Solution to
Fifth International Mathematics Assessment for Schools
Round 1 of Junior Division

1. What is the value of $\sqrt{(-2)^8}+(1+2+2^2+\cdots+2^{2015})^0+|-16|$?

(A) 0 (B) 32 (C) 33

(D) $2^{2016}-1$ (E) $2^{2016}+31$

【Solution】

The value is $16+1+16=33$.

Answer：(C)

2. Someone set the alarm clock for 1:30 pm and fell asleep at 12:35 pm. When he was awaken by the alarm clock, for how long had this person been sleeping?

(A) 1hour and 5 minutes (B) 55 minutes (C) 95 minutes

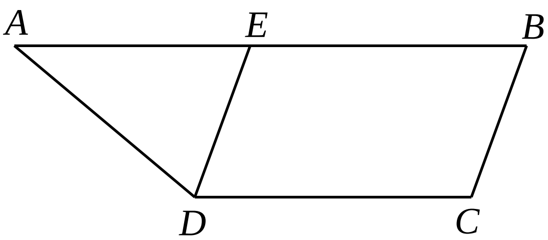
(D) 105 minutes (E) 11 hours and 5 minutes

【Solution】

This person slept for 25 minutes before 1:00 pm and 30 minutes after 1:00 pm, for a total of 55 minutes.

Answer：(B)

3. In a quadrilateral *ABCD*, $AB//DC$, $BC//ED$, $AD=AE$ and $\angle C=110°$. What is the measure of $\angle A$?



(A) 20° (B) 35° (C) 40°

(D) 55° (E) 70°

【Solution】

Since *BCDE* is a parallelogram, $\angle DEB=\angle C=110°$. Hence $\angle DEA=70°$.

It follows that $\angle A=180°-2\angle DEA=180°-2\times70°=40°$.

Answer：(C)

4. In a sale, each dress is reduced to 49% of its price and if two dresses are purchased at the same time, both are reduced to 45% of their prices. Lily buys two dresses together and pays 90 dollars for both. By doing this instead of buying them separately, how many dollars has she saved?

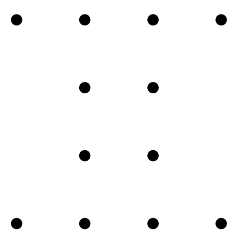
(A) 10 (B) 8 (C) 6 (D) 4 (E) 3.6

【Solution】

The price of both dresses together is $90\div45\%=200$ dollars. Lily will have to pay $200\times49\%=98$ dollars if she buys them separately. Hence she has saved 8 dollars.

Answer：(B)

5. Sixteen points are arranged in a 3 cm by 3 cm formation. Four of them are removed, leaving behind twelve points as shown in the diagram. If we choose three of these twelve points as vertices of a triangle, what is the largest possible area of this triangle, in cm^2?



(A) 9 (B) $\frac{9}{2}$ (C) 3 (D) 2 (E) $\frac{3}{2}$

【Solution】

It is clear that we should not choose any of the four central points. Then two of the vertices are in the same row while the third one is in the opposite row. The height is 3 cm and the largest possible base is also 3 cm. Hence the largest possible area of this triangle is $\frac{1}{2}\times 3\times 3=\frac{9}{2}$ cm^2.

Answer：(B)

6. Class A has 17 students more than Class B, which has 15 students less than Class C. Of the following five numbers, which can be the total number of students in these three classes?

(A) 150 (B) 151 (C) 152 (D) 153 (E) 154

【Solution】

If class B has no students, then the total number of students is 17+15=32. We can add 1 student to each class at a time and maintain the differences. Hence when the total number of student is divided by 3, the remainder must be 2. Of the five numbers given, only 152 has this property.

Answer：(C)

7. From 0, 1, 2, 3, 4 and 5, we choose two different numbers x and y. What is the largest possible value of $2(x+y)^2+(x-y)^2$?

(A) 75 (B) 163 (C) 175 (D) 187 (E) 200

【Solution】

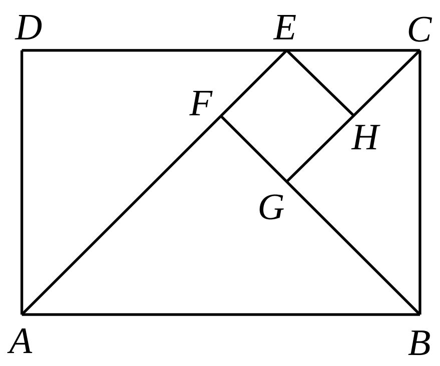
Since $2(x+y)^2+(x-y)^2=3x^2+3y^2+2xy$, we should make x and y as large as possible. The largest possible value of the expression is $3\times 5^2+3\times 4^2+2\times 4\times 5=163$.

