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Solution to
Fifth International Mathematics Assessment for Schools
Round 1 of Middle Division

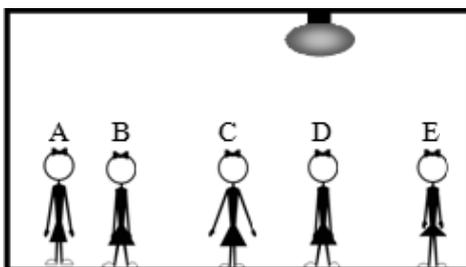
1. What is the value of $2015 - 116$?
 (A) 1889 (B) 1890 (C) 1898 (D) 1899 (E) 1989

【Suggested Solution】

We have $2015 - 116 = 2015 - 15 - 1 - 100 = 2000 - 100 - 1 = 1900 - 1 = 1899$.

Answer : (D)

2. The diagram shows five children of equal height standing in a row under a street lamp. Which of them casts the shortest shadow?



- (A) A (B) B (C) C (D) D (E) E

【Suggested Solution】

Since D is nearest to the lamp, her shadow is the shortest.

Answer : (D)

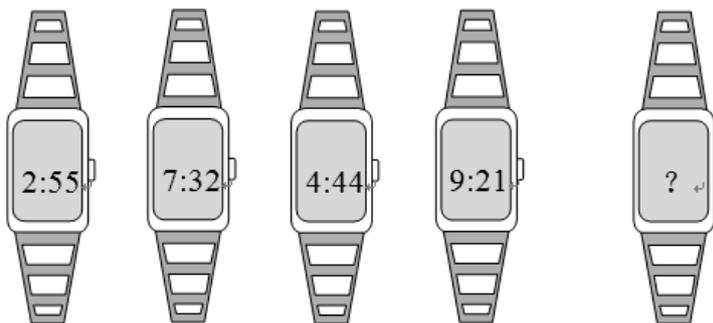
3. Anne ate half of a box of biscuits and then half of the remaining biscuits. If only 1 biscuit is left, how many biscuits were there originally?
 (A) 4 (B) 6 (C) 8 (D) 10 (E) 12

【Suggested Solution】

The last biscuit is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ of all the biscuits, there were $1 \div \frac{1}{4} = 4$ biscuits originally.

Answer : (A)

4. The sum of the digits displaced on each of the five watches is the same. Which of the following can be the time displayed on the last watch?



- (A) 6 : 29 (B) 8 : 31 (C) 9 : 37 (D) 10 : 00 (E) 11 : 57

【Suggested Solution】

The constant digit sum is 12, and only 8:31 has this property.

Answer : (B)

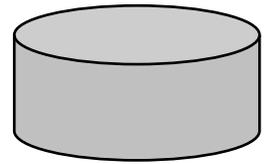
5. Eight pop bottles can be traded in for a pencil. What is the maximum number of pencils that can be obtained from 34 pop bottles?
 (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

【Suggested Solution】

Since 34 is more than 4 times 8 but less than 5 times 8, at most 4 pencils can be obtained.

Answer : (D)

6. What is the minimum number of straight cuts required to divide a cylindrical cake into eight identical pieces?
 (A) 1 (B) 2 (C) 3 (D) 4 (E) 5



【Suggested Solution】

Since each straight cut can at most double the number of pieces, 3 cuts are required. These are sufficient as a third cut can be along the plane halfway between the top and the bottom of the cylinder.

Answer : (C)

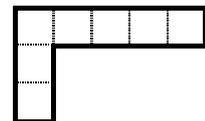
7. The product of two two-digit multiples of 10 is 2000. What is their sum?
 (A) 120 (B) 105 (C) 100 (D) 90 (E) 80

【Suggested Solution】

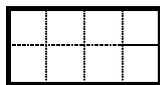
If we divide the two numbers by 10, they become single-digit numbers with product $20 = 4 \times 5$. Hence their sum is $40 + 50 = 90$.

Answer : (D)

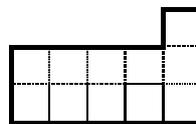
8. Which of the following figures can be combined with the given one to form a square?



