International Mathematics Assessments for Schools

2016 JUNIOR DIVISION FIRST ROUND PAPER

Time allowed：75 minutes

When your teacher gives the signal, begin working on the problems.

INSTRUCTION AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. No calculators, slide rules, log tables, math stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 20 multiple-choice questions, each with 5 choices. Choose the most reasonable answer. The last 5 questions require whole number answers between 000 and 999 inclusive. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a mathematics assessment, not a test; do not expect to answer all questions.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.

THE ANSWER SHEET

1. Use only pencils.
2. Record your answers on the reverse side of the Answer Sheet (not on the question paper) by FULLY filling in the circles which correspond to your choices.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places. So please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The IMAS reserves the right to re-examine students before deciding whether to grant official status to their scores.

Questions 1-10, 3 marks each

1. What is the value of $\sqrt{(-20)^2 + 16^2 - 15^2}$?
   （A）−19　　　（B）11　　　（C）21　　　（D）51　　　（E）61

2. The table below summarizes the results of a test in a certain class. What is the total score of this class?

Summary of the results of a test			
No. of students	The highest score	The lowest score	The average score
42	100	16	84.5

   （A）672　　（B）3528　　（C）3549　　（D）4200　　（E）4872

3. A three-digit number is not divisible by 24. When divided by 24, the quotient is a and the remainder is b. What is the minimum value of $a+b$?
   （A）5　　　（B）6　　　（C）7　　　（D）8　　　（E）9

4. In the trapezium *ABCD*, *AB* is parallel to *CD*. *E* and *F* are points on *AD* and *BC* respectively such that *EF* is also parallel to *AB*. The area, in cm^2, of triangles *BAF*, *CDF* and *BCE* are 8, 7 and 18 respectively. What is the area, in cm^2, of *ABCD*?
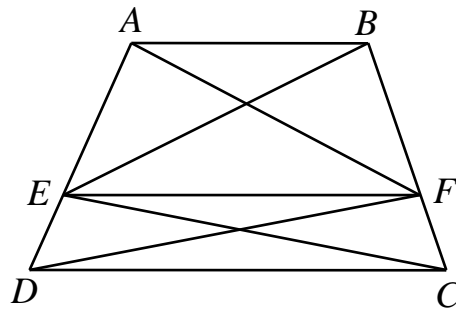

   （A）30　　（B）32　　（C）33　　（D）35　　（E）36

5. What is the value of the negative number *x* which satisfies $|x-3| = |3x|+1$?

   （A）−2　　（B）−1　　（C）$-\dfrac{2}{3}$　　（D）$-\dfrac{1}{2}$　　（E）$-\dfrac{1}{4}$

6. The radius of each wheel of Rick's bicycle is 25 cm. He rides to school at a constant speed and arrives after 10 minutes. During this time, each wheel makes 160 revolutions per minite. Of the following five distances, which is closest to that between Rick's home and school?
    （A）1 km   （B）1.5 km   （C）1.8 km   （D）2 km   （E）2.5 km

7. How many two-digit numbers are there such that at least one digit is divisible by 3?
    （A）48   （B）54   （C）60   （D）66   （E）80

8. The chart below shows the sale figures of a certain merchandise in 2014 and 2015 by the season. How many more items were sold in 2015 than in 2014?

Sales charts of a merchandise

items
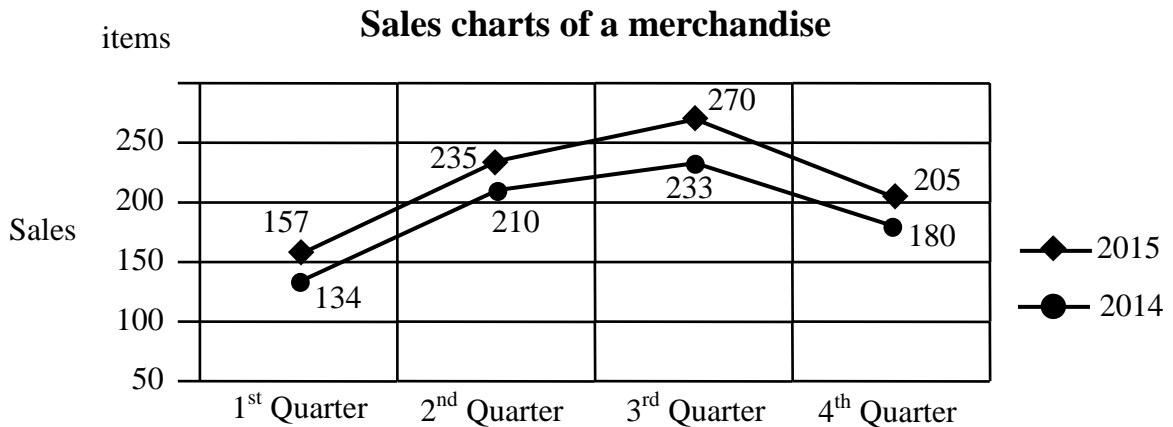
（A）23   （B）48   （C）85   （D）90   （E）110

9. *ABC* is an equilateral triangle. *D* is a point inside such that *BCD* is a right isosceles triangle. The altitude *BE* of *ABC* intersects *CD* at *F*. What is the measure, in degrees, of ∠*CFE* ?
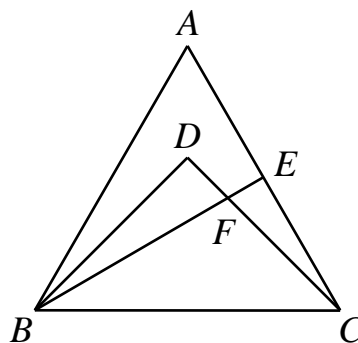
    （A）75°   （B）70°   （C）65°   （D）60°   （E）55°

