Solution to
Sixth International Mathematics Assessment for Schools
Round 1 of Middle Division

1. What is the simplified value of $25 \times 99 + 55 \times 5$?

    (A) 2750   (B) 2850   (C) 2900   (D) 2950   (E) 3000

【Solution】

$25 \times 99 + 55 \times 5 = 25 \times (99 + 11) = 25 \times 110 = 2750$.

Answer: (A)
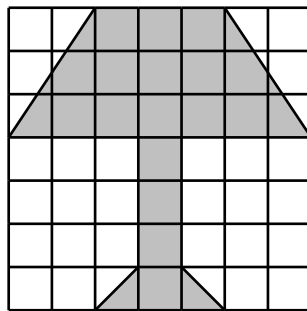
2. In the arithmetic expression $\overline{1A} + \overline{B1} = 100$, what is the value of the digit B?

    (A) 5   (B) 6   (C) 7   (D) 8   (E) 9

【Solution】

We know the ones digit of 100 is 0, so the value of the digit A must be 9. Since $100 - 19 = 81$, then $B = 8$.

Answer: (D)

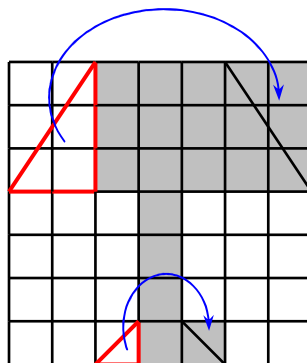3. As shown in the figure below, there is a 7 × 7 grid paper where each small square has an area of 1 cm², what is the area, in cm², of the shaded figure ?

    (A) 18   (B) 19   (C) 20   (D) 21   (E) 22

【Solution】

The above shaded part can be assumed to assemble by exactly 20 small squares, so the area of the shaded figure is 20 cm².
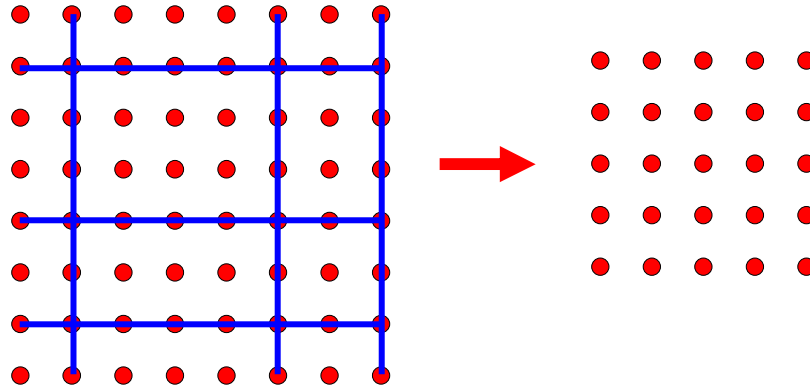

Answer: (C)

4. Students are arranged in a square formation with 8 rows and 8 columns. Now, 3 rows and 3 columns are removed, how many students were reduced?

    (A) 9   (B) 24   (C) 27   (D) 39   (E) 45

【Solution】

After removing 3 rows and 3 columns, the formation becomes a 5×5 square, then the number of students reduced is $8\times8-5\times5=39$.
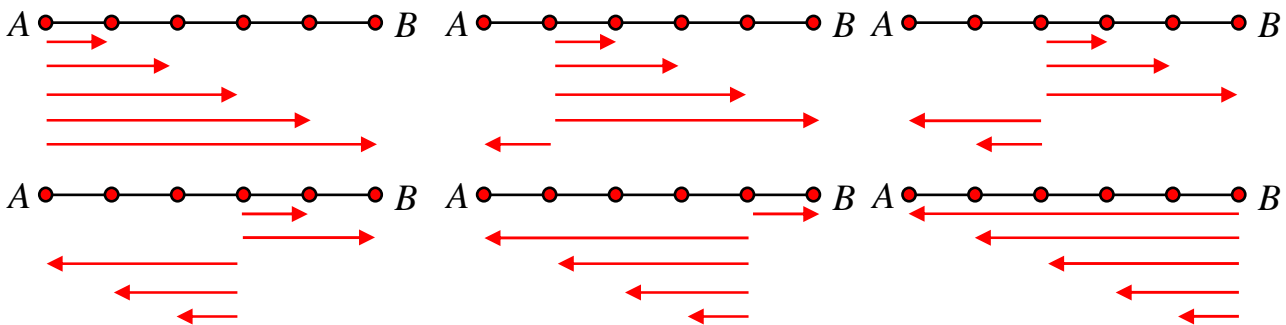

