Solution Key to Second Round of 4th IMAS Upper Primary Division

1. What is the simplified value of $32\times37\times75$?

(A) 88075 (B) 88800 (C) 88200 (D) 74000 (E) 80800

【Suggested Solution】

$32\times37\times75=8\times4\times25\times3\times37=800\times111=88800$.

Answer: (B)

2. In a gymnastics competition, an athlete receives a score from each of the seven judges. After the highest score and the lowest score have been removed, the average of the remaining five scores is the actual score for that athlete. If the seven judges give scores of 9.2, 9.5, 9.3, 9.6, 9.1, 9.6 and 9.4 to an athlete, what is the actual score for this athlete?

(A) 9.3 (B) 9.38 (C) 9.4 (D) 9.42 (E) 9.5

【Suggested Solution】

After the highest score 9.6 and the lowest score 9.1 were removed, the average of the remaining score of the 5 judges is $(9.2+9.3+9.4+9.5+9.6)\div5=9.4$ score.

Answer: (C)

3. There are 4 children in the first row, and each subsequent row has one more child than the preceding row. If there are 39 children altogether, how many children are in the last row?

(A) 5 (B) 6 (C) 9 (D) 15 (E) 35

【Suggested Solution #1】

From the given information, we know that $4+5+6+7+8+9=39$, so there must be 9 children on the last row.

【Suggested Solution #2】

According to the problem, we know that after the first row there are $39-4=35$ children. Since there will be 5 children seated on the second row, so after the second row there will be $35-5=30$ children.

Then there will be 6 children seated on the third row, so after the third row there will be $30-6=24$ children.

Then that there will be 7 children seated on the fourth row, so after the fourth row there will be $24-7=17$ children.

It follow that there will be 8 children seated on the fifth row, then after the fifth row there will be $17-8=9$ children. And there will be 9 children seated on the sixth row, so there must be six rows only. Thus, there will be 9 children on the last row.

【Suggested Solution #3】

Assume there are still 3 extra rows before the first row such that there are 1, 2 and 3 children, then the total number of children are $39+1+2+3=45$, and we know that $1+2+3+4+5+6+7+8+9=45$, hence there are 9 children on the last row.

Answer: (C)

4. When 200 is subtracted from the square of a positive integer n, the difference is a three-digit multiple of 4. How many different values can n take?

(A) 8 (B) 9 (C) 16 (D) 17 (E) 32

【Suggested Solution】

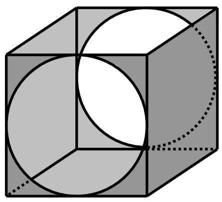
Since 200 is also a multiple of 4, then the sum of the required three-digit number and 200 will be an even perfect square number and it must not be less than 300 and greater than 1199.

We know that $16^2 = 256 \le 300 \le 324 = 18^2$ and $34^2 = 1156 \le 1199 \le 1296 = 16^2$.

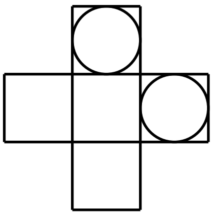
Hence, three-digit numbers that meet the conditions are $18^2 - 200 = 124$, $20^2 - 200 = 200$, $22^2 - 200 = 284$, $24^2 - 200 = 376$, $26^2 - 200 = 476$, $28^2 - 200 = 584$, $30^2 - 200 = 700$, $32^2 - 200 = 824$ and $34^2 - 200 = 956$. There are a total of 9 three-digit numbers satisfying the requirement of the problem.

Answer: (B)

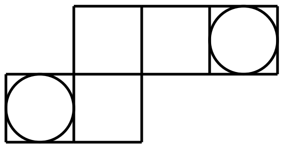
5. From a cubical box without the top, a circle is removed from each of two opposite faces. Which of the following shapes can be folded to form such a box?