Answer：(B)

8. Divide the rectangle $ABCD$ into four isosceles right triangles and one square, as in the diagram left. If the area of square $EFGH$ is 100 cm^2, what is the area of rectangle $ABCD$, in cm^2?

(A) 750 (B) 1000 (C) 1100

(D) 1200 (E) 1600



【Solution】

The side length of $EFGH$ is 10 cm. The length of a leg of CEH is also 10 cm. Hence the length of a leg of BCG is 20 cm, the length of a leg of ABF is 30 cm and the length of the hypotenuse of ADE is 40 cm. It follows that the length of a leg of AE is $20\sqrt{2}$ cm. Hence the total area of $ABCD$ is

$$100+\frac{1}{2}\times(100+400+900+800)=1200 \text{ cm}^2.$$

Answer：(D)

9. A group of students are staying in a hotel. If five of them share a room, then there is no room for six of them. If six of them share a room, there are just enough rooms, one of which has less than six students. Of the following five numbers, which cannot be the number of students?

（A）46 （B）51 （C）56 （D）61 （E）66

【Solution】

If the students occupy x rooms altogether, then there are $5x+6$ students.

If 6 students share a room except for one shared by y students, then y = 1, 2, 3, 4 or 5. We have $5x+6=6(x-1)+y$, which simplifies to $x+y=12$. Hence x = 11, 10, 9, 8 or 7 so that $5x+6=$61, 56, 51, 46 or 41, and cannot be 66.

Answer：(E)

10. In a pentagon, one angle is $48°$. The second angle is three times as large as the first. The third angle is $30°$ less than the second. The fourth angle is $10°$ less than the fifth. What is the measure of the fourth angle, in degrees?

（A）112 （B）122 （C）132 （D）142 （E）152

【Solution】

The second angle is $144°$. The third angle is $114°$. Let the fourth angle be x degrees. Then the fifth angle is $x+10$ degrees. From $48+144+114+x+(x+10)=540$, $x=112$.

Answer：(A)

11. There are three shirts, three pairs of trousers and three pairs of shoes. Of each type, one is red, one is black and one is white. In how many different ways can we choose one of each type so that something white is chosen?

（A）8 （B）9 （C）18 （D）19 （E）27

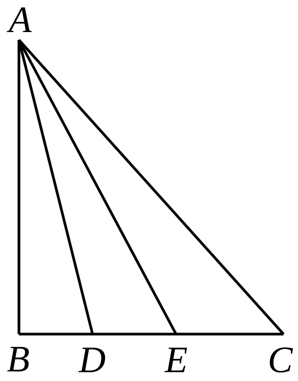
【Solution】

The number of different choices is $3\times3\times3=27$. Of these, $2\times2\times2=8$ ways result in nothing white being chosen. Hence the number of desired ways is $27-8=19$.

Answer：(D)

12. In triangle ABC, AB is perpendicular to BC. D and E are points on BC such that $\angle BAD=\angle DAE=\angle EAC$ and $\angle ADC-\angle C=56°$. What is the measure of $\angle BAC$?

（A）42° （B）45° （C）51° （D）60° （E）84°



【Solution】

Let $\angle BAD=\angle DAE=\angle EAC=x$.

Then $\angle C=90°-3x$ and $\angle ADC=\angle C+56°=146°-3x$.

Summing the angles of triangle ADC, we have $(146-3x)+(90-3x)+2x=180$, which yields $x=14$.

Hence $\angle BAC=42°$.

Answer：(A)

13. If $\dfrac{a}{b}=a+1$ and $\dfrac{b}{a}=a-1$, what is the value of $\dfrac{b^2}{(a-1)^2}$?

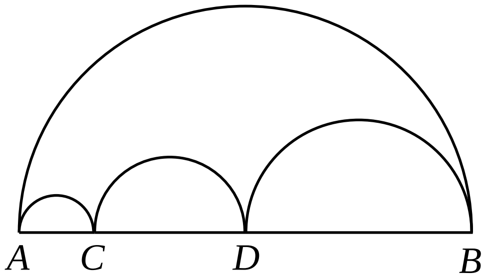
（A）1 （B）2 （C）3 （D）4 （E）5

【Solution】

Multiplication yields $1=\frac{a}{b}\times\frac{b}{a}=(a+1)(a-1)=a^2-1$. Hence $a^2=2$. Since $\frac{b}{a}=a-1$, we have $\frac{b}{a-1}=a$ so that $\frac{b^2}{(a-1)^2}=a^2=2$.