10. In how many ways can 36 be expressed as the sum of two prime numbers, the first larger than the second?
    （A）1   （B）2   （C）3   （D）4   （E）5

Questions 11-20, 4 marks each

11. Every student in a class is either in the mathematics club or the language club, and one third of them are in both. If there are 22 students in the language club, 4 less than the number of students in the mathematics club, how many students are there in this class?
    （A）12　　　（B）18　　　（C）24　　　（D）30　　　（E）36

12. The average of a group of numbers is 5. A second group contains twice as many numbers and its average is 11. What is the average when the two groups are combined?
    （A）6　　　（B）7　　　（C）8　　　（D）9　　　（E）10

13. What is the value of x^y if $\sqrt{x-1}+\sqrt{1-x}+y=2016$?

    （A）2015　　（B）2016　　（C）$\dfrac{1}{2016}$　　（D）1　　　（E）0

14. Each of A and B goes to the gymnasium 3 or 4 times a week. After n weeks, A has been there 57 times while B has been there only 47 times. What is the value of n?
    （A）15　　　（B）16　　　（C）17　　　（D）18　　　（E）19

15. D is a point on AB such that $AD=1$ and $BD=2$. How many points C are there in the plane such that both ACD and BCD are isosceles triangles?
   （A）2　　　（B）4　　　（C）5　　　（D）6　　　（E）8

16. From a 5×5 square piece of paper, two 2×4 rectangles are cut off along the grid lines. In how many different ways can this be done?
    （A）6　　　（B）9　　　（C）12　　　（D）18　　　（E）24

17. The number a is 5 more than its reciprocal. What is the value of $(a^2-1)^2-125a$?

    （A）5　　　（B）25　　　（C）125　　　（D）$\dfrac{1+\sqrt{21}}{2}$　　（E）$5\sqrt{21}$

18. With each vertex of a parallelogram *ABCD* as centre, a circle is drawn. Exterior common tangents are then drawn, as shown in the diagram below. If the perimeter of *ABCD* is 36 cm and the radius of each circle is 2 cm, what is the maximum area, in cm^2, of the figure enclosed by the circular arcs and tangents?
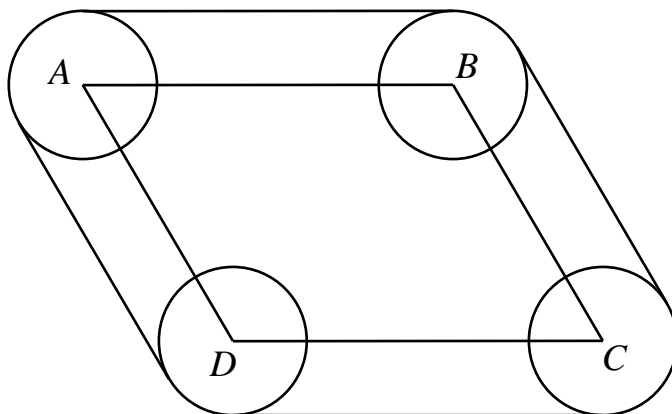
(A) $117 + 4\pi$　(B) $144 + 4\pi$　(C) $153 + 4\pi$　(D) $144 + 12\pi$　(E) $153 + 12\pi$

19. What is the smallest positive integer with 12 positive divisors such that it is relatively prime to $(2016^3 - 2016)$?
    (A) 7007　　(B) 9163　　(C) 26741　　(D) 39083　　(E) 52877

20. At most how many right triangles can be formed by five lines on the plane?
   (A) 4　　　(B) 5　　　(C) 6　　　(D) 7　　　(E) 8

Questions 21-25, 6 marks each

21. The International Article Number has 13 digits *ABCDEFGHIJKLM*. Here *M* is a check digit. Let $S = A + 3B + C + 3D + E + 3F + G + 3H + I + 3J + K + 3L$. If *S* is a multiple of 10, then *M* is chosen to be 0. Otherwise it is chosen to be $M = 10 - t$ where *t* is the remainder obtained when *S* is divided by 10. The Code for a certain Article Number is 6901020□09017. What is the missing digit?

22. What is the largest three-digit number which can be expressed as the sum of the cubes of three different positive integers?

23. The diagram shows a quadrilateral *ABCD* with $\angle CDA = 150°$. The bisector of $\angle DAB$ is perpendicular to *BC* and the bisector of $\angle ABC$ is perpendicular to *CD*. What is the measure, in degrees, of $\angle BCD$?
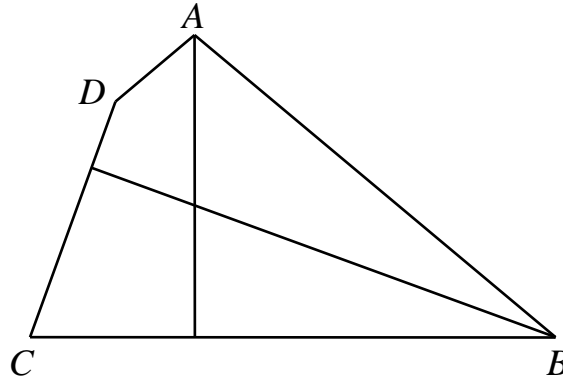
24. Let a and b be positive real numbers such that $a^2 = b(b+1)$ and $b^2 = a+1$. What is the value of $\dfrac{1}{a} + \dfrac{1}{b}$?

25. Each blouse cost 40 dollars, each skirt 70 dollars and each pair of shoes 80 dollars. Fanny bought at least one item of each kind, and spent at most 800 dollars. A outfit consisted of one item of each kind, and two outfits were different if they differed in at least one item. At most how many different outfits could there be?

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