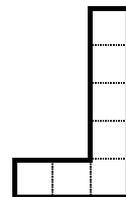
(A)



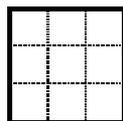
(B)



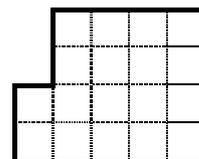
(C)



(D)



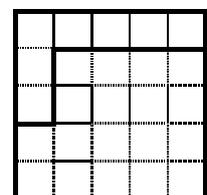
(E)



【Suggested Solution】

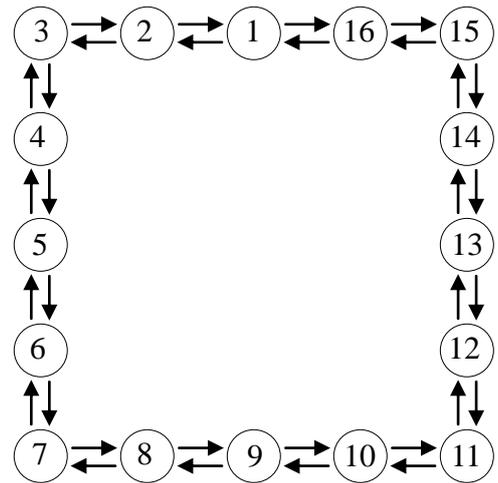
Of the five figures, the only one which can combine with the given one to form even a rectangle is (A) or (E), but the former yields a rectangle which is not a square.

Answer : (E)



9. The subway system of a certain city consists of 16 stations in a closed loop, with trains going in both directions. The fare is 1 dollar for a ride of 1 station, 2 dollars for a ride of 2 or 3 stations, 3 dollars for a ride of 4 or 5 stations, 4 dollars for a ride of 6 or 7 stations, and 5 dollars for a ride of 8 or more stations. What is the minimum cost for going from Station #2 to Station #14?

- (A) 1 (B) 2 (C) 3
 (D) 4 (E) 5



【Suggested Solution】

We can go from Station #2 through Stations #1, #16, and #15 to get to Station #14. This ride of 4 stations costs 3 dollars.

Answer : (C)

10. Climbing the stairs, a boy takes 60 seconds to go from the 1st floor to the 4th floor. Assuming that the distance between consecutive floors is the same and the boy continues to climb at the same uniform speed as before, how many more seconds will he take to reach the 8th floor?

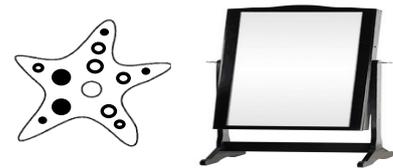
- (A) 60 (B) 80 (C) 100 (D) 110 (E) 120

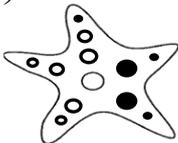
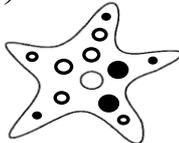
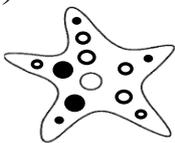
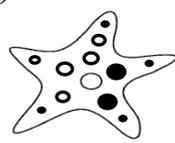
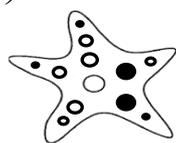
【Suggested Solution】

Going from the 1st floor to the 4th floor requires the climbing of 3 floors. Thus the climbing of each floor requires $60 \div 3 = 20$ seconds. Going from the 4th floor to the 8th floor requires the climbing of 4 floors. Hence the number of additional seconds required is $20 \times 4 = 80$ seconds.

Answer : (B)

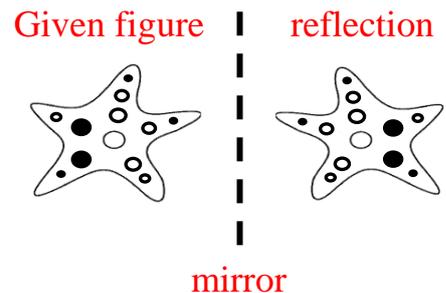
11. Which of the following figures is the mirror reflection of the given figure?



- (A)  (B)  (C)  (D)  (E) 

【Suggested Solution】

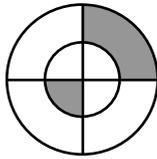
In the given figure, the “head” leans to the right, and the “right arm” have small black dots at the tip. Figures (A), (C) and (E) are the only one leaning to the left, and Figures (E) are the only one with the “left arm” have small black dots at the tip.



