Solution to
Seventh International Mathematics Assessment for Schools
Round 1 of Middle Primary Division

1. What is the value of $19\times1+19\times3+19\times5+19\times7+\cdots+19\times19$?

(A) 1900 (B) 1919 (C) 2900 (D) 2919 (E) 3800

【Solution】

$$\begin{aligned}19\times1+19\times3+19\times5+19\times7+\cdots+19\times19&=19\times(1+3+5+7+\cdots+19)\\&=19\times\frac{(1+19)\times10}{2}\\&=19\times100\\&=1900\end{aligned}$$

Hence (A).

Answer：(A)

2. If $(\Delta\times2-1)\times2=2018$, then what is the value of Δ?

(A) 502 (B) 503 (C) 504 (D) 505 (E) 506

【Solution】

$$\begin{aligned}(\Delta\times2-1)\times2&=2018\\ \Delta\times2-1&=2018\div2=1009\\ \Delta\times2&=1009+1=1010\\ \Delta&=1010\div2=505\end{aligned}$$

Hence (D).

Answer：(D)

3. Five students sit along a circle and starts to call out some numbers one-by-one. Student A calls out "1", B calls out "2", C calls out "3", D calls out "4", E calls out "5" and then it returns back to Student A who calls out "6" and so on, where each student increases the previously called number by one and calls it out. Which student calls out the number "99"?

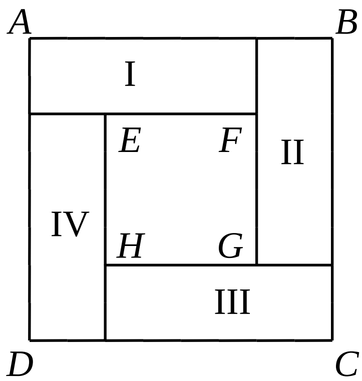
(A) A (B) B (C) C (D) D (E) E

【Solution】

Taking division with remainders, we get $99=5\times19+4$. It is student D who calls 99.
Hence (D).

Answer：(D)

4. In the figure shown below, four rectangles of the same size, denoted by I, II, III and IV, are placed together, where *ABCD* and *EFGH* are both squares. If rectangle I has a perimeter of 20 cm, then what is the perimeter, in cm, of *ABCD*?



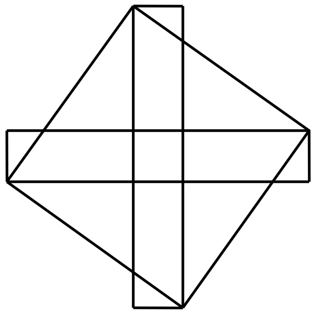
(A) 40 (B) 60 (C) 80
(D) 100 (E) 120

【Solution】

The side length of square *ABCD* is exactly the same as the sum of the length and width of rectangle I. So, the perimeter of *ABCD* is $(20 \div 2) \times 4 = 40$ cm. Hence (A).

Answer：(A)

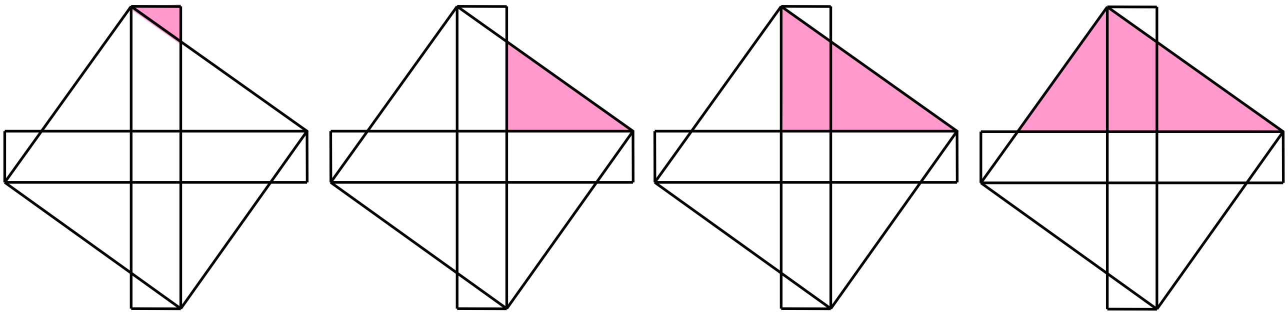
5. How many triangles in total are there in the figure below?



