meaning of the problem, we must form a three-digit number where the digit in each place is selected from each set and the sum of all the digits formed when divided by 3 will give a remainder of 0, that is; the arrangement of all the three-digit numbers that formulate is always divisible by 3. Hence, we just need to find all those three-digit numbers from the above that are divisible by 2; or the digit in the units place is an even number.

【Method #1】

Using Listing Method, we have the following numbers that are divisible by 2:
132, 312, 126, 216, 162, 612, 192, 912, 156, 516, 138, 318, 186, 816, 168, 618, 198, 918, 432, 342, 234, 324, 426, 246, 462, 642, 264, 624, 924, 294, 942, 492, 354, 534, 456, 546, 654, 564, 954, 594, 384, 834, 348, 438, 468, 648, 486, 846, 684, 864, 498, 948, 894, 984, 732 372, 726, 276, 762, 672, 792, 972, 756, 576, 738, 378, 786, 876, 768, 678, 978, 798. All these can compose a total of 72 three-digit numbers divisible by 6.

【Method #2】

To form a three-digit number that meets the requirements of the problem, we have the following three cases:

Case 1: when the digit in the units place is selected from the first set (the only possibility is the number 4) while when the digit in the tens place is selected from the second set and the digit in the hundreds place is selected from the third set, then there will be 3×3 possible ways to form three-digit numbers, but the digit in the tens and hundreds place can be interchanged in order to form another three-digit that satisfies the requirement of the problem, so we have a total of $1\times(3\times3)\times2=18$ ways to form three-digit numbers.

Case 2: when a digit in the units place is selected from the second set (we can choose 2 or 8 as the units digit) while when the digit in the tens place is selected from first set and the digit in the hundreds place is selected from the third set, then there are 3×3 ways to form such kind of three-digit number. Since the digit in the tens and hundreds place can be interchanged to form new three-digit number, so we have a total of $2\times(3\times3)\times2=36$ ways to form three-digit numbers.

Case 3: when the digit in the units place is selected from the third set (the only possibility is only the number 6) while when the digit in the tens place is selected from the second set and the digit in the hundreds place is selected from the third set, then there are 3×3 possible ways to form three-digit number. Likewise, digits in the tens and hundreds place can be interchanged to form a new three-digit number, then we have a total of $1\times(3\times3)\times2=18$ ways to form three- digit numbers.

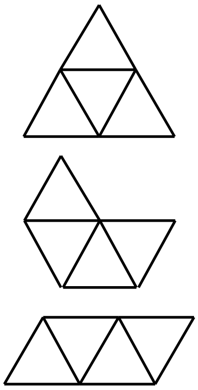
Thus, there are a total of $18+36+18=72$ three-digit numbers divisible by 6.

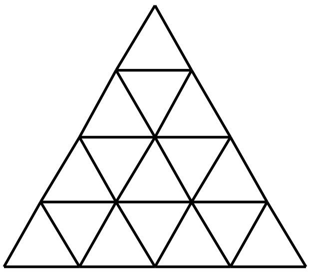
Answer: 72 three-digit numbers

【Marking Scheme】

- Is able to show that the three-digit number is formed, no matter how the arrangement may be, all these three digits will still be divisible by 3, thus will be rewarded 5 marks.
- Is able to explain the digit in units place of a three-digit number formed which must always be an even number, thus will be rewarded 5 marks.
- Is able to use method #1 to list down all the 72 three-digit numbers correctly, thus will be rewarded 10 marks. If some of those enumerated three-digit numbers are incorrect, no marks will be rewarded; is able to use method #2 to present the digit in units place which must be selected from the first, second and third sets correctly, each will be rewarded 3 marks; lastly, is able to write down the total numbers of three-digit numbers correctly, 1 mark will be rewarded.

15. Each side of an equilateral triangle is divided into 4 equal parts by 3 points, and these points are joined by lines parallel to the sides of the triangle, dividing into 16 small equilateral triangles. A tetriamond is a shape formed by 4 small equilateral triangles joined edge to edge.



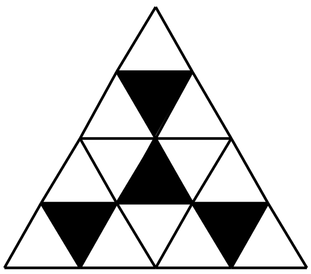


(a) Show that if 4 of the small triangles are painted, then it will be impossible to fit any tetriamond inside the large triangle without covering up any part of the painted small triangles.(4 marks)

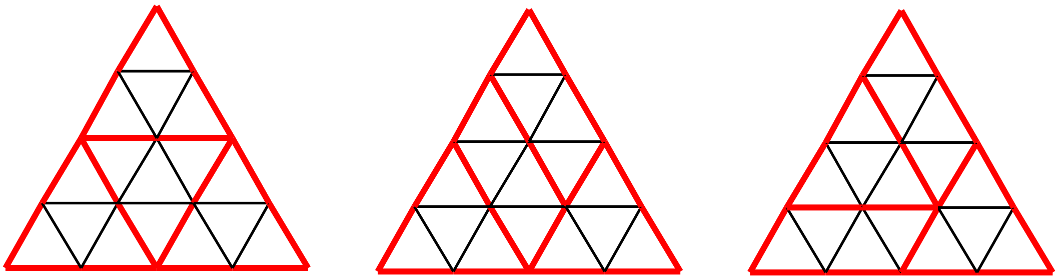
(b) Prove that if 3 of the small triangles are painted, then it is always possible to fit any tetriamond inside the large triangle without covering up any part of the painted small triangles.(16 marks)

【Suggested Solution】

(1) Let us paint black the 4 of the small triangles as figure below, then we cannot put any of the tetriamond in the big triangle by preventing covering any of black-shaded small triangles.



(2) We can use anyone of the three figures below to partition the given big triangle into four regions, because we just need to paint black the three small triangles, then there will be a certain region whose small triangles that have not been painted black, so the four small triangles in this region will surely become a tetriamond.



【Marking Scheme】

(1) In the first sub-problem: if the position of small triangles are correctly painted black, then reward it 4 marks.

(2) For sub-problem:

- The big triangle is correctly partitioned into four regions as the key, reward 8 marks.
- Is able to explain that there is one region without having shaded black, reward 4 marks.
- Is able to explain that the four small triangles in a certain is tetriamond, then no matter which three of the four small triangles are shaded, it is possible to put the tetriamond completely inside the big triangles and none of them will be covered by the black triangles. Reward 4 marks.