Answer: (D)

5. A train travels between City A to City B. There are 4 other stops in-between these two cities. How many different kinds of tickets are there? (Note: The ticket from City A to City B and City B to City A must not be the same).

    (A) 8   (B) 12   (C) 20   (D) 24   (E) 30

【Solution】

From the given information, there are 6 train stations; each station must issue different tickets to any one of the other 5 train stations. Hence, there are $6\times5=30$ different kinds of train tickets.

Answer: (E)

6. In the figure shown below, a square is embedded in a rectangle. If $AB=9\,\text{cm}$ and $CD=15\,\text{cm}$, then what is the perimeter, in cm, of the rectangle?

    (A) 48   (B) 50   (C) 52   (D) 54   (E) 56
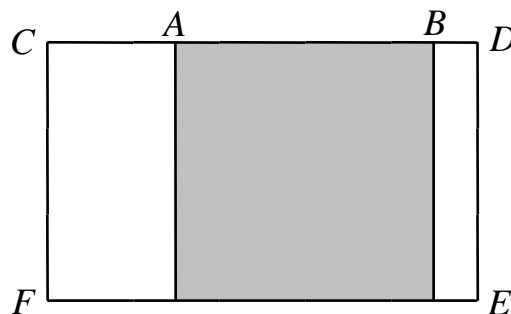
From the given figure, we know the width of the given rectangle is also the side length of the given square. Hence, the sum of length and width of the given rectangle is $CD + AB = 15 + 9 = 24$ cm. Hence, the perimeter of the rectangle is $24 \times 2 = 48$ cm.
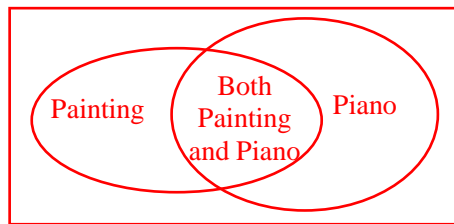
Answer: (A)

7. In a class, 22 students take painting course, 28 students take piano course, 10 students take both courses and 5 students take none of the courses. Find the number of students in the class.
   (A) 40　　　　(B) 45　　　　(C) 50　　　　(D) 55　　　　(E) 65

【Solution】
From the given information, there are $22 + 28 - 10 = 40$ students participated in extra-curricular activities. Thus, there are $40 + 5 = 45$ students in the class.
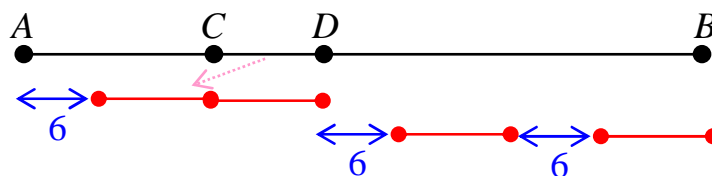

Answer: (B)

8. As shown in the figure below, points C and D are on segment AB. The length of AC is 6 cm longer than the length of CD. The length of BD is twice as the length of AC. How much, in cm, is BD longer than AD?
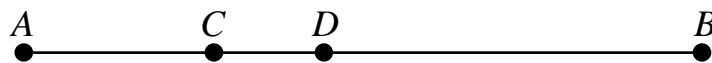
   (A) 3　　　　(B) 6　　　　(C) 9　　　　(D) 12　　　　(E) 15

【Solution 1】

From the above figure, we know that the length of BD is 6 cm longer than the length of AD.