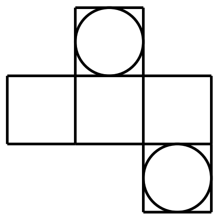
(A)



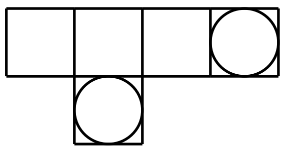
(B)



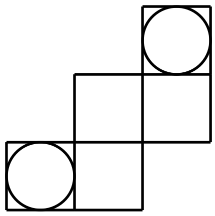
(C)



(D)

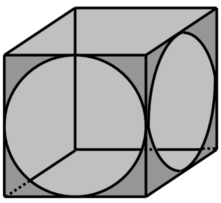


(E)

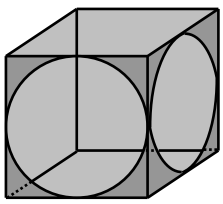


【Suggested Solution】

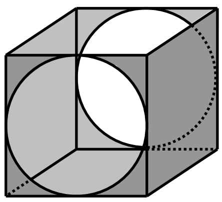
(A)



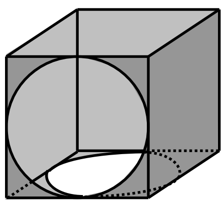
(B)



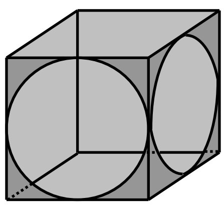
(C)



(D)



(E)



Let us fold the expanded view of each 5 options in the given problem in the form of boxes and then rotate them, we able to observe that among the 5 available figures, only option (C) meets the requirements, while other options when folded as boxes, the two circles are in the adjacent surface.

Answer: （C）

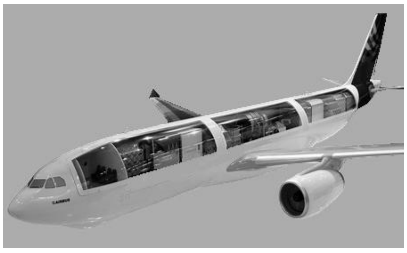
6. If a computer printer is sold at a 10% off discount, a profit of \$220 can still be made. However, if it is sold at a 20% off discount, there will be a loss of \$100. What is the list price of this computer printer?

【Suggested Solution】

From the given information, we know the list price of the printer is $(220+100)\div(0.9-0.8)=3200$ dollars.

Answer: \$3200

7. There are three storage compartments in an airplane. The maximum weight which can be stored in them are 10, 16 and 8 tons respectively. The maximum volume which can be stored in them are 66, 84 and 51 m^3 respectively. The airplane is used to transport grain, each ton of which has volume 6 m^3. If the actual weight of grain in each compartment must be in the same proportion to the maximum weight allowed in that compartment, what is the maximum weight of grain the airplane can transport at a time?



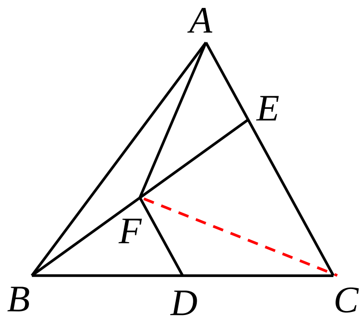
	Front	Middle	Back
maximum weight/tons	10	16	8
maximum volume /m^3	66	84	51

【Suggested Solution】

Since the maximum volume of the three storage compartments are $66>6\times10$, $84<6\times16$ and $51>6\times8$, then we know the middle storage compartment can store at most $84\div6=14$ tons of grain, while the front storage compartment can store at most $10\times\frac{14}{16}=8.75$ tons of grain and the rear storage compartment can store at most $8\times\frac{14}{16}=7$ tons of grain. Therefore, the airplane can store at most a total of $8.75+14+7=29.75$ tons of grain.

Answer: 29.75 tons

8. In triangle *ABC*, *D* is the midpoint of *BC*. *E* is an arbitrary point on *CA*, and *F* is the midpoint of *BE*. If the area of triangle *ABC* is 120 cm^2 and the area of the quadrilateral *AFDC* is 80 cm^2, what is the area, in cm^2, of triangle *BDF*?



【Suggested Solution】

Connect FC, let S represent the area. From the given information, we have

$S_{\triangle ABF} = S_{\triangle AFE}$, $S_{\triangle BCF} = S_{\triangle FCE}$. It follows $S_{\triangle AFC} = \frac{1}{2} S_{\triangle ABC} = 60\,\text{cm}^2$.

Hence, $S_{\triangle BDF} = S_{\triangle FDC} = S_{AFDC} - S_{\triangle AFC} = 80 - 60 = 20\,\text{cm}^2$.