(A) 4　(B) 8　(C) 12　(D) 14　(E) 16

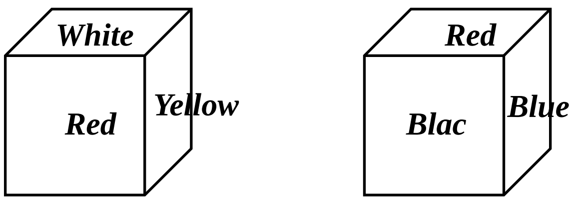
【Solution】

Refer to the figure below, there are four identical triangles for each case. Thus, the total number of triangles is $4 \times 4 = 16$. Hence (E).



Answer：(E)

6. The six faces of a cube are colored using six different colors namely red, blue, yellow, green, black and white in some order. Turn the cube arbitrarily and it shows the two possible scenarios below. What is the color opposite to the face of green?



(A) Red　(B) Yellow　(C) Blue　(D) Black　(E) White

【Solution】

The red face is adjacent to the yellow, blue, black and white faces. Therefore, its opposite face is colored green. Hence (A).

Answer：(A)

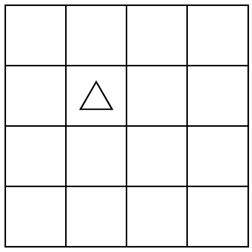
7. Consider every positive integers whose digits do not include 2 and that sum of its digits is equal to 3 and arrange all such integers in increasing order. What is the sum of the three smallest integers that satisfy the conditions?

(A) 36　(B) 63　(C) 144　(D) 206　(E) 414

【Solution】
Since $3=1+1+1$, we can arrange such numbers in increasing order as 3, 30, 111, 300, ⋯, etc. Therefore, the sum of the three smallest numbers is $3+30+111=144$. Hence (C).

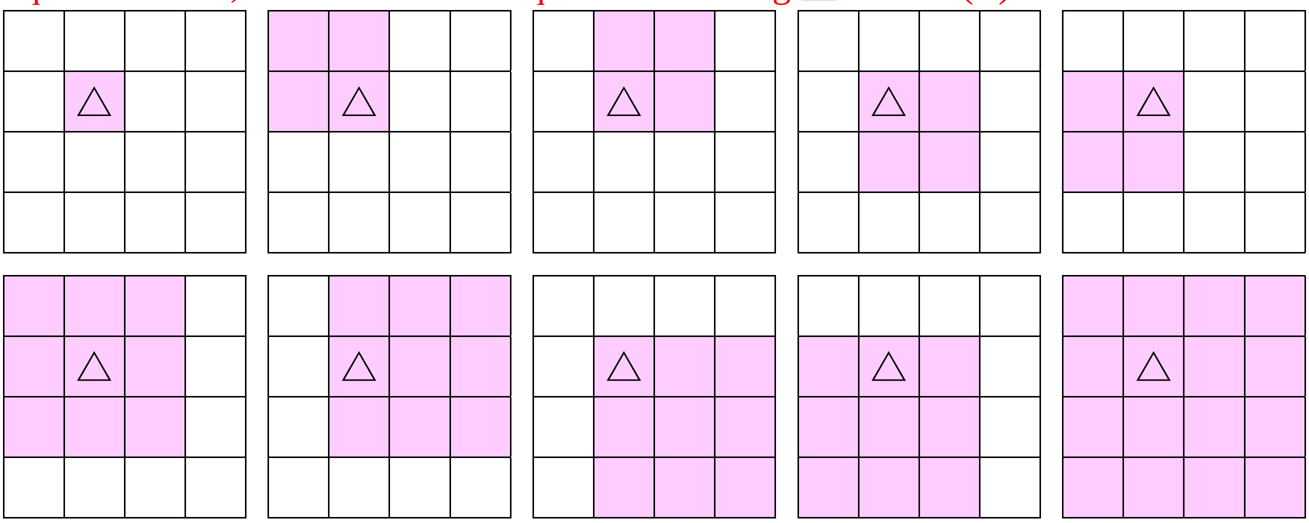
Answer：(C)

8. In the 4×4 square table shown below, a △ is placed on the second row and second column. What is the total number of squares with sides falling on the grid lines and containing △?