【Solution 2】
Let $CD = a$, then $AC = a + 6$, $BD = 2AC = 2(a + 6) = 2a + 12$.

Therefore, $BD - AD = (2a + 12) - (a + 6 + a) = 6$ cm.

Answer: (B)

9. The sum of four consecutive odd integers is 72. What is the largest integer among the four integers?
   (A) 15　　　　(B) 17　　　　(C) 19　　　　(D) 21　　　　(E) 23

【Solution】
From the given information, we know the average of these four consecutive odd integers is $72 \div 4 = 18$, so the four consecutive odd integers are 15, 17, 19, 21.

Therefore, the largest odd integer is 21.

Answer: (D)

10. Each of the digits 1, 2, 3, 4, 5 and 6 are printed on one of the 6 cards. Choose two different cards to form a 2-digit number. List down all possible 2-digit numbers in increasing order on a sheet of paper, what is the 21st number in the list?
(A) 43   (B) 45   (C) 46   (D) 51   (E) 61

【Solution】
From the given information, among all 2-digit numbers there are 5 of them whose tens digit is 1 and they are: 12, 13, 14, 15, 16. Similarly, there are also 5 possible numbers whose tens digit is either 2, 3 and 4. Hence, the 21st number of the list is 51.

Answer: (D)

11. If $(\Delta \div 2 - 2) \times 2 + 2 = 222,$ then what is the value of the symbol Δ?
(A) 56   (B) 224   (C) 228   (D) 876   (E) 884

【Solution】
Since $\Delta \div 2 - 2 = (222 - 2) \div 2 = 220 \div 2 = 110,$
then $\Delta = (110 + 2) \times 2 = 112 \times 2 = 224.$

Answer: (B)

12. Positive integers are arranged in the array as shown below, what is the sum of all the integers located on the fourth row?

$$1$$
$$2 \quad 3 \quad 4$$
$$5 \quad 6 \quad 7 \quad 8 \quad 9$$
$$\ldots$$

(A) 75   (B) 81   (C) 89   (D) 91   (E) 189

【Solution】
According to the pattern, the sum of the fourth row is
$$10 + 11 + 12 + 13 + 14 + 15 + 16 = 91.$$

Answer: (D)

13. Define a new operation "$*$" such that $4 * 2 = 86$, $6 * 3 = 189$, $8 * 4 = 3212$ and $9 * 3 = 2712$. Find $10 * 2$.
(A) 128   (B) 208   (C) 2008   (D) 2012   (E) 2020

【Solution】
From the given information, we deduce that the result of the operation is the product followed by the sum. Hence, $10 \times 2 = 20$, $10 + 2 = 12$, and we have $10 * 2 = 2012$.

Answer: (D)

14. The average score of Peter in several math exams is 88. On the last exam, he must get 98 in order to raise his average score to 90. How many exams has he taken in total?
(A) 3   (B) 4   (C) 5   (D) 6   (E) 7

【Solution 1】
In order to increase the average score, $98 - 90 = 8$ is distributed to previous exam scores. The previous average exam increases by 2, thus, there are $8 \div 2 = 4$ previous exams, so he took a total of $4 + 1 = 5$ exams.

15. How many different ways are there to place 2 identical black balls and 2 identical white balls in three different boxes A, B and C such that each box should contain at least one ball?