Answer: 20 cm^2

9. The number sentences $1000 - 991 = 9$ and $1001 - 994 = 7$ are examples in which the difference between a four-digit number and a three-digit number is a one-digit number. How many such number sentences are there, including these two examples?

【Suggested Solution】

Clearly, the least four-digit number is 1000 and largest four-digit number is 1008, otherwise the difference of four-digit number and three-digit number will be greater than or equal to $1009 - 999 = 10$.

When the four-digit number is 1000, the possible three-digit numbers that can be used are from 991 to 999, so there are 9 number sentences.

When the four-digit number is 1001, the possible three-digit numbers that can be used are from 992 to 999, so there are 8 number sentences.

When the four-digit number is 1002, the possible three-digit numbers that can be used are from 993 to 999, so there are 7 number sentences.

When the four-digit number is 1003, the possible three-digit numbers that can be used are from 994 to 999, so there are 6 number sentences.

When the four-digit number is 1004, the possible three-digit numbers that can be used are from 995 to 999, so there are 5 number sentences.

When the four-digit number is 1005, the possible three-digit numbers that can be used are from 996 to 999, so there are 4 number sentences.

When the four-digit number is 1006, the possible three-digit numbers that can be used are from 997 to 999, so there are 3 number sentences.

When the four-digit number is 1007, the possible three-digit numbers that can be used are from 998 to 999, so there are 2 number sentences.

When the four-digit number is 1008, the only possible three-digit numbers that can be used is 999, so there is 1 number sentence only.

Answer: 45 number sentences

10. An ant starts from the top left corner square of a 3×5 chessboard. It moves from a square to the adjacent square in the same row or column. After visiting every square exactly once, it ends up at the square in the middle row second column from the right. How many different paths can the ant follow?

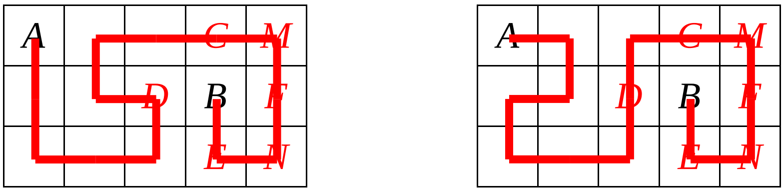
A			C	M
		D	B	F
			E	N

【Suggested Solution】

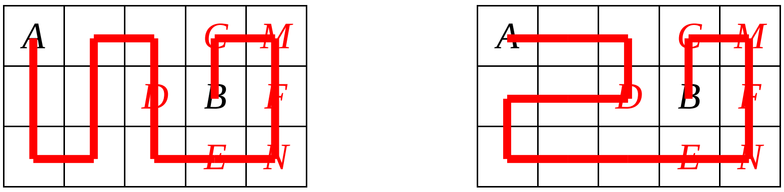
Let us label all the squares surrounding square *B* as shown. Based on the conditions of the problem, we know the ant must pass through squares *C*, *D*, *E* or *F* before reaching square *B*.

Case 1: If the last square before the ant reaches square *B* is the square *F* and the only possibility to reach *F* is passing thru square *M* or square *N*, and from square *M* or square *N*, no matter it is moving to any squares, it is impossible to return back to the initial square, which is the square *A*. So the ant cannot reach square *B* via square *F*.

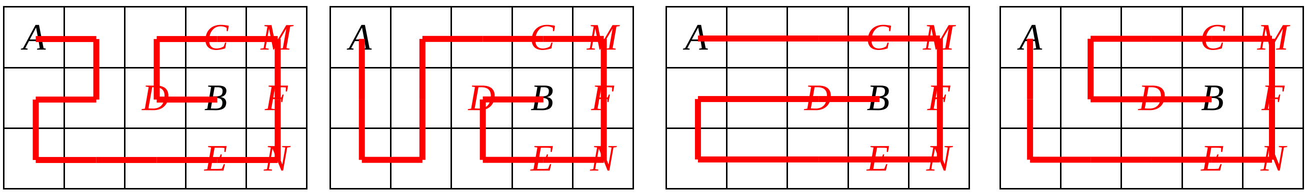
Case 2: If it is the last square allowing the ant to reach *B* is via square *E*, before reaching square *E* the ant must pass thru square *N* only. Since there is no other square connecting to square *E*, then the square that can connect square *N* is the square *F*, and following the same pattern, continuing to trace back, we know there are two different paths, as shown below.