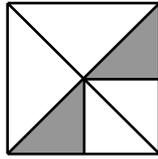
Answer : (E)

12. Of the following figures, which one **does not** have exactly $\frac{1}{4}$ of its area shaded?

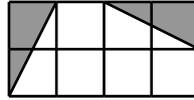
(A)



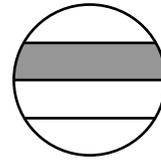
(B)



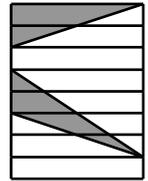
(C)



(D)



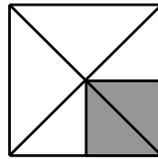
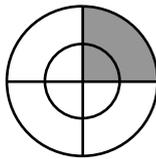
(E)



【Suggested Solution】

In Figure (D), the two strips in the middle have equal area, and are bigger than the strips on the outside. Hence the shaded area is greater than $\frac{1}{4}$ of the whole figure. It

is easy to verify that in each of the other figures, the shaded area is exactly $\frac{1}{4}$ of the whole figure.



Answer : (D)

13. A, B and C share 36 grapes. If A gives 10 to B and B then gives 8 to C, each has the same number of grape. How many more grapes than C did A have initially?



(A) 14

(B) 16

(C) 18

(D) 20

(E) 22

【Suggested Solution】

At the end, each has $36 \div 3 = 12$ grapes. Initially, A had $12 + 10 = 22$ grapes while C had $12 - 8 = 4$. Hence A had $22 - 4 = 18$ more grapes than C.

Answer : (C)

14. The total weights of three groups of four animals are recorded as shown. Each animal of the same type has the same weight. What is the weight of each sheep?

$$\begin{array}{ccccccccc}
 \text{Sheep} & + & \text{Sheep} & + & \text{Monkey} & + & \text{Tiger} & = & 20 \\
 \text{Sheep} & + & \text{Monkey} & + & \text{Monkey} & + & \text{Tiger} & = & 17 \\
 \text{Sheep} & + & \text{Monkey} & + & \text{Tiger} & + & \text{Tiger} & = & 15
 \end{array}$$

- (A) 2 (B) 4 (C) 5 (D) 7 (E) 9

【Suggested Solution】

Combining all three records, we see that the total weight of four animals of each type is $20 + 17 + 15 = 52$. Hence the total weight of one animal of each type is $52 \div 4 = 13$. It follows that each sheep weighs $20 - 13 = 7$.

Answer : (D)

15. A number is larger than another number by 416. The larger number is 9 times as large as the smaller number. What is the sum of these two numbers?

- (A) 468 (B) 500 (C) 520 (D) 530 (E) 572

【Suggested Solution】

Eight times the smaller number is 416, so that it is 52. The sum is ten times the smaller number. Hence it is 520.

Answer : (C)

16. Every student consumes water at a constant rate. If a jug containing 420 litres of water is just sufficient to supply 20 students for 7 days, for how many days can the same jug supply 28 students?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

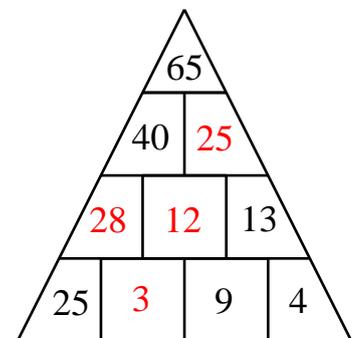
【Suggested Solution】

The number of days is inversely proportional to the number of students. Hence the number of days is $\frac{20}{28} \times 7 = 5$.

Answer : (C)

17. In the number triangle shown, every number not on the bottom row is equal to the sum of the two numbers below it. What is the number represented by the “?” mark?

- (A) 3 (B) 4 (C) 8
(D) 16 (E) 26



Answer : (A)

18. Three cups of equal value are the door prizes won by five people. Since the cups form a set, it is decided that a husband and wife pair among the five friends should get them. In return, the couple offers 1800 dollars to be shared equally by the other three friends. If each person has received an equal share, what is the value, in dollars, of each cup?

- (A) 900 (B) 1000 (C) 1200 (D) 1300 (E) 1500

【Suggested Solution】

Each equal share is worth $1800 \div 3 = 600$ dollars, so that the total worth of the three cups is 3000 dollars. It follows that each cup is worth $3000 \div 3 = 1000$ dollars.