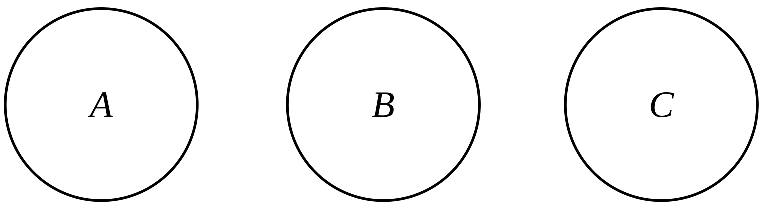
(A) 8 (B) 10 (C) 11 (D) 12 (E) 14

【Solution】
Enumerating the squares according to the different sizes containing △. There are 1 such 1×1 square; 4 such 2×2 squares; 4 such 3×3 squares and 1 such 4×4 square. In total, there are 10 such squares containing △. Hence (B).



Answer：(B)

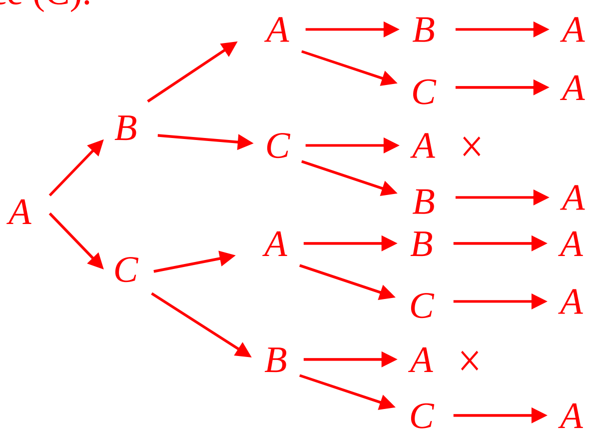
9. In the figure below, a frog jumps between the three circles. In each jump, it goes from one circle into another circle. It is known that the frog starts from A and ends at A after 4 jumps. How many different paths can the frog have?



(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

【Solution 1】

One can enumerate the following 6 ways: *ABABA*, *ABACA*, *ABCBA*, *ACABA*, *ACACA*, *ACBCA*. Hence (C).



【Solution 2】

Denote three circles between the start circle and end circle as X, Y, Z:

$$A \to X \to Y \to Z \to A$$

Since X and Z are neither *A*, one distinguishes two cases as below:

(i) if Y is *A*, then X or Z each has two choices *B* or *C*, so there are $2\times1\times2=4$ ways of jump.

(ii) if Y is not *A*, then X is determined by Y, as well as Z is also determined. There are two choices of Y so there are $2\times1\times1=2$ ways of jump.

Totally there are $4+2=6$ ways of jump. Hence (C).

Answer：(C)

10. In 1202 A.D., Italian mathematician Fibonacci (1170～1250) wrote in his book 《Liber Abaci》 the following interesting problem: Exactly two months after their birth, a pair of rabbits will give birth to a new pair (one male and one female) and then give birth to a pair each month after that. There is only one pair of new born rabbits at the beginning. If the rabbits never die, how many pair of rabbits are there after exactly 12 months?

(A) 144 (B) 233 (C) 234 (D) 235 (E) 377

【Solution】

By iteration, we get the following sequence:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \cdots$$

where the 13th number is 233. The sequence is the so-called Fibonacci sequence, where each term (starting from the third) is the sum of the two terms before it. Hence (B).

Answer：(B)

11. Define "*" as an operation such that $4*2=82$, $6*3=183$, $8*4=324$, $9*3=276$ and $9*5=454$. What is the value of $10*2$?

(A) 55 (B) 125 (C) 202 (D) 208 (E) 2002

【Solution】

From the examples it shows that the result is the products followed by the difference. As $10\times2=20$, $10-2=8$, we have $10*2=208$. Hence (D).

Answer：(D)

12. Bob got a score of 94 on foreign language test, and his average score on the native language and math tests is 97. What is his average score on these three tests?

（A）94　（B）94.5　（C）95　（D）95.5　（E）96

【Solution 1】

His total score of native language and math is $97\times 2=194$, and total score of three subjects is $194+94=288$, so the average of three is $288\div 3=96$. Hence (E).

【Solution 2】

Since the average score on native language and math is 97 and the score for foreign language is 94, the scores on native language and math are totally $(97-94)\times 2=6$ more than the score for foreign language. Thus the average score of the three tests is $6\div 3=2$ more than the score for foreign language, i.e. the average score of the three tests is $94+2=96$. Hence (E).

Answer：（E）

13. Color the surfaces of a $5\times 5\times 5$ wooden cube red, then cut it into smaller $1\times 1\times 1$ cube pieces. How many of these smaller cubes have exactly two red faces?

