Solution Key to Second Round of IMAS 2015/2016
Upper Primary Division

1. What is the value of $666+669+699+999$?

(A) 2433 (B) 2970 (C) 2973 (D) 3030 (E) 3033

【Suggested Solution】

$666+669+699+999=666+670+700+1000-3=3036-3=3033$ 。

Or direct computation yields $666+669+699+999=3033$.

Answer: (E)

2. The positive integers *a*, *b*, *c* and *d* are such that

$$\frac{1}{a-2013}=\frac{1}{b+2014}=\frac{1}{c-2015}=\frac{1}{d+2016}.$$

Which of the following orderings of these four numbers is correct?

(A) $b<d<a<c$ (B) $d<b<a<c$ (C) $d<a<b<c$ (D) $d<b<c<a$ (E) $b<d<c<a$

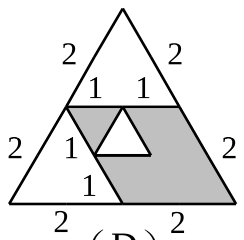
【Suggested Solution】

From $a-2013=b+2014=c-2015=d+2016$, we see clearly that $d<b$ and $a<c$. Also, $a>a-2013=b+2014>b$, hence $d<b<a<c$ 。

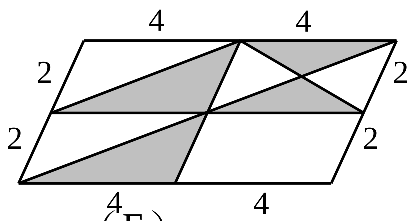
Answer: (B)

3. In which of the following diagrams is the total area of the shaded parts equal to the total area of the unshaded parts?

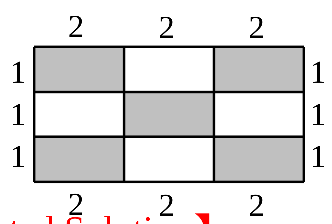
(A)



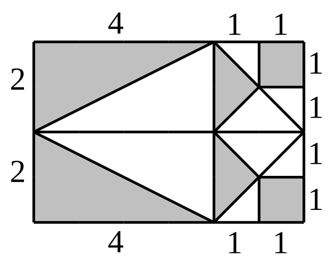
(B)



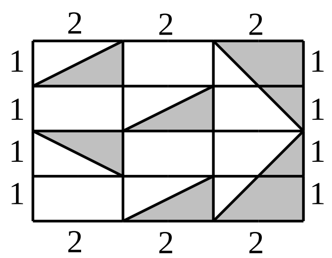
(C)



(D)



(E)

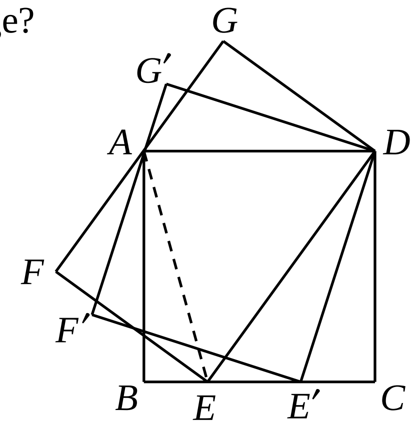


【Suggested Solution】

Figure (B) has an extra white parallelogram. Figure (C) has an extra black rectangle. Figure (E) has four extra white rectangles. Figure (A) has two white equilateral triangle and one black equilateral triangle of equal sizes, and a fourth one of the same size but partly white and partly black. Hence the only possible answer is Figure (D), and this is in fact the case.

Answer: (D)

4. E is a variable point on the side BC of a square $ABCD$. $DEFG$ is a rectangle with FG passing through A. As the point E moves from B towards C, how does the area of $DEFG$ change?



(A) Remaining constant　(B) Steadily increasing　(C) Steadily decreasing　(D) Increasing and then decreasing　(E) Decreasing and then increasing

【Suggested Solution】

The area of triangle ADE is constant since it has a fixed base AD and a constant altitude. The area of $DEFG$ is always twice the area of ADE since they have a common base DE and the same altitude.