Case 3: If it is the last square allowing the ant to reach *B* is via square *C*, before reaching square *C*, the ant must pass thru square *M* only. Since there is no other square connecting to square *C*, then square that can connect square *M* is the square *F*, and following the same pattern, continuing to trace back, there are also two different paths, as shown below.



Case 4: If it is the last square allowing the ant to reach square *B* is via square *D*, with the same pattern as discussed above, there are 4 different paths from square *A* to square *B* as the diagrams below.



Therefore, the ant has 8 different paths to move from *A* to *B*.

Answer: 8 paths

11. Lily's eight-digit telephone number is divisible by 3 and 5. Micky can only remember its first six digits, which are 8, 9, 2, 0, 1 and 5 in that order. What is the maximum number of times Micky has to dial before connecting to the correct telephone number?

【Suggested Solution】

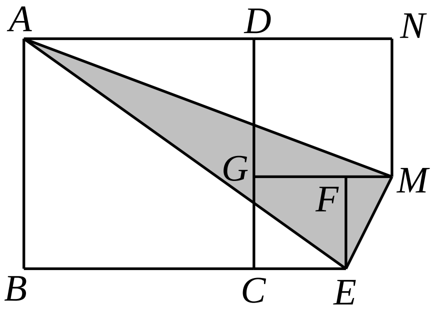
Since the telephone number 892015□□ must be divisible by 5, then its units' digit must be either 0 or 5.

When the units' digit is 0, then 892015□0 must be divisible by 3, it follows that 8 + 9 + 2 + 0 + 1 + 5 + □ + 0 = 25 + □ is also divisible by 3, this implies only 2, 5 or 8 can be filled in the tens' position □.

When the units' digit is 5, then 892015□5 is divisible by 3, it follows that 8 + 9 + 2 + 0 + 1 + 5 + □ + 5 = 30 + □ is also divisible by 3, this implies only 0, 3, 6 or 9 can be filled in the tens' position □.

Answer: 7 times

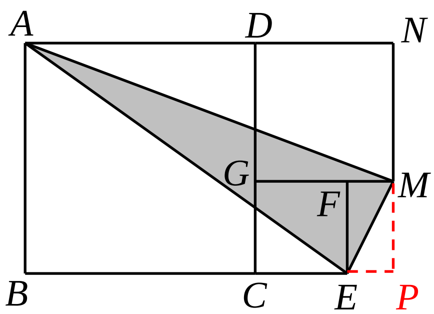
12. *ABCD*, *CEFG* and *DGMN* are squares, with *G* on *CD* and *F* on *MN*. If *AB* = 10 cm and *MN* = 6 cm, what is the area of triangle *AME*, in cm^2?

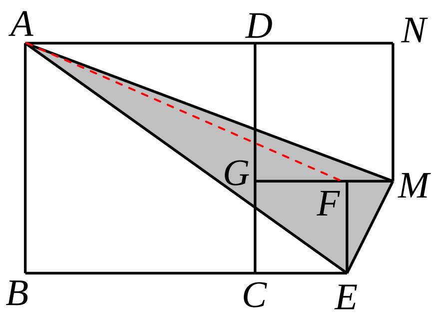


【Suggested Solution #1】

Extend segments *BC*, *NM* so they intersect at *P* as shown in diagram on the left below. Then *ABPN* is a rectangle and $AN = 10 + 6 = 16$ cm, the side length of square *CEFG* is $10 - 6 = 4$ cm, so that $BE = 10 + 4 = 14$ cm, $EP = 6 - 4 = 2$ cm. Hence

$$S_{\triangle AEM} = S_{ABPN} - S_{\triangle ABE} - S_{\triangle EPM} - S_{\triangle AMN}$$
$$= 10 \times 16 - \frac{1}{2} \times 10 \times 14 - \frac{1}{2} \times 2 \times 4 - \frac{1}{2} \times 16 \times 6 = 38\text{ cm}^2$$