（A）36　（B）48　（C）61　（D）90　（E）98

【Solution】

Unit cubes with two red faces are exactly those adjacent to edges of the large cube but not at the corner. There are $5-2=3$ such unit cubes on each edge of the large cube, who has 12 edges. The total number of unit cubes with two red faces is then $12\times 3=36$. Hence (A).

Answer：（A）

14. A palindrome number is a positive integer that is the same when read forwards or backwards. The numbers 909 and 1221 are examples of palindromes. How many palindrome numbers are there between 10 and 1000?

（A）90　（B）99　（C）100　（D）106　（E）108

【Solution】

A two-digit number is a palindrome number if the two digits are the same but non-zero, so there are 9 of them. A three-digit number is a palindrome number if the first digit is equal to the third and non-zero, there are 90 of them. 1000 is not a palindrome number. Totally there are $9+90=99$ of palindrome numbers between 10 and 1000. Hence (B).

Answer：（B）

15. In the month of February of some year, there are more Saturdays than any other days in a week. What day is the last day of this month?

（A）Wednesday　（B）Thursday　（C）Friday

（D）Saturday　（E）Sunday

【Solution】

February in a flat year has 28 days, which is a multiple of 7. Each day in a week appears four times in a flat year February. For a leap year February, the first day or the last day is the same day in a week, which appear 5 times. Hence (D).

Answer：（D）

16. Four students namely Annie, Benny, Charlie and Deany all paid the same amount of money to buy some number of notebooks together. After distributing the notebooks, Annie, Benny and Charlie got 6, 7 and 11 notebooks more than Deany, respectively. As such, to be fair, Annie, Benny and Charlie gave back a total of \$48 to Deany. What is the price of each notebook?

（A）2 （B）6 （C）8 （D）12 （E）16

【Solution】

Totally A, B, C takes $6+7+11=24$ notebooks more than D. D takes $24\div 4=6$ notebooks less than average. He is paid back \$48. Each notebook counts for $48\div 6=8$ dollars. Hence (C).

Answer：（C）

17. A train left town A at 8:30 AM some day and arrived at town B at 1:50 AM of the next day. There is no time difference between the two places. How long did the train travelled for the trip?

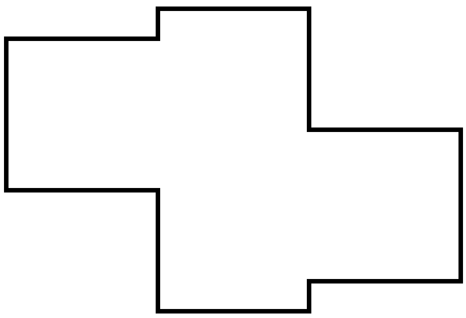
（A）5 hours 20 minutes （B）10 hours 20 minutes （C）15 hours 20 minutes （D）16 hours 20 minutes （E）17 hours 20 minutes

【Solution】

The train used 15 hours 30 minutes on the first day; and 1 hours 50 minutes on the next day. The total time is 17 hours 20 minutes. Hence (E).

Answer：（E）

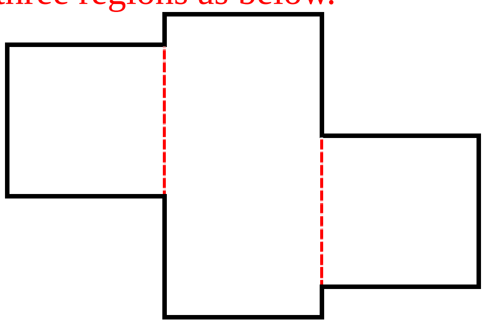
18. Mike placed 4 identical squares, each with side length 5 cm and are non-overlapping, to form a new figure as shown below. Find the perimeter, in cm, of this new figure.



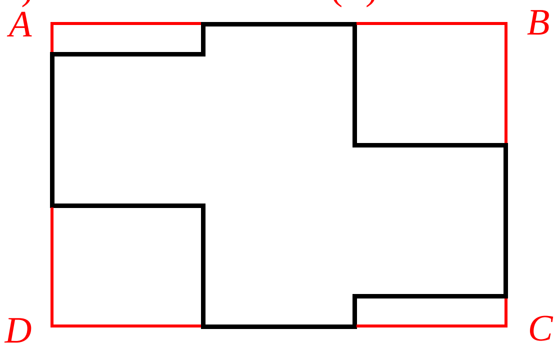
（A）15 （B）20 （C）30 （D）45 （E）50

【Solution】

The figure can be cut into three regions as below.



