7^{th} International Mathematics Assessments for Schools
(2017-2018)

Junior Division Round 2

Time: 120 minutes

Printed Name: ________________ *Code*: __________ *Score*: __________

Instructions:

- Do not open the contest booklet until you are told to do so.
- Be sure that your name and code are written on the space provided above.
- Round 2 of IMAS is composed of three parts; the total score is 100 marks.
- Questions 1 to 5 are given as a multiple-choice test. Each question has five possible options marked as A, B, C, D and E. Only one of these options is correct. After making your choice, fill in the appropriate letter in the space provided. Each correct answer is worth 4 marks. There is no penalty for an incorrect answer.
- Questions 6 to 13 are a short answer test. Only Arabic numerals are accepted; using other written text will not be honored or credited. Some questions have more than one answer, as such all answers are required to be written down in the space provided to obtain full marks. Each correct answer is worth 5 marks. There is no penalty for incorrect answers.
- Questions 14 and 15 require a detailed solution or process in which 20 marks are to be awarded to a completely written solution. Partial marks may be given to an incomplete presentation. There is no penalty for an incorrect answer.
- Use of electronic computing devices is not allowed.
- Only pencil, blue or black ball-pens may be used to write your solution or answer.
- Diagrams are not drawn to scale. They are intended as aids only.
- After the contest the invigilator will collect the contest paper.

The following area is to be filled in by the judges;
the contestants are not supposed to mark anything here.

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total Score	Signature
Score																	
Score																	

Junior Division Round 2

Questions 1 to 5, 4 marks each

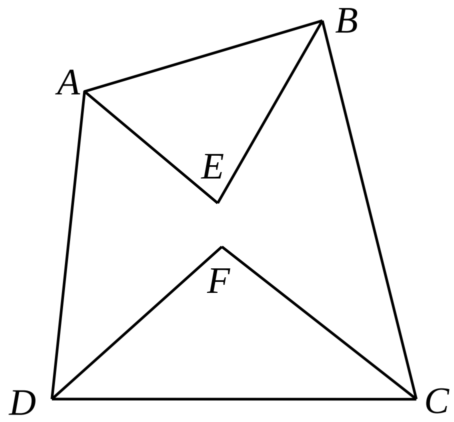
1. Among all the expressions listed below, how many are negative numbers?

$$(1000-1)^1,\ (1000-2)^2,\ \cdots,\ (1000-n)^n,\ \cdots,\ (1000-2018)^{2018}.$$

（A）509　（B）510　（C）1009　（D）1018　（E）1019

Answer：______________

2. In convex quadrilateral *ABCD*, bisectors of $\angle DAB$ and $\angle ABC$ intersect at *E*, bisectors of $\angle BCD$ and $\angle CDA$ intersect at *F*, as shown in the figure below. If $\angle AEB = 80°$, what is the angle measure, in degrees, of $\angle DFC$?



（A）80　（B）90　（C）100　（D）110　（E）Undetermined.

Answer：______________

3. Two numbers *m* and *n*, which may be equal, are taken from the set 1, 2, 3, 4, 5, 6, 7, 8 and 9. Which number below is not a possible value of $10(m+n)-mn$?

（A）19　（B）55　（C）72　（D）79　（E）83

Answer：______________

4. If *a* and *b* are real numbers, which of the following expressions below must be non-negative?

（A）a^2+b^2+a+b　（B）$a^{2018}+b^{2017}$　（C）$a^4b^4+a^2b^2-1$　（D）$a^3b^3-2a^2b^2+ab$　（E）$a^2b^2+2ab+1$

Answer：______________

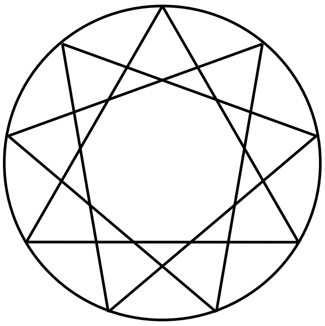
5. The product of the sum and arithmetic mean of n integers is 2018. Which of the following statements below is true?