【Suggested Solution #2】

Connect *AF* as in the diagram at the right. From the given information, the side length of square *CEFG* is $10 - 6 = 4$ cm, $FM = 6 - 4 = 2$ cm. Then $S_{\triangle AFM} = \frac{1}{2} \times 2 \times 6 = 6\text{ cm}^2$,

$S_{\triangle FEM} = \frac{1}{2} \times 2 \times 4 = 4\text{ cm}^2$, $S_{\triangle AEF} = \frac{1}{2} \times 4 \times (10 + 4) = 28\text{ cm}^2$. Therefore,

$$S_{\triangle AEM} = S_{\triangle AFM} + S_{\triangle FEM} + S_{\triangle AEF} = 6 + 4 + 28 = 38\text{ cm}^2.$$

Answer: 38 cm^2

13. The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 are divided into two groups. The sum of all the numbers in one group is *n*, while the product of all the numbers in the other group is also *n*. What is the maximum value of *n*?

【Suggested Solution】

When $n=42$: divide the given numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 into two groups such that the first group composed of 1, 2, 3, 4, 5, 8, 9 and 10 with a sum of 42 while the second group composed of 6 and 7 with a product of 42. It meets the condition.

When $n=43,\ 44,\ 46$ and 47: the number *n* containing the prime number (such as 43, 11, 23 or 47) is greater than 10, and this causes difficulty to compose the second group that produces a product of *n*.

When $n=45$: since $45=3\times3\times5=5\times9$, so the numbers in the second must contain 5 and 9 only, while the numbers in the first group will then be 1, 2, 3, 4, 6, 7, 8, 10 with a sum of 41, which does not meet the condition of the problem.

When $n\ge48$: the sum of all numbers in the first group is not less than 48, then the sum of all numbers in the other group cannot be greater than $1+2+\cdots+10-48=7$, it is clear that the product of this group cannot be greater than 48, which is a contradiction!

Thus, the maximum value of *n* is 42.

Answer: 42

【Remark】

In considering this case that the second group has two numbers *a* and *b* only, then from the given information we have $55-(a+b)=ab$, after the rearrangement we have $(a+1)(b+1)=56=7\times8$, so the only possibilities are $a=6$, $b=7$ or $a=7$, $b=6$. It follows that $n=42$.

14. We increase by 1 each of three prime numbers, not necessarily distinct. Then we form the product of these three sums. How many numbers between 1999 to 2021 can appear as such a product?

【Suggested Solution #1】

Assume the three prime numbers as $p\le q\le r$, we consider the different cases for *p*, *q* and *r*.

Case 1: If $p=q=2$, then $n=9(r+1)$, so *n* is divisible by 9. Thus, between 1999 to 2021, only 2007 and 2016 is divisible by 9.

- When $n=2007$, it follows $r+1=223$, so that $r=222$. But 222 is not a prime number, therefore no numbers satisfy the condition when *n* = 2007.
- When $n=2016$, it follows $r+1=224$, so that $r=223$, which is a prime number, therefore 2016 satisfies the condition.

Case 2: If $p=2$, $2<q\le r$, then $n=3(q+1)(r+1)$ with both $q+1$ and $r+1$ are even number greater than 2, it follows *n* is divisible by 3 and 4, so that the possible value of *n* must be 2004 and 2016. From case 1, we know 2016 satisfies the condition of the problem.

- When $n=2004$, then $(q+1)(r+1)=668=2^2\times167$, so 668 cannot be expressed as the product of even numbers greater than 2. Hence, 2004 did not meet the condition of the problem.

Case 3: If $2<p\le q\le r$, then each of $p+1, q+1$ and $r+1$ is even number greater than 2, so the value of n must be divisible by 8, it follows that the possible values of n are 2000, 2008 and 2016. From the above cases, 2016 meets the requirements of the problem.

- When $n=2000$, then $(p+1)(q+1)(r+1)=2^4\times5^3$, it follows only of $p+1$, $q+1$ or $r+1$ is a multiple of 4. Suppose $r+1$ is not a multiple of 4, neither $2\times5-1=9$, $2\times5^2-1=49$, nor $2\times5^3-1=249$ is a prime number, so that 2000 did not meet the condition of the problem.
- When $n=2008$, then $(p+1)(q+1)(r+1)=2^3\times251$, ，so that only one of $p+1$, $q+1$ or $r+1$ is an even number greater than 2. Hence, 2008 did not meet the condition of the problem.