The left and right part must be covered by one square each and the middle region is covered by two. The region has the same length of circumference as the minimum rectangles covering it, which is $5 \times 3 = 15$ cm wide and $5 \times 2 = 10$ cm high. The circumference is $(15 + 10) \times 2 = 50$ cm. Hence (E).



Answer：(E)

19. Adam owns an old watch, which is slower than a normal watch by the same amount of time for each hour. At 8 o'clock in the morning, the old watch reads 8 o'clock. At 9 o'clock in the morning, it reads 8:58. What time does it read when the real time is 4 o'clock in the afternoon?

(A) 3：42 (B) 3：44 (C) 3：46 (D) 4：08 (E) 4：16

【Solution】

The watch is 2 minutes slower every hour. There are 8 hours from 8 am to 4 pm. The watch is then $2 \times 8 = 16$ minutes behind. It reads 3：44. Hence (B).

Answer：(B)

20. Identical equilateral triangles are placed together into two figures as shown below. One can cover Figure B using 5 pieces of Figure A. How many different ways can we do the covering?

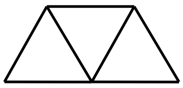


Figure A

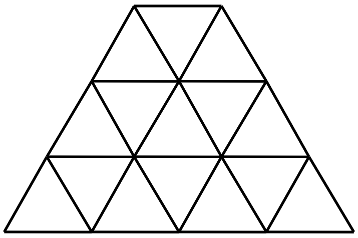


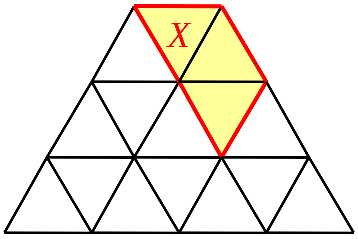
Figure B

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

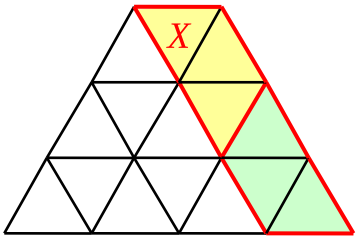
【Solution】

The triangle X can be covered in three ways:

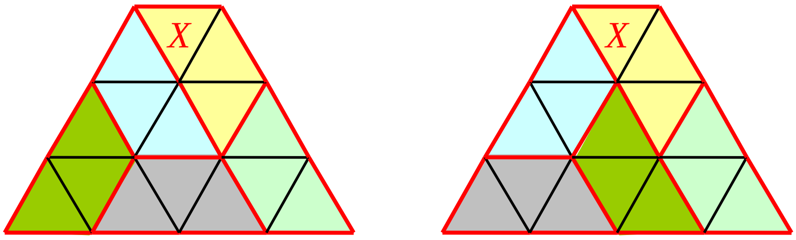
Case (i)



Then another piece of A is determined as below

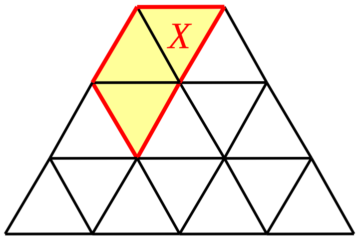


The blank triangle area can be covered in two ways:



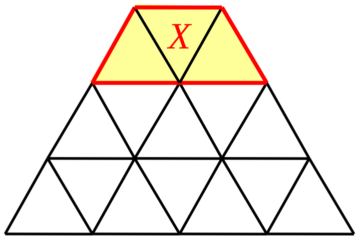
This case contributes two cases in total

Case (ii)

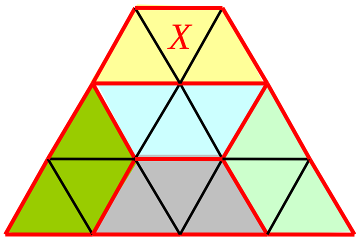


This case is symmetric to case (i), so it contributes two cases.

Case (iii)



The blank area has only one covering method.

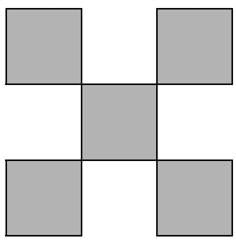


This case contributes 1.

In total, there are $2+2+1=5$ ways of covering. Hence (C).

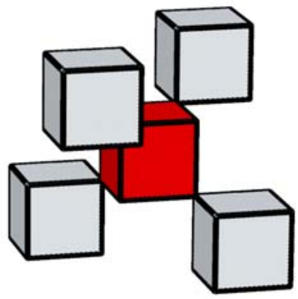
Answer：（C）

21. Put some unit cubes into a 3D model such that in the model, each cube touches some other cubes in at least one point. It is known that the model looks like the figure below from three directions of upright front, left and top. What is the least number of unit cubes needed to make such a model?