Answer : (B)

19. What is the maximum value of the expression $() \times () + () - ()$ if a different number chosen from 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 is used to fill each bracketed space?

- (A) 81 (B) 88 (C) 91 (D) 97 (E) 99

【Suggested Solution】

Clearly, the maximum value is achieved by maximizing first the product and then the sum, and finally subtracting the smallest possible value. Hence this value is $9 \times 10 + 8 - 1 = 97$.

Answer : (D)

20. How many of the following statements are correct?

- A : The sum of your age on your birthday in 2015 and your birth year must be 2015.
 B : If today is Tuesday, then 217 days later it will again be Tuesday.
 C : If we multiply 2015 by 8888 and then add 2015, the sum must be odd.
 D : If the sum of the ages of three people is odd, then the sum of their ages a year later must be even.

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

【Suggested Solution】

All four statements are obviously correct.

Answer : (E)

21. What is the total number of digits used to write down the first 31 positive integers?

【Suggested Solution】

Each of the 31 numbers has a units digit. Each of 22 of them also has a tens digit. Hence the total number of digit is $31 + 22 = 53$.

Answer : 053

22. There are tokens of weight 1 gram and tokens of weight 4 grams. At least how many tokens must be used in order to balance an object of weight 103 grams, if tokens may also be placed on the same pan as the object?

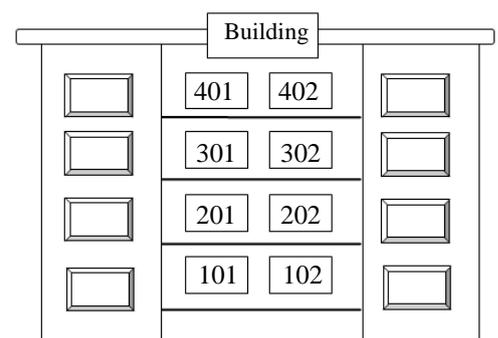


【Suggested Solution】

We must use 25 tokens of weight 4 grams just to reach 100 grams. We may add 3 tokens of weight 1 gram, bringing the total to 28. However, we can save 1 token by using 26 tokens of weight 4 grams while placing a token of weight 1 gram with the object. Thus the minimum number is 27.

Answer : 027

23. There are two classrooms on each of four floors in the school building, x01 and x02 where x is the floor number. Martial Arts and Drama are on the first floor. Calligraphy and Fine Arts are on the same floor. Music is directly above Fine Arts and Painting is directly above Calligraphy. Modeling is directly above Painting. Neither Martial Arts nor Music is in an odd-numbered room. There is a



room for Dancing, what is its room number ?

【Suggested Solution】

Modeling, Painting and Calligraphy are in the same room on the fourth, third and second floors respectively, all in odd-numbered classrooms since Music is not. Now Music and Fine Arts are on the third and second floors respectively. It follows that Dancing is in classroom 402.

Building	
401 Modeling	402 Dancing
301 Painting	302 Music
201 Calligraphy	202 Fine Arts
101 Martial Arts	102 Drama

Answer : 402

24. During a certain period, each morning and afternoon is either wet or dry. The total number of wet mornings and wet afternoons is 9. There are 8 dry mornings and 7 dry afternoons. A wet afternoon always follows a dry morning. In how many days during this period are both morning and afternoon dry?

【Suggested Solution】

The total number of mornings and afternoons is $9 + 7 + 8 = 24$, so that there are 12 of each. Since the 4 wet mornings and 5 wet afternoons must occur in 9 different days, the number of days in which both the morning and the afternoon are dry is 3.

	1	2	3	4	5	6	7	8	9	10	11	12
Morning	dry	wet	wet	wet								
Afternoon	wet	wet	wet	wet	wet	dry						

Answer : 003

25. A standard deck of cards consists of two Jokers and thirteen cards in each of the four suits: Spades ♠, Hearts ♥, Diamonds ♦ and Clubs ♣. What is the minimum number of cards one must draw at random to ensure that six cards of the same suit are drawn?



【Suggested Solution】

Without six cards of the same suit, the most we can draw are 22 cards, consisting of both Jokers and five cards of each suit. When we draw 23 cards, we must have six cards of the same suit.

Answer : 023