Answer：(B)

14. C and D are points on AB such that $AC : CD : DB = 1 : 2 : 3$. Semicircles are drawn on the same side of AB with respective diameters AB, AC, CD and DB. What fraction of the area of the largest semicircle is the total area of the other three semicircles?



(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{13}{36}$ (D) $\frac{7}{12}$ (E) $\frac{7}{18}$

【Solution】

We may take $AB = 12$ so that $AC = 2$, $CD = 4$ and $DB = 6$. Then the desired fraction is $\frac{\frac{1}{2}\pi\times(1^2+2^2+3^2)}{\frac{1}{2}\pi\times6^2}=\frac{7}{18}$.

Answer：(E)

15. Each coin is worth either 1 dollar, 5 dollars or 10 dollars. Their total worth is 60 dollars. They may be divided into three, four or five piles of equal worth. What is the minimum number of coins?

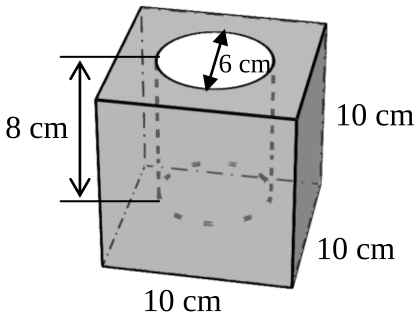
(A) 6 (B) 11 (C) 15 (D) 16 (E) 20

【Solution】

When divided into five piles each worth 12 dollars, we see that we must have at least 10 coins each of which is worth 1 dollar. Hence the number of coins is at least 15. If it is exactly 15, each of the other five coins is worth 10 dollars, but these 15 coins cannot be divided into four piles each worth 15 dollars. It follows that there must be at least 16 coins. If we change one of the coins worth 10 dollars into two coins each of which is worth 5 dollars, all divisions are realizable. Hence the minimum number of coins is 16.

Answer：(D)

16. From a cube of side length 10 cm, a cylinder with diameter 6 cm and depth 8 cm is hollowed out. What is the volume, in cm^3, of the remaining part of the cube? Take $\pi=3.14$.



(A) 426.08 (B) 517.46 (C) 573.94
(D) 717.46 (E) 773.92

【Solution】

The desired volume is $10^3-\pi\times 3^2\times 8=1000-72\pi=773.92\ \text{cm}^3$.

Answer：(E)

17. If a, b and c are all positive integers, which of the following numbers can be the value of $(a+b+c)(a+b-c)(a-b+c)(-a+b+c)$?

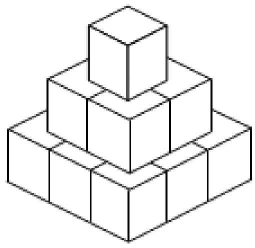
(A) 24 (B) 54 (C) 48 (D) 60 (E) 100

【Solution】

The four factors are either all odd or all even. For any of the given numbers to be the value of their product, they must all be even, so that the product must be a multiple of $2^4=16$. Thus 48 is the only possibility, and is realizable with $a = b = c = 2$.

Answer：(C)

18. A three-layer structure consists of 14 unit cubes. The bottom layer consists of 9 cubes in a 3 by 3 configuration. The middle layer consists of 4 cubes in a 2 by 2 configuration. The top layer consists of a single cube. The exposed surface area of this structure is painted, including the bottom. What is the total area of the unpainted surface of the individual cubes?



(A) 20 (B) 31 (C) 42 (D) 53 (E) 64

【Solution】

Looking from the top or from the bottom, we can see 9 unit cubes. Looking from any side, we can see 1 + 2 + 3 unit cubes. Hence the exposed surface area is 18 + 24 = 42. Subtracting this from the total surface area of the individual cubes, we have $14\times 6-42=42$.

Answer:(C)

19. In an election between four candidates, they are supported respectively by 11, 12, 13 and 14 of the first 50 voters. Six more votes are to be cast, each for one of the four candidates. In how many ways can the candidate currently with 13 supporters become the uncontested winner?

(A) 16 (B) 17 (C) 18 (D) 19 (E) 20

【Solution】

In order to be the uncontested winner, our favourite candidate must receive at least 2 of the last 6 votes. Let the final vote count be $15+x$, $14+y$, $12+z$ and $11+t$, with $x+y+z+t=4$.

If $x = 4$, then $(y, z, t) = (0, 0, 0)$. There is only 1 possibility.

If $x=3$, then $(y, z, t) = (1, 0, 0)$ or its permutations. There are 3 possibilities.

If $x=2$, then $(y, z, t) = (1, 1, 0), (2, 0, 0)$ or their permutations. There are 6 possibilities.

If $x=1$, then we must have $y = 0$ or 1.