    (A) 6   (B) 9   (C) 12   (D) 15   (E) 18
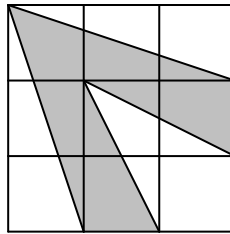
【Solution】
The ball distribution in three boxes falls into three cases:

(i) BB, W, W; (ii) WW, B, B; (iii) BW, B, W;

For cases (i) or (ii) one needs to choose between three boxes to get the box with two balls. For case (iii) one needs to arrange a sequence of the three boxes corresponding to two different ball contents. So there is a total of $3+3+3\times 2=12$ ways.
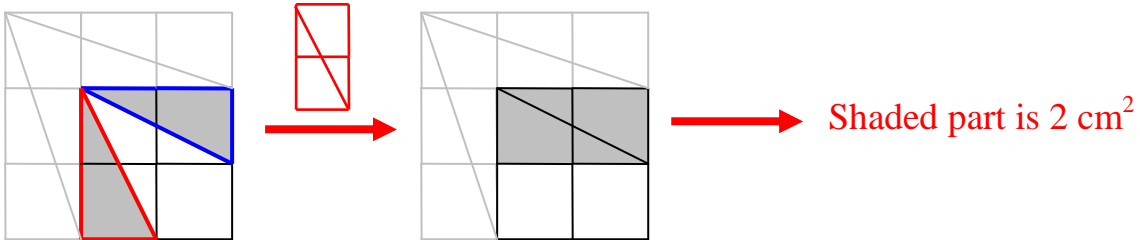
Answer: (C)

16. As shown in the figure below, there is a 3×3 grid paper where each small square has an area of 1 cm^2. Find the area, in cm^2, of the shaded part.
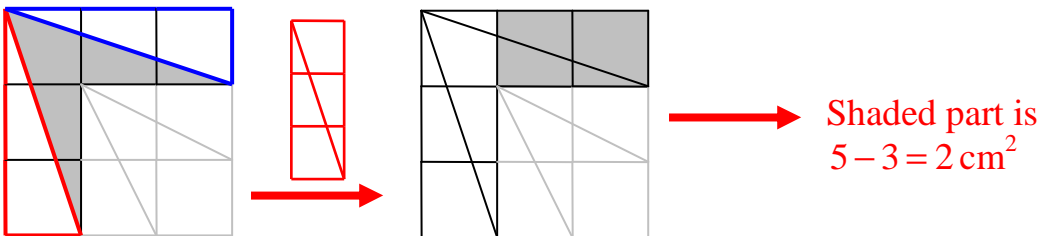

    (A) 3   (B) 4   (C) 5   (D) 6   (E) 7

【Solution】
(i)

Shaded part is 2 cm^2

(ii)

Shaded part is $5-3=2$ cm^2

One computes by cut and join to get $2+2=4$ cm^2.
Using the Cutting and Pasting Method, we can easily find the area of shaded part is $2+2=4$ cm^2.
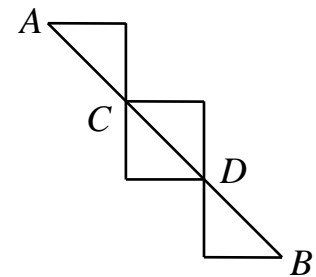
Answer: (B)

17. The sum of the digits of a 2-digit number is equal to 9. If the 2-digit number is multiplied by 5, the sum of all digits is still 9. How many such 2-digit numbers are there?

(A) 4   (B) 5   (C) 7   (D) 8   (E) 9

【Solution】
Let us list down all 2-digit numbers whose sum of all digits equals 9, there are 9 of them: 18, 27, 36, 45, 54, 63, 72, 81 and 90. After multiplying each of them by 5, we have 90, 135, 180, 225, 270, 315, 360, 405 and 450. We discovered that all of them still have the property. Hence, there are a total of 9 such numbers.

Answer: (E)

18. As shown in the figure below, an ant starting from point A crawl along the path to point B. It is required that both point C and point D can only pass once, how many different paths can the ant crawl?

(A) 6   (B) 8   (C) 10
(D) 12   (E) 18

【Solution】
From the given figure, we know there are 2 paths from A to C; 3 ways from C to D; 2 ways from D to B. Thus, the total number of paths from A to B is $2 \times 3 \times 2 = 12$.