Answer: (A)

5. Jerry and George are jogging along a circular path. If Jerry runs another 400 m, he will have completed 2 laps. If George runs another 500 m, he will have completed 3 laps. The total distance they have covered is 100 m more than 4 laps. What is the length, in m, of 1 lap?

(A) 1000　(B) 900　(C) 800　(D) 750　(E) 700

【Suggested Solution】

Since 2+3=5 laps minus 400+500=900 m is equal to 4 laps plus 100 m, each lap is 900+100=1000 m.

Answer: (A)

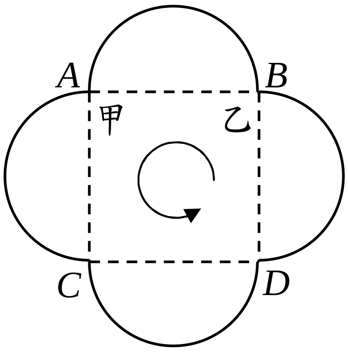
6. A worker makes 6000 dollars in basic wages plus overtime payment. His overtime payment is two-thirds of his basic wage. How much, in dollars, is his basic wage?

【Suggested Solution】

Suppose the wage is 3 dollars. Then the overtime is 2 dollars and the total income would have been 3+2=5 dollars. Since 6000÷5=1200, the wage is 1200×3=3600 dollars.

Answer: 3600 dollars.

7. The diagram shows a path consisting of four semi-circular arcs. Each arc is of length 100 m and uses a different side of a square as its diameter. Initially, Jane is at A and Yves at B. They start walking counter-clockwise at the same time. Jane's speed is 120 m per minute and Yves's is 150 m per minute. Each pauses for 1 second whenever they are at the points A, B, C or D. How many seconds after starting will Yves catch up for the first time with Jane?



【Suggested Solution】

When Yves catches up with Jane, she has turned one more corner and hence run 1 second less. In 1 second, Jane covers 120÷60=2 m. So Yves has to make up 102 m. Since she gains 150-120=30 m per minute or 0.5 m per second of running time, Jane has been running for 204 seconds. During this time, she has covered 2×204=408 m, which means she has turned 5 corners. Hence the desired time is 204+5=209 seconds.

Answer: 209 seconds

8. There are three kinds of bottles, holding 0.4 L, 0.6 L and 1 L respectively. The total capacity of several bottles, at least one each kind, is 18 L. How many possible values of the number of bottles holding 0.6 L are there if there is at least one bottle of each kind?

【Suggested Solution】

First take away one bottle of each kind, so that the relevant total capacity is now reduced to 16. If we use an equal number of 0.4 L bottles and 0.6 L bottles, the number of 0.6 L bottles can be any number from 0 to 16 inclusive. Since three 0.4 L bottles can be replaced by two 0.6 L bottles and we have up to sixteen 0.4 L bottles, enough for five replacements, the number of 0.6 L bottles can stretch to16+5×2=26. Thus there are 27 possible values.

Answer: 27

9. Marion chooses three different non-zero digits and form all possible three-digit numbers with them. If m is the sum of these numbers and n is the sum of the digit-sums of these numbers, what is the value of $\frac{m}{n}$?

【Suggested Solution】

Each of the three digits is used 6 times, 2 times as the hundreds digit, 2 times as the tens digit and 2 times as the units digit. Hence the sum of the six three-digit numbers is 2×(100+10+1)=222 times the sum of the three digits. The sum of the digit-sums of these numbers is 6 times the sum of the three digits. Hence $\frac{m}{n}=37$.

Answer: 37

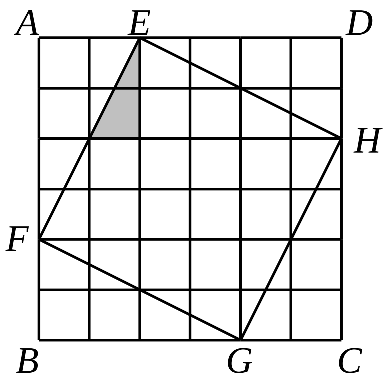
10. How many of the integers from 100 to 999 inclusive have the property that the sum of the units digit and the hundreds digit is equal to the tens digit?