Thus, from all the cases above, there is only one number between 1999 to 2021 meeting all the conditions of the problem and that is 2016.

【Suggested Solution #2】

Let us assume the three prime numbers represented as $p\le q\le r$ and deeply analyze these three numbers.

Since each value of $p+1$, $q+1$ and $r+1$ must be equal to 3 or even number greater than 3, so the value of $(p+1)(q+1)(r+1)$ will be either multiple of 3 or multiple of 8. Between 1999 to 2021, we will deal with 2000, 2001, 2004, 2007, 2008, 2010, 2013, 2014, 2016, 2019 in order to find out which of them satisfies the condition of the problem.

We know that $2000=2^4\times5^3$, is not a multiple of 3, now express 2000 as the product of three even numbers such that each of them is greater than 3:

$$2000=2^2\times(2\times5)\times(2\times5^2)=4\times10\times50$$
$$=(2\times5)\times(2\times5)\times(2^2\times5)=10\times10\times20$$

But $10=9+1$, 9 is not a prime number, so has not met the condition.

For the case of $2001=3\times23\times29$, but neither 23 nor 29 is an even number greater than 3, so has not met the condition.

For the case of $2004=2^2\times3\times167$, in the three factors except the prime number 3, the other two numbers are both neither even number greater than 3, so has not met the condition.

For the case of $2007=3^2\times223$, in the three factors except the prime number 3 the other two numbers are both neither even number greater than 3, so has not met the condition.

For the case of $2008=2^3\times251$, all the factors are not even numbers of greater than 3, so has not met the condition.

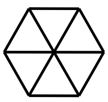
For the case of $2010 = 2 \times 3 \times 5 \times 67$, in the three factors except the prime number 3 the other two numbers are both neither even number greater than 3, so has not met the condition.
For the case of $2013 = 3 \times 11 \times 67$, in the three factors except the prime number 3 the other two numbers are both neither even number greater than 3, so has not met the condition.
For the case of $2014 = 2 \times 19 \times 53$, all the factors are not even numbers of greater than 3, so has no met the condition.
For the case of $2016 = 2^5 \times 3^2 \times 7$, we now express 2016 as the product of three even numbers in which some are greater than or equal to 3: $2016 = 3 \times 4 \times 168$, where $3 = 2 + 1$, $4 = 3 + 1$, $168 = 167 + 1$. But 2, 3 and 167 are prime numbers, so it meets the condition of the problem. Hence, 2016 is the solution.
For the case of $2019 = 3 \times 673$, in the three factors except the prime number 3 the other two numbers are both neither even number greater than 3, so has not met the condition.
Based on all the cases above, we conclude that between 1999 to 2021, there is one that meets the condition of the problem and that is 2016.

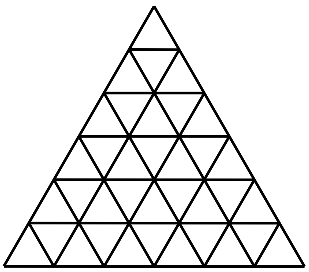
Answer: 2016

【Marking Scheme】

(1) Indicate each value of $p+1$, $q+1$ and $r+1$ is only even number greater than or equal to 3 will be rewarded 5 marks.
(2) Indicate the value of $(p+1)(q+1)(r+1)$ is only multiple of 3 or multiple of 8 will be rewarded 5 marks.
(3) Aside from the discussion of the case consists of 2016, other numbers that has not met the conditions of the problem will be rewarded 5 marks.
(4) Only the correct answer without solutions will be rewarded 5 marks. In case, more than one answers are given, NO marks must be rewarded.

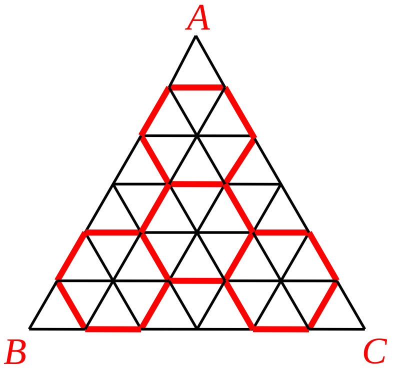
15. Each side of an equilateral triangle is divided into 6 equal parts by 5 points, and these points are joined by lines parallel to the sides of the triangle, dividing into 36 small equilateral triangles. A regular hexagon is the same size as 6 of the small equilateral triangles put together. What is the maximum number of such hexagons that can along the grid line fit inside the large equilateral triangle without overlap?





