Solution Key to Second Round of IMAS 2015/2016
Middle Primary Division

1. The operation * is such that $2*6=26-2-6=18$ and $7*3=73-7-3=63$. If a and b are non-zero digits such that $a*b=36$, what is the value of a?

（A）3　（B）4　（C）6　（D）7　（E）9

【Suggested Solution】

From the examples, it is reasonable to conclude that $a*b=(10a+b)-a-b=9a$.

From $9a=36$, we have $a=4$.

Answer: （B）

2. In how many different ways can three identical balls be distributed among three distinct boxes, if we allow some boxes to be empty?

（A）6　（B）10　（C）12　（D）18　（E）27

【Suggested Solution】

Add three balls in, use six 0s to represent the balls and two 1s to represent the dividing lines separating the balls going into different boxes. The first 1 has 5 positions to choose from. Then the second 1 has 4 choices. However, since the two 1s are not distinguished, the total number of ways is $5\times4\div2=10$. After this, remove 1 ball from each box for allow some boxes to be empty.

Answer: （B）

3. The diagram shows the playing board of a game where we start on square 1. We move forward 2 squares and backwards 1 square. Then we move forward 3 squares and backwards 2 squares, and so on. Each time, we move forward one square more than the last time, and the backward move is one square shorter than the forward move. Moving from one square to the next, in either direction, is called a step. How many steps does it take to reach square 11 for the first time?

1	2	3	4	5	6	7	8	9	10	11

（A）30　（B）32　（C）34　（D）36　（E）38

【Suggested Solution】

After the first pair of moves, the farthest we reach is square 3. After each subsequent couple, we reach another two squares farther. Hence it takes $(11-3)\div2=4$ more couples of moves, except that in the last couple, we do not have to move back. Hence the total number of steps is $(2+1)+(3+2)+(4+3)+(5+4)+6=30$.

Answer: （A）

4. A beam balance has a 1 g token, a 4 g token and a 7 g token. Each token may be placed on the same pan as the object being weighed or on the other pan, or not at all. How many different positive integral weights can be measured exactly using this balance and the three tokens?

（A）8　（B）9　（C）10　（D）11　（E）12

【Suggested Solution】

Using only one token, it can balance 1, 4 and 7. Using two tokens, they can balance $1+4=5$, $1+7=8$, $4+7=11$, $4-1=7-4=3$ and $7-1=6$. Using all three tokens, they can balance $1+4+7=12$, $7-1-4=2$, $1+7-4=4$ and $7+4-1=10$. Note that 4 has been balanced before. Thus the total number of ways is 11.

Answer: （D）

5. There are three kinds of bottles, holding 0.4 L, 0.6 L and 1 L respectively. The total capacity of several bottles is 10 L. How many possible values of the number of bottles holding 0.6 L are there if there is at least one bottle of each kind?

（A）9　　（B）10　　（C）11　　（D）12　　（E）13

【Suggested Solution】

First take away one bottle of each kind, so that the relevant total capacity is now reduced to 8. If we use an equal number of 0.4 L bottles and 0.6 L bottles, the number of 0.6 L bottles can be any number from 0 to 8 inclusive. Since three 0.4 L bottles can be replaced by two 0.6 L bottles and we have up to eight 0.4 L bottles, enough for two replacements, the number of 0.6 L bottles can stretch to 8+2×2=12. Thus there are 13 possible values.

Answer: （E）

6. A three-digit number has digit-sum 18. The tens digit is 2 less than the hundreds digit and the units digit is 2 less than the tens digit. What is this three-digit number?

【Suggested Solution】

The tens digit must be $18\div3=6$ so that the three-digit number must be 864.

Answer: 864

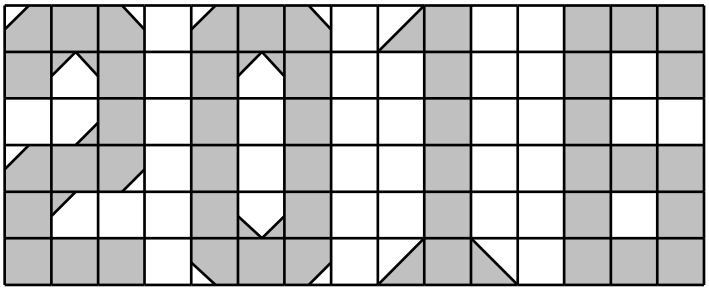
7. The diagram shows a paper strip with four possible dividing lines. We may cut along none, any or all of these lines. How many different sizes can the piece containing the circle have?