【Suggested Solution】

The tens digit can be any except 0. The units digit may be any digit less than the tens digit. Hence the total number is $1+2+3+4+5+6+7+8+9=45$.

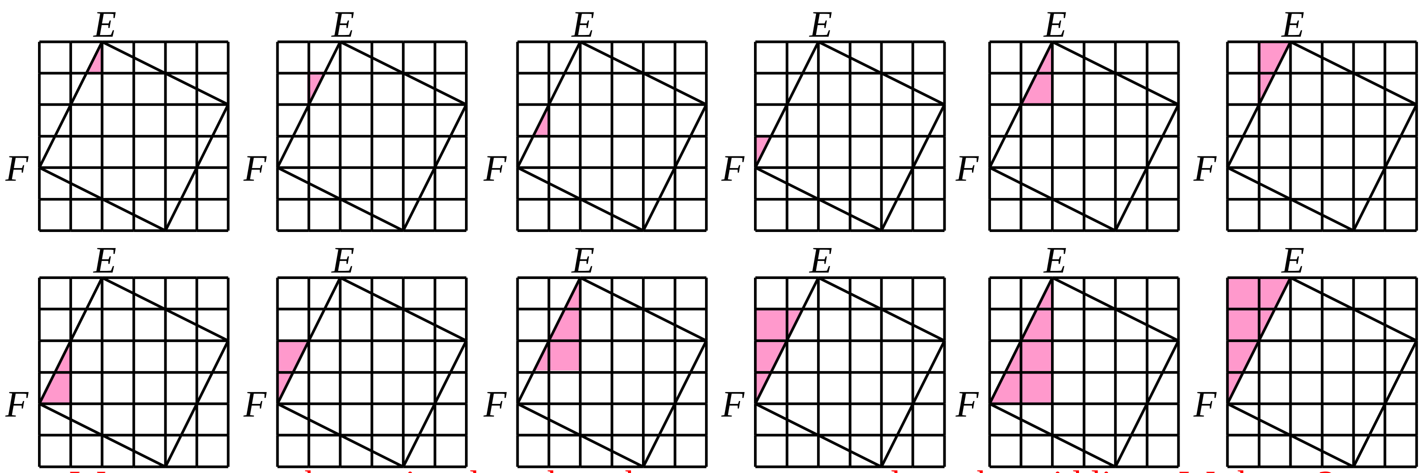
Answer: 45

11. The diagram shows a shaded triangle in a 6 by 6 board. How many triangles are there such that their edges are all grid lines of the board or the edges of *EFGH*, and their angles are equal respectively to the angles of the shaded triangle? You should also count the shaded triangle.

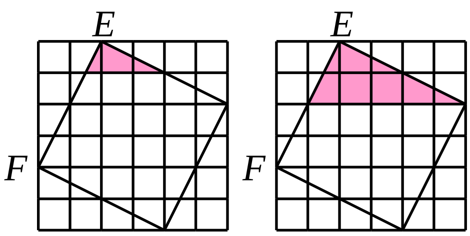


【Suggested Solution】

We first count those triangles whose hypotenuses are along the sides of *EFGH*. Each region outside *EFGH* has 6 lattice points, each of which can serve as the right-angle vertex of such a triangle. By symmetry, there are 24 such triangles outside *EFGH*, and by symmetry again, there are also 24 such triangles inside *EFGH*.