Answer: (D)

19. Arrange black and white beads according to the pattern shown in the figure below. If the total number of beads is 97, how many black beads are there?

○●○○●○●○○●○● ⋯

(A) 38   (B) 39   (C) 40   (D) 41   (E) 42

【Solution】
From the given figure, we discovered the beads are arranged as: W, B, W, W, B; which is in a period of 5. Since $97 = 5 \times 19 + 2$, there are 19 complete periods with a remaining of first two beads in a period. There are two black beads in each period and one black beads among the first two beads of a period. Thus, the total number of black beads is $2 \times 19 + 1 = 39$.

Answer: (B)

20. There are 6 persons A, B, C, D, E and F attend a meeting. Some of them shake hands with one another. In the final count up, A shakes 5 hands, B shakes 4, C shakes 3, D shakes 2 and E shakes 1. How many does F shake?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

【Solution】
A, B, C, D and E totally shake $5 + 4 + 3 + 2 + 1 = 15$ hands. Since each hand shaking is counted twice, the total number of hands shaking is even. F should shake 1, 3 or 5 hands. If F shakes once, he shakes A; but B should shake A, C, D, E and now E shakes twice, a contradiction. If F shakes 5 hands, both A and F shake hands with E, also a contradiction. So F can only shake 3 hands and below is a possible situation:

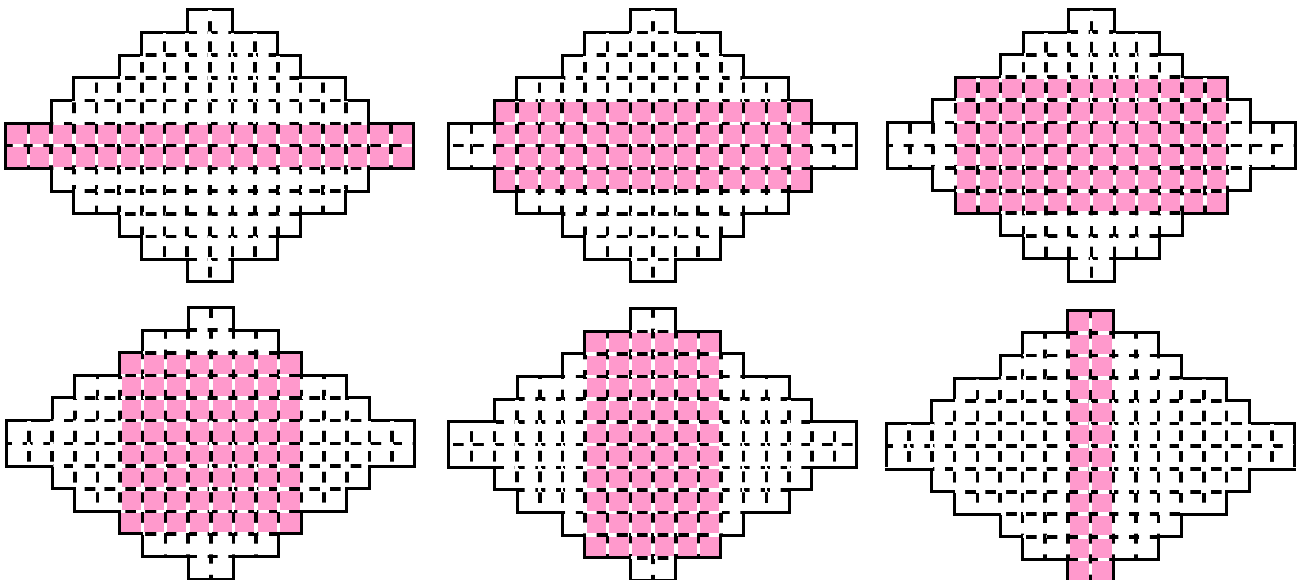|	A	B	C	D	E	F
A		○	○	○	○	○
B	○		○	○	×	○
C	○	○		×	×	○
D	○	○	×		×	×
E	○	×	×	×		×
F	○	○	○	×	×	