【Suggested Solution】

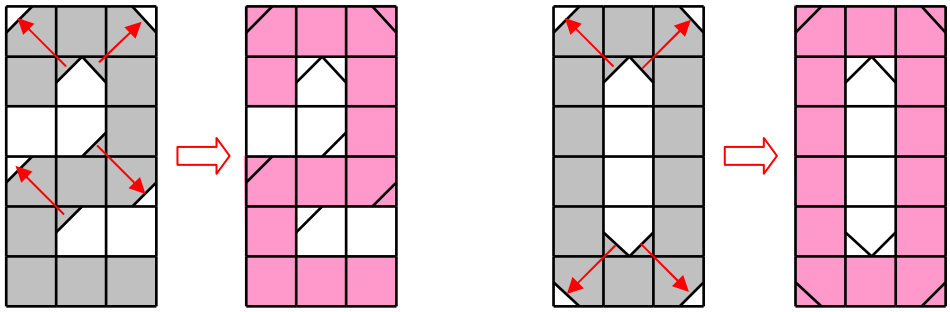
There are 3 dividing line, including the end of the strip, which can serve as the left edge of the piece containing the circle. The same can be said about the right edge. Hence the total number of different sizes is $3\times3=9$.

Answer: 9 sizes

8. The diagram shows a 15 by 6 board on which the number 2016 are shaded. Each square is 1 cm by 1 cm. Each long diagonal joins opposite vertices of a square while each short diagonal joins midpoints of adjacent sides of a square. What is the total area, in cm^2, of the shaded regions?



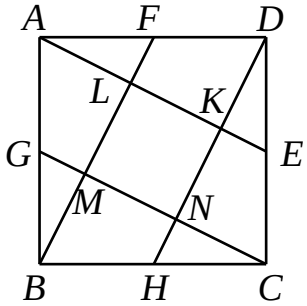
【Suggested Solution】



As shown in the diagram, both 2 and 0 can be transformed so that they consist of complete squares like 6. The respective numbers of squares are 13, 14 and 14, while 1 consists of 6 complete squares and 3 half-squares. The total area is 48.5 cm^2.

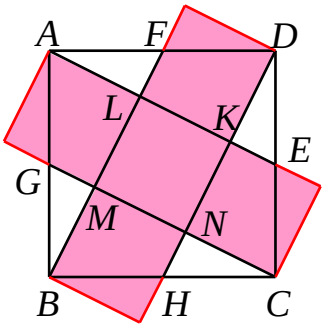
Answer: 48.5 cm^2

9. In the diagram, *E*, *F*, *G* and *H* are the respective midpoints of the sides *CD*, *DA*, *AB* and *BC* of a square *ABCD*. If the area of the square *KLMN* is 4 cm^2, what is the area, in cm^2, of *ABCD*?



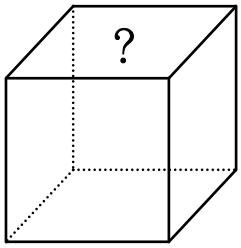
【Suggested Solution】

If we rotate triangle *BMG* about *G*, it can combine with *ALMG* to form a square. Similar transformations can be performed on the other sides of *ABCD*. Hence the area of *ABCD* is $4\times5=20\,cm^2$.



Answer: 20 cm^2

10. A cube is placed on the table, and a number is written on each face. The sum of the two numbers on opposite faces is always 16. Honey can see three faces, and the numbers on them add up to 24, and Lily can see the top face and the other two lateral faces, and the numbers on them add up to 26. What is the number on the top face?

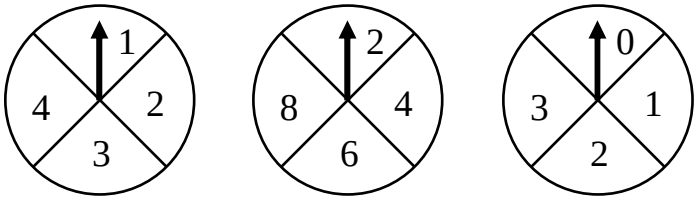


【Suggested Solution】

Between them, Honey and Lily see all four lateral numbers once and the top number twice. Since the sum of the four lateral numbers is $16\times 2=32$, the top number is $\frac{50-32}{2}=9$.