【Suggested Solution #1】

We can put four congruent regular hexagons in the given equilateral triangle as the diagram shown.



We will now do the proving that no more than four congruent regular hexagons can be placed inside the given equilateral triangle. We consider how many small equilateral triangles cannot be used as part of a regular hexagon. By observation, there is one side of some regular hexagon which coincides with the side *BC* of equilateral triangle.

Suppose none of one side of regular hexagon lies on side *BC* of equilateral triangle, then none of the 11 small triangular grids in the lower row of equilateral triangle will be part of regular hexagon.

Suppose one side of a regular hexagon lies on side *BC* of equilateral triangle, then there are 8 small triangular grids in the lower row of equilateral triangle that will not be part of regular hexagon.

Suppose one side of two regular hexagon lies on side *BC* of equilateral triangle, then there are 5 small triangular grids in the lower row of equilateral triangle that will not be part of regular hexagon.

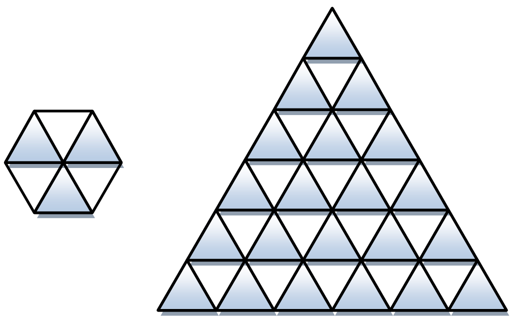
According to Principle of Symmetry, there are at least 5 small triangular grids in each side of $\triangle ABC$ that are not part of regular hexagon.

Deduct those small triangular grids that are repeated, we know there are at least $5\times3-3=12$ small triangular grids in the given diagram that are not part of $\triangle ABC$.

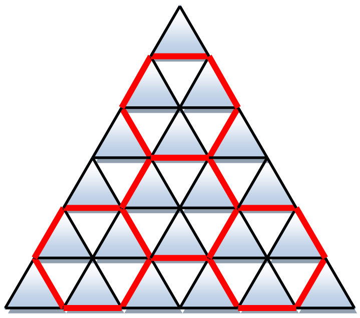
Since there are 36 small triangular girds in the diagram, so there are three that cannot exceed $(36-12)\div6=4$ regular hexagons which can fill in the given equilateral triangle.

【Suggested Solution #2】

Let us first paint every two small neighboring triangular grips by white color and blue color alternately as shown in the diagram below.



We have exactly 15 white color triangular grips inside the big equilateral triangle and each regular hexagon will contain exactly three (3) white color triangular grips, so we can exactly put five (5) regular hexagons. Observe all the white color triangular grips that are parallel to lowest base (parallel to the horizontal base of equilateral triangle) of the given equilateral triangle, we note that there will be five (5) white color triangular grips in the lowest row of equilateral triangle, then there will be at most two white color triangular grips in the lowest base (parallel to the horizontal base of equilateral triangle) of each regular hexagon, and so on. The last row of small triangular grips in the given equilateral triangle we can at most put are two(2) regular hexagon on it, then at least one (1) white color small triangular grip cannot be part of regular hexagon. Hence, in the given equilateral triangle there are at most 14 white color small triangular grips will be in some of regular hexagons, that is; we can put exactly four (4) regular hexagons in $\triangle ABC$ as shown below.



Answer: 4 regular hexagons

【Marking Scheme】

(1) Able to indicate that there are at least 5 small triangular grids in each side of $\triangle ABC$ are not part of any regular hexagon will be rewarded 10 marks.

(2) Able to indicate that at least 12 small triangular grids not belonging to part of any regular hexagons will be rewarded 5 marks.

(3) Able to paint two neighboring small triangular grids that were inside the $\triangle ABC$ in alternate color will rewarded 10 marks.

(4) Indicate at least 1 white color small triangular grid cannot fit in the given equilateral triangle will rewarded 5 marks.

(5) Indicate the correct answer with the correct diagram will be awarded 5 marks.