（A）Minimum of n is 1 （B）Minimum of n is 2 （C）Minimum of n is 1009
（D）Minimum of n is 2018 （E）No such n exists.

Answer：________________

Questions 6 to 13, 5 marks each

6. Rotate an equilateral triangle inscribed in a circle 40 degrees clockwise and counter-clockwise, as shown in the figure below. How many triangles are there in the figure?

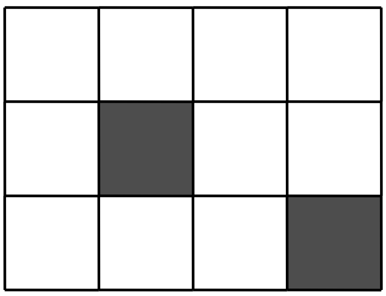


Answer：________ triangles

7. Consider a four-digit number $\overline{abcd}$ where a and d are both non-zero. If the last two digits in the sum of $\overline{abcd}$ and $\overline{dcba}$ are 58, what is the maximum possible value of $\overline{abcd}$?

Answer：________________

8. A rectangle is divided into 12 unit squares such that 10 are white and 2 are black, as shown in the figure below. To form a centrally symmetric picture by adding some white squares but no black squares, what is the least number of white squares needed?



Answer：________ white unit squares

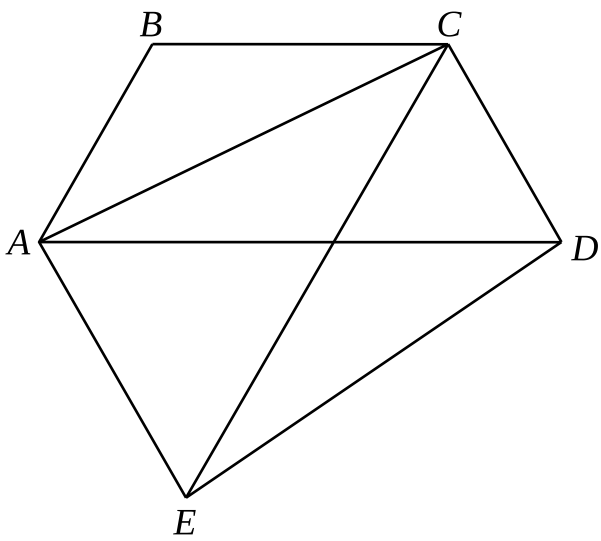
9. A three-digit number is said to be "lucky" if it is divisible by 6 and by swapping its last two digits will give a number divisible by 6. How many "lucky" numbers are there?

Answer： ________________

10. Find the value of x such that both x and $\sqrt{2017-99\sqrt{x}}$ are integers.

Answer： ________________

11. In the figure below, quadrilaterals $ABCD$ and $ABCE$ are both isosceles trapezoids, where $AB//CE$ and $BC//AD$. If $AC = DE$, what is the measure, in degrees, of $\angle ABC$?



°

Answer： ________________

12. Place $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, $\cdots$, $\sqrt{100}$ into several groups such that the sum of each group is not more than 10. Find the least number of groups needed to attain this kind of an arrangement?

Answer： ________________

13. There is a sequence of five positive integers. Each number right after the first term is at least twice the number before it. If the sum of the five numbers is 2018, what is the least possible value of the last number?

Answer： ________________

Questions 14 to 15, 20 marks each
(Detailed solutions are needed for these two problems)

14. Let a, b, c and d be four positive integers such that $\frac{b}{a}$, $\frac{c}{b}$, $\frac{d}{c}$ are simplified fractions and $\frac{b}{a}+\frac{c}{b}+\frac{d}{c}$ is an integer. Prove that $d \geq a-1$.

15. In the figure below, ABC is a right isosceles triangle where $AB = AC$. Let D be an exterior point such that $BD = \sqrt{2}AD$. Prove that $\angle ADC + \angle BDC = 45°$.