【Solution】

It is obvious the model is contained in a cube of edge length 3 and contains some of the 27 unit cubes of the large cube. The unit cube in the center has to appear in the model. The cubes at the center of faces of the large cube must not appear. The cubes at the middle of edges of the large cube must not appear. Moreover, one of the two unit cubes at any adjacent corners of the large cube must appear. It is then obvious by pigeonhole principle, one needs at least 5 unit cubes. One such construction is in the figure below.



Answer：005

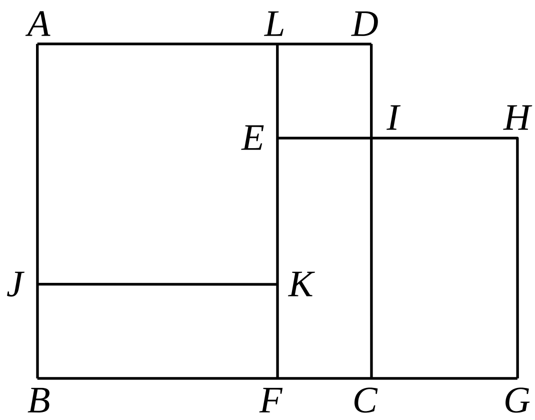
22. How many ways can we divide 6 students into 3 groups so that each group has exactly 2 students?

【Solution】

Denote the six persons as A, B, C, D, E, F. There are five possibilities for the person in the same group as A. Without loss of generality, assume A and B are in the same group. Then there are 3 possibilities for the person in the same group as C. The last group is also determined. Totally there are $5\times3=15$ ways of division.

Answer：015

23. In the figure below, *ABCD*, *EFGH* and *AJKL* are squares. The area of *AJKL* is 2018 cm^2. If rectangles *EFCI* and *JBFK* both have an area of 1360 cm^2, then what is the area, in cm^2, of *CGHI*?



【Solution】

Since $DL = AD - AL = AB - AJ = BJ$ and *EFCI* has the same area as *JBFK*, we have $EF = JK$. Square *AJKL* and *EFGH* are of the same area. Then *CGHI* has area $2018-1360=658$ cm^2.

Answer：658

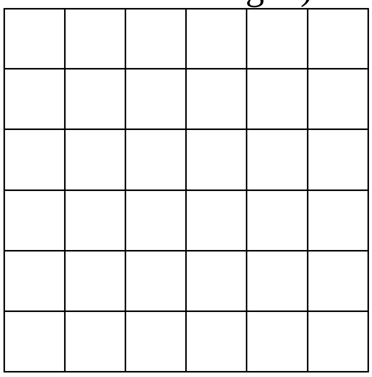
24. Three pairs of red, four pairs of yellow and five pairs of white socks are placed in a bag. Now, blindly take a sock out each time. How many socked are needed to be taken out to guarantee having six pairs of socks? (Note: Two socks of the same color are considered a pair)

【Solution】

Since two socks of the same color forms a pair, one needs 12 socks. Now there are three colors, there might be two single socks, so at least one needs to take out $12+1+1=14$ socks.

Answer：014

25. Cut the 6×6 square table below into rectangles along grid lines such that no two rectangles are identical. What is the maximum number of rectangles one can get? (Note: A square is considered a rectangle.)

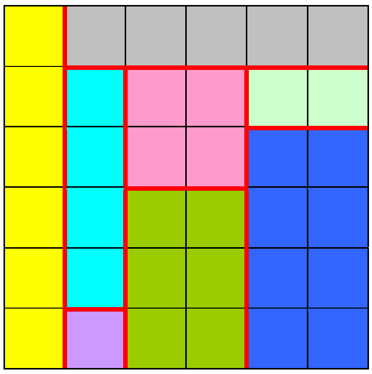


【Solution】

For more rectangles, the rectangles we get need to be as small in area as possible. There one rectangle of area 1, 2 or 3; two rectangles of area 4(1×4 and 2×2); one of area 5; two of area 6(1×6 and 2×3). These are 7 rectangles of smallest area. Since

$$1+2+3+4+4+5+6+6=31<6\times6=36<1+2+3+4+4+5+6+6+7=38,$$

There are at most 8 different rectangles, while the figure below shows one such example.



Answer：008