Answer: (C)

21. The shape enclosed by solid lines in the figure below is composed of unit squares. What is the maximum area of a rectangle that can be cut from the shape along grid lines?
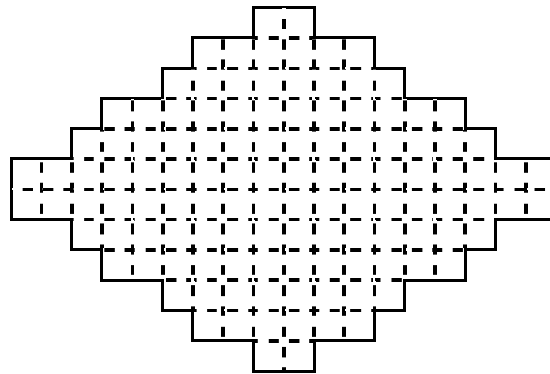
【Solution】
Compute the area of rectangles of following shapes: $2\times18=36$, $4\times14=56$, $6\times12=72$, $8\times8=64$, $10\times6=60$, $12\times2=24$. The maximal area is 72.

Answer: 072

22. A box is filled with blue and green beads. The number of the blue beads is twice as many as the green beads. Now take 6 blue beads and 4 green beads each time. When the green beads are emptied, 6 blue beads are left. Please find the total number of beads.

23. The teacher gave three students the same amount of money to buy stationery. A bought 3 pens and has \$3 left. B bought 5 crayons and has \$5 left. C bought 7 pencils and has \$3 left. If the price for each item is an integer, what is the least total amount of money the teacher gave them?

【Solution】
A bought 3 pens and left \$3, the amount of money one student got is divisible by 3. B bought 5 crayons and left \$5, the amount of money is also divisible by 5, thus divisible by 15. C bought 7 pencils and left \$3, then one compute $15 = 7 \times 2 + 1$, $30 = 7 \times 4 + 2$, $45 = 7 \times 6 + 3$. The least amount the teacher gave to a single student is 45, totally for three students is $45 \times 3 = 135$.

Answer: 135

24. Two students are computing $A \times B$. The first student mistakenly took the last digit of A as 2 instead of 7 and got a product of 418.The second student mistakenly the tens digit of A as 3 instead of 2 and got a product of 703. What is the value of B?

【Solution 1】
The first student got the last digit as 2 instead of 7, the product is $5 \times B$ less than the correct one; The second student got a result $10 \times B$ more than the correct one. We have $B = (703 - 418) \div (5 + 10) = 19$.

【Solution 2】
From the given information, we know that A is 27. Hence $B = 418 \div 22 = 19$.

Answer: 019

25. The students in a research class are clustered into two groups: the morning and afternoon sessions. A student takes part in exactly one group in each session (the two groups in each session can be different and the number of students in each group can be different). Each group has at least one student and at most 4 students. Each student reports the number of students in the group he or she belongs to in two sessions. One finds that no two students report the same pair of numbers (with order, for example, (1, 4) and (4, 1) are different). What is maximum number of students in the class?

Consider the two numbers reported by a student as an ordered pair. The first number represents number of students in the morning session, the second number represents the afternoon session. Since maximum number of students in a group is 4, there is a total of 16 combinations as follows:

(1, 1)	(1, 2)	(1, 3)	(1, 4)
(2, 1)	(2, 2)	(2, 3)	(2, 4)
(3, 1)	(3, 2)	(3, 3)	(3, 4)
(4, 1)	(4, 2)	(4, 3)	(4, 4)

The pairs in row k ($1 \le k \le 4$) correspond to all students in a size k group in the morning, the total number of which must be divisible by k, similarly for the kth column. There are at most 3 students on the third row and the third column. If the pair (3, 3) is removed the remaining 15 pairs satisfy the condition. So the maximum number of students is 15.

Must remove 1

Must remove 1

Answer: 015