International Mathematics Assessments for Schools

2017 JUNIOR DIVISION FIRST ROUND PAPER

Time allowed：75 minutes

When your teacher gives the signal, begin working on the problems.

INSTRUCTION AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. No calculators, slide rules, log tables, math stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 20 multiple-choice questions, each with 5 choices. Choose the most reasonable answer. The last 5 questions require whole number answers between 000 and 999 inclusive. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a mathematics assessment, not a test; do not expect to answer all questions.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.

THE ANSWER SHEET

1. Use only pencils.
2. Record your answers on the reverse side of the Answer Sheet (not on the question paper) by FULLY filling in the circles which correspond to your choices.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places. So please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The IMAS reserves the right to re-examine students before deciding whether to grant official status to their scores.

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Questions 1-10, 3 marks each

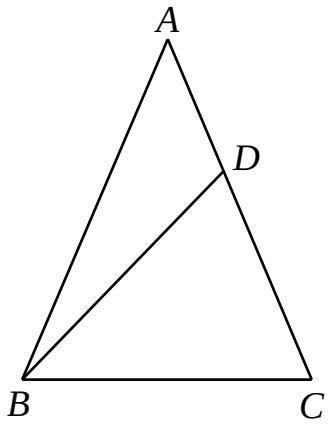
1. What is the simplified value of $\sqrt{(-18)^2} - 1^{2016} - (-1)^{2017}$?
   （A）−20　（B）−18　（C）0　（D）16　（E）18

2. Which of the following numbers is the sum of four consecutive positive integers?
   （A）2016　（B）2017　（C）2018　（D）2019　（E）2020

3. In a supermarket, 3 kg of pear costs $16.26, while 2 kg of apple costs $13.62. How much does 1 kg of apple cost more than 1 kg of pear?
   （A）0.61　（B）1.39　（C）1.42　（D）1.81　（E）2.64

4. The value of the faction $\frac{m}{n}$ increases by 1 when the numerator increases by 2017. Find the value of n.
   （A）1　（B）2016　（C）2017　（D）2018　（E）Uncertain

5. In the figure below, triangle ABC is an isosceles triangle with $AB = AC$. Point D is on AC with $BD = BC$. If $\angle ABD = 21°$, what is the measure, in degrees, of $\angle BAC$?



   （A）21　（B）38　（C）42　（D）46　（E）54

6. What is the sum of all the prime divisors in the final result of $2^3 + 0^3 + 1^3 + 7^3$ (repeated prime divisors are counted only once)?
   （A）7　（B）12　（C）13　（D）16　（E）64

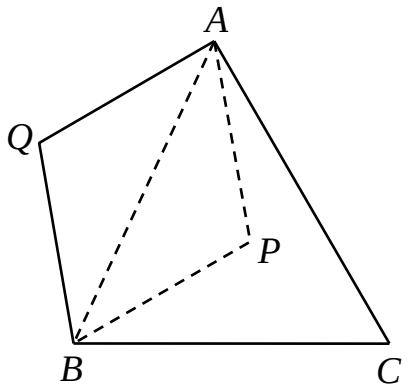
7. Let *a*, *b*, *c*, *d*, *e* and *f* are distinct digits such that the expression $\overline{ab}+\overline{cd}=\overline{ef}$. What is the least possible value of $\overline{ef}$?

（A）30　（B）34　（C）36　（D）39　（E）41

8. How many integers *x* satisfy $|2x+1|\le 8$?

（A）3　（B）4　（C）6　（D）8　（E）9

9. In the figure below, *AQBC* is a convex quadrilateral with $QA=QB$ and $\angle C=60°$. Triangle *QAB* is folded along *AB* to form triangle *PAB*. It is also known that $\angle PBC=30°$ and $\angle PAC=20°$. What is the measure, in degrees, of $\angle AQB$?



（A）100　（B）110　（C）120　（D）130　（E）140

10. If all the edges of a rectangle are integers, then which of the following **CANNOT** be the length of its diagonals?

（A）5　（B）6　（C）$\sqrt{41}$　（D）$\sqrt{53}$　（E）10

Questions 11-20, 4 marks each

11. The length of 2 altitudes on adjacent sides of a parallelogram are 2 cm and 3 cm respectively with perimeter of the parallelogram is 18 cm. What is the area, in cm^2, of the parallelogram?

（A）9.6　（B）10　（C）10.5　（D）10.8　（E）12

12. Three students are to participate in four games. Each student participates in at least one game and each game has exactly one student participating. In how many ways can this be done?

（A）12　（B）18　（C）24　（D）30　（E）36

13. The sum of a, b and c is negative, while the product of these three numbers is positive. If $x=\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}$, what is the value of $x^{2017}-2017x^2+36$?

（A）−1982　（B）−1981　（C）−1980　（D）1980　（E）1982

14. Given two positive integers, $\frac{4}{7}$ of the first is exactly $\frac{2}{5}$ of the second. What is the minimum sum of these two integers?

（A）10　（B）14　（C）15　（D）35　（E）17

15. Tom starts working at 9∶00 in the morning and finishes at 5∶00 in the afternoon. How many more degrees does the minute hand rotates than the hour hand does on the clock during this period?

（A）120　（B）1200　（C）1320　（D）2640　（E）2880

16. The price criteria of the subway ticket of a city is as follows: \$2 for within 4 km, \$1 more per 4 km for distances between 4 km and 12 km, \$1 more per 6 km for distances over 12 km. It costs \$8 to take subway from station *A* to station *B*. Which of the following is closest to the distance between *A* and *B*?