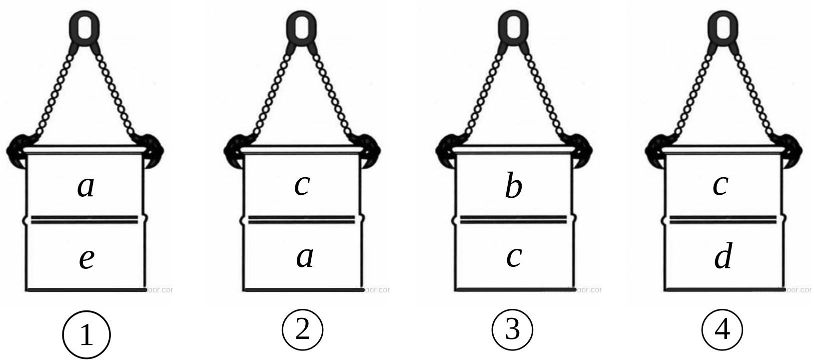
We now count those triangles whose hypotenuses are along the grid lines. We have 2 with right-angle vertex at E, and by symmetry 8 in all.



The total count Is 24+24+8=56.

Answer: 56

12. There are 26 toys be distributed into five boxes, with a, b, c, d and e toys in them respectively, where a, b, c, d and e are positive integers. The diagram shows four combinations of two boxes at a time. The total number of toys in the two boxes exceeds 11 in all but the second case. How many different distributions are there?

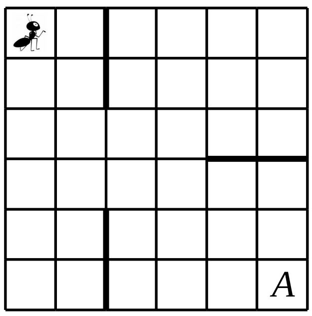


【Suggested Solution】

Let a, b, c, d and e be the respective number of toys in these five boxes.
In the three combinations in which the numbers of toys exceed 11, each box appears once, except that the one containing c toys appears twice. The total number of toys in these six boxes is at least $3\times(11+1)=36$. It follows that c is at least 10. Since $a+c$ is at most 11, we must have $a=1$ and $c=10$, so that $b+d+e=15$. Now $a+e$ is at least 12. Hence e is at least 11 and $b+d$ is at most 4. Both $b+c$ and $c+d$ are also at least 12, so that each of b and d is at least 2. It follows that we must have $b=d=2$ and $e=11$. This is the only possible distribution.

Answer: 1 distribution

13. The diagram shows a 6 by 6 board with three barriers. An ant is at the top left corner and wishes to reach the bottom right corner. It may only crawl between squares which share a common side, and only towards the bottom or the right, It cannot pass through any barrier. How many different paths can it follow?



【Suggested Solution】

The diagram below shows the number of ways the ant can reach each square on the board according to the rules.

1	1	0	0	0	0
1	2	0	0	0	0
1	3	3	3	3	3
1	4	7	10	10	10
0	0	7	17	27	37
0	0	7	24	51	88

Answer: 88 paths

14. What is the largest integer n such that there is a multiple of 4 less than n^2 but greater than $n^2+\frac{2016}{n^2}$?

【Suggested Solution】

If n is even, then n^2 itself is a multiple of 4, and $\frac{2016}{n^2}$ must be greater than 4. (5 points) If n is odd, then n^2 is 1 more than a multiple of 4, and $\frac{2016}{n^2}$ must be greater than 3. (5 points)

Now 2016÷3=672 and the largest square below 672 is 625. (5 points) Hence n is at most 25.

For n=25, the next multiple of 4, namely 628, falls in the required range. Hence the maximum value of n is 25. (5 points)

Answer: 25

15. Each of the integers from 1 to 16 inclusive is put in a different square of a 4 by 4 table. For any two squares sharing a common side, the sum of the numbers in them is recorded. What is the maximum value of the smallest one among the recorded numbers?

【Suggested Solution】

The diagram shows that the minimum sum can be as large as 15. (6 points) We now prove that it cannot be larger. If 1 is not in a corner, then it has at least three neighbors. One of them is at most 14, so that the minimum sum is at most 1+14=15. (6 points) Hence we may assume that 1 is at a corner and has two neighbors. Clearly, 2 should not be a neighbor of 1. Then 2 also has at least two neighbors. One of these four is at most 13, so that the minimum is at most 2+13=15. (8 points)

1	16	6	9
15	2	13	7
3	14	5	10
12	4	11	8

Answer: 15