If $y=0$, then $(z, t) = (3, 0), (2, 1)$ or their permutations. There are 4 possibilities.

If $y=1$, then $(z, t) = (2, 0), (1, 1)$ or their permutations. There are 3 possibilities.

Finally, if $x=0$, we must have $y=0$ and either $(z, t) = (1, 3)$ or $(2, 2)$.

Hence the total number of possibilities is $1+3+6+7+2=19$.

Answer：(D)

20. Let x, y and z be distinct positive prime numbers such that $x+y+z$ and $x^2+y^2+z^2$ are also prime numbers. What is the minimum value of $x+y+z$?

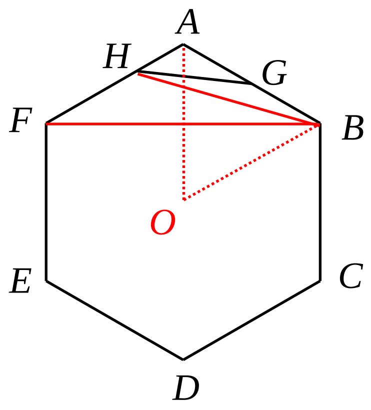
(A) 17 (B) 19 (C) 23 (D) 29 (E) 31

【Solution】

We may assume that $x<y<z$. We cannot have $x = 2$ as otherwise $x+y+z$ is even. If $x>3$, then $x^2 \equiv y^2 \equiv z^2 \equiv 1 \pmod 3$ so that $x^2+y^2+z^2$ is divisible by 3. Hence $x=3$. We cannot have $x+y+z=17$ since either $x = y = 3$ or $y = z = 7$. If $x+y+z=19$, then (x, y, z) must be (3, 5, 11), but $x^2+y^2+z^2$ ends in 5. Hence the minimum value of $x+y+z$ is 23, and this can be realized with (x, y, z) = (3, 7, 13).

Answer：(C)

21. *ABCDEF* is a regular hexagon. *G* is the midpoint of *AB* and *H* is the point on *AF* such that *FH*=2*AH*. If the area of triangle *AHG* is 1 cm². What is the area, in cm², of *ABCDEF*?



【Solution】

Since *AG* = *GB*, the area of triangle *AHG* is half that of triangle *AHB*. Since *FH* = 2*AH*, the area of triangle *AHB* is one-third that of triangle *AFB*. It follows that the area of *AFB* is 6 cm². By symmetry, the area of triangle *CDE* is the same. Since *BCEF* is a rectangle with the same base as *AFB* but double the height, its area is four times as large. Hence the area of *ABCDEF* is $6\times 6=36\,\text{cm}^2$.

Answer：036

22. What is the value of *abc* where *a*, *b* and *c* are positive real numbers such that $a(b+c)=48$, $b(c+a)=70$ and $c(a+b)=88$?

【Solution】

Adding the three equations yields $2(ab+bc+ca)=206$. Hence $ab+bc+ca=103$. Subtracting each of the three equations from this, we have $ab=15$, $bc=55$ and $ca=33$. It follows that the square of *abc* is the product of these three numbers. Since each of *a*, *b* and *c* is positive, *abc* = 165.

Answer：165

23. What is the value of b^a where a and b are real numbers such that $b=\sqrt{a^2-6a+b}+|b-9|+9$?

【Solution】

Since $b-9=\sqrt{a^2-6a+b}+|b-9|\ge|b-9|$, we have $b\ge 9$. Hence $\sqrt{a^2-6a+b}=0$. If $b>9$, we have $a^2-6a+b=(a-3)^2+(b-9)>0$, which is a contradiction. Hence $b=9$. From $a^2-6a+9=0$, we have $a=3$ so that $b^a=9^3=729$.

Answer：729

24. What is the maximum value of a if $a^2 \mid (10\times11\times12\times\cdots\times19)$?

【Solution】

Since $10\times11\times12\times\cdots\times19=2^8\times3^4\times5^2\times11\times13\times17\times19$, the maximum value of a is $2^4\times3^2\times5^1=720$.

Answer：720

25. For any permutation of 1, 2, 3, 4, 5, 6, 7 and 8, add the second number to the first, multiply the sum by the third number, add the fourth number to the product, multiply the sum by the fifth number, and so on. What is the minimum value of the final sum?

【Solution】

Since only additions and multiplications are involved, the value of the expression grows progressively. Clearly, the three multipliers, namely, the third, the fifth and the seventh numbers, should be 1, 2 and 3 but in reverse order, while the other five numbers should be in increasing order. Hence the minimum value is $(((4+5)\times3+6)\times2+7)\times1+8=81$.

Answer：081