（A）12 km　（B）18 km　（C）24 km　（D）36 km　（E）48 km

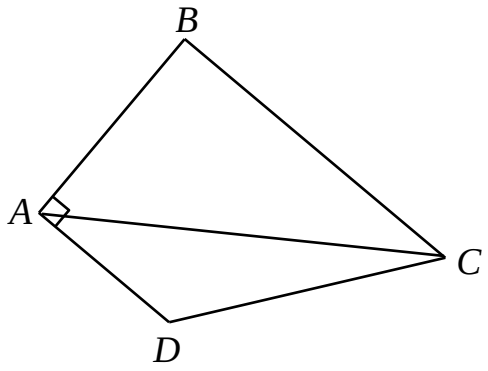
17. Consider a 3-digit number $\overline{abc}$, where a, b and c are distinct digits, so that their sum is 7. How many such three-digit numbers are there?

（A）6　（B）12　（C）14　（D）18　（E）22

18. If positive numbers x and y satisfy $x^2-y^2=2$. What is the simplified value of $x\sqrt{2+y^2}-y\sqrt{x^2-2}$?

（A）2　（B）$2\sqrt{2}$　（C）4　（D）$4\sqrt{2}$　（E）Uncertain

19. In the figure below, $ABCD$ is a quadrilateral where $AB = 4$ cm, $BC = 6$ cm, $CD = 5$ cm, $DA = 3$ cm and $\angle BAD = 90°$. Find the length, in cm, of AC.



(A) 5 (B) 7 (C) $\sqrt{34}$ (D) $3\sqrt{5}$ (E) $2\sqrt{13}$

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20. If $a = \frac{1}{3}$ and $b = \frac{1}{4}$, find the numerical value of $a^3 + b^3 - a^2b - ab^2$.

(A) $\frac{7}{1728}$ (B) $\frac{7}{1718}$ (C) $\frac{5}{1718}$ (D) $\frac{5}{1728}$ (E) $\frac{5}{1628}$

Questions 21-25, 6 marks each

21. Let a, b and c be positive integers satisfy $a^2 + bc = \frac{19}{a} + b + c$. Find the sum of a, b and c.

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22. The operation 「$\otimes$」 satisfies:

(i) For all x and y, $x \otimes y = (x-1) \otimes (y-1) + x + y$;

(ii) For all x, $x \otimes 1 = 1$.

Find the value of $(3 \otimes 3) \otimes 3$.

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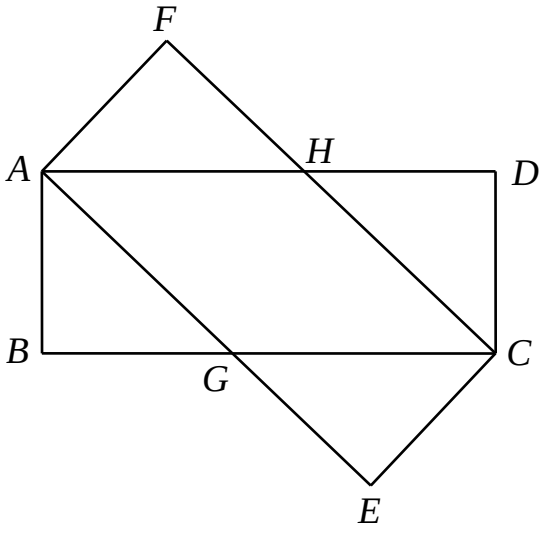
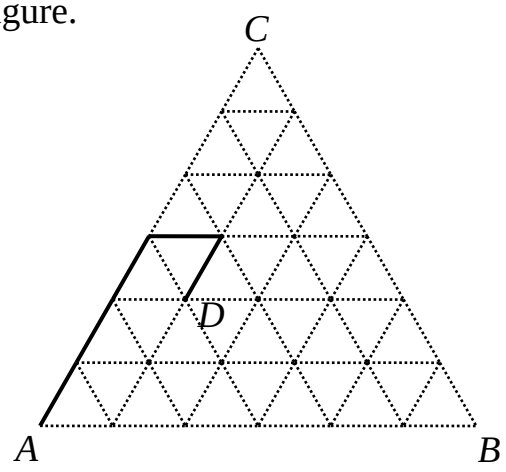
23. In the figure below, *ABCD* and *AFCE* are congruent rectangles with $AB = AF = 20\,\text{cm}$ and $AD = AE = 50\,\text{cm}$. Find the area, in cm^2, of *AGCH*.



24. In the figure below, the side length of equilateral triangle *ABC* is 6 cm. Each side is divided into 6 equal segments and connects corresponding dividing points to get an equilateral network. Call a point "reachable" if it can be connected to *A* by a broken line of length 5 cm along the grid lines without passing any lattice point twice. For example, point *D* in the figure is reachable. Find the number of reachable points in the figure.



25. The students in a research class are clustered into two groups: the morning and afternoon sessions. A student takes part in exactly one group in each session (the two groups in each session can be different and the number of students in each group can be different). Each group has at least one student and at most 8 students. Each student reports the number of students in the group he or she belongs to in two sessions. One finds that no two students report the same pair of numbers (with order, for example, (1, 4) and (4, 1) are different). What is maximum number of students in the class?

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