Answer: 9

11. The diagram shows three wheels of fortune, generating a three-digit number. The hundreds digit comes from the wheel on the left, the tens digit from the wheel in the middle, and the units digit from the wheel on the right. A prize is awarded if this three-digit number is divisible by 6. How many different winning numbers are there?



【Suggested Solution】

The units digit must be even. If it is 0, we may have 120, 180, 240, 360, 420 or 480. If it is 2, we may have 162, 222, 282, 342 or 462. The total is 11.

Answer: 11

12. Daniel finished reading a comic book in 16 days. Then Ray starts reading the same book, and gets to the halfway point of it in 5 less days. If Daniel reads 6 pages per day more than Ray, how many pages does this book have?

【Suggested Solution】

Ray can finish the book in $2\times(16-5)=22$ days. So in 176 days, Daniel can read the book 11 times while Ray can read it 8 times. Daniel has read $176\times 6=1056$ pages more than Ray. Hence the book has $1056\div 3=352$ pages.

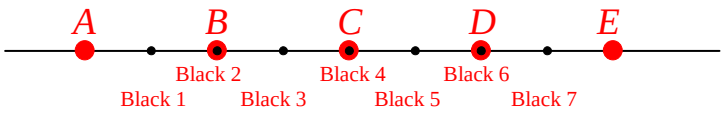
Answer: 352 pages

13. There are five different red points in the plane. The midpoint of each segment joining two red points is painted black. What is the minimum number of black points?

【Suggested Solution】

With three red points, we have 3 black points. When we add a fourth red point, consider the two red points closest to it. The two black points generated by them with the new red point must be new, bringing the total to 5. The same argument shows that when a fifth red point is added, we must have at least 7 black points. Now if the red points are at 1, 3, 5, 7 and 9 on the number lines, the black points are at 2, 3, 4, 5, 6, 7

and 8, and there are exactly 7 of them.



Answer: 7

14. In how many ways can the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 be divided into three groups of three such that the sum of the numbers in the second group is 1 less than the sum of the numbers in the first group but 1 more than the sum of the numbers in the third group?

【Suggested Solution】

The sum of all 9 numbers is 45. Hence the sums of the groups are 16, 15 and 14.
The second group must be one of (6, 5, 4), (7, 5, 3), (7, 6, 2), (8, 4, 3), (8, 5, 2), (8, 6, 1), (9, 4, 2) or (9, 5, 1). (2 points)
If it is (6, 5, 4), the first group may be (9, 6, 2) or (8, 7, 1). (2 points)
If it is (7, 5, 3), the first group must be (9, 6, 1). (2 points)
If it is (7, 6, 2), the first group may be (9, 4, 3) or (8, 5, 3). (2 points)
If it is (8, 4, 3), the first group may be (9, 6, 1) or (9, 5, 2). (2 points)
If it is (8, 5, 2), the first group may be (9, 6, 1), (9, 4, 3) or (7, 6, 3). (2 points)
If it is (8, 6, 1), the first group (9, 5, 2), (9, 4, 3) or (7, 5, 4). (2 points)
If it is (9, 4, 2), the first group may be (8, 7, 1), (8, 5, 3) or (7, 6, 3). (2 points)
If it is (9, 5, 1), the first group may be (8, 6, 2) or (7, 6, 3). (2 points)
The total number of ways is $1+2+2+2+3+3+3+2=18$. (2 points)

Answer: 18

15. Consider all integers between 100 and 999 inclusive such that the hundreds digit is equal to the units digit. If S is the sum of all such numbers, what is the digit-sum of S?

【Suggested Solution】

The total number of integers of this form is $9\times10=90$. (5 points) Form 45 pairs as follows. Consider 101. Its partner 999 is chosen where the tens digit is obtained by subtraction from 9 $(9-0=9)$ and the other digits are obtained by subtraction from 10 $(10-1=9)$. In this way,111 is paired with 989, 222 with 878, and so on. The sum of the two numbers in each pair is always 1100. (10 points)
Hence $S=45\times1100=49500$ and the digit-sum of S is 18. (5 points)

Answer: 18