Solution to
Seventh International Mathematics Assessment for Schools
Round 1 of Upper Primary Division

1. What is the least common denominator of these three fractions: $\frac{1}{4}$, $\frac{5}{6}$ and $\frac{3}{8}$?

（A）12　（B）16　（C）24　（D）48　（E）96

【Solution】

Least common multiple of three denominators 4, 6 and 8 is 24. The three fractions are respectively expanded to $\frac{6}{24}$, $\frac{20}{24}$ and $\frac{9}{24}$. Hence (C).

Answer：（C）

2. What is the largest possible integer that can be placed in the □ below such that the inequality is satisfied?

$$9\times\square<2018$$

（A）202　（B）212　（C）218　（D）224　（E）230

【Solution】

$\square<\frac{2018}{9}=224\frac{2}{9}$, the largest integer is 224. Hence (D).

Answer：（D）

3. Bob got a score of 94 on foreign language test, and his average score on the native language and math tests is 97. What is his average score on these three tests?

（A）94　（B）94.5　（C）95　（D）95.5　（E）96

【Solution 1】

His total score of native language and math is $97\times2=194$, and total score of three subjects is $194+94=288$, so the average of three is $288\div3=96$. Hence (E).

【Solution 2】

Since the average score on native language and math is 97 and the score for foreign language is 94, the scores on native language and math are totally $(97-94)\times2=6$ more than the score for foreign language. Thus the average score of the three tests is $6\div3=2$ more than the score for foreign language, i.e. the average score of the three tests is $94+2=96$. Hence (E).

Answer：（E）

4. If we add 6 to the denominator of $\frac{2}{3}$, then what number should be added to its numerator such that the value of the fraction remains unchanged?

（A）3　（B）4　（C）5　（D）6　（E）7

【Solution 1】

$\frac{2}{3}=\frac{6}{9}=\frac{2+4}{3+6}$. Hence (B).

【Solution 2】

The denominator is 1.5 times of numerator in the original fraction, in order to remain its value, when we add 6 to the denominator, should add $6 \div 1.5 = 4$ to its numerator. Hence (B).

Answer：（B）

5. There are 240 kg of flour in a restaurant, which is planned to be used for 8 days. After some modifications in the menu recipe, daily consumption of flour is now reduced by 6 kg. How many days will the stock of flour last?

（A）10　　（B）12　　（C）16　　（D）20　　（E）24

【Solution】

Original daily consummation is $240 \div 8 = 30$ kg. After changing the recipe, the daily consummation is $30 - 6 = 24$ kg. The flour will be used for $240 \div 24 = 10$ days. Hence (A).

Answer：（A）

6. A train left town A at 8:30 AM some day and arrived at town B at 1:50 AM of the next day. There is no time difference between the two places. How long did the train travelled for the trip?

（A）5 hours 20 minutes　（B）10 hours 20 minutes（C）15 hours 20 minutes
（D）16 hours 20 minutes（E）17 hours 20 minutes

【Solution】

The train used 15 hours 30 minutes on the first day; and 1 hours 50 minutes on the next day. The total time is 17 hours 20 minutes. Hence (E).

Answer：（E）

7. Three hundred pairs of shoes were shipped to a department store. The shoes were packed into 2 wooden boxes and 9 carton boxes. Each wooden box contains the same number of shoes, while each carton box also contains the same number of shoes. If 3 carton boxes contain the same number of shoes as 1 wooden box, how many pairs of shoes does each wooden box contain?

（A）24　　（B）30　　（C）45　　（D）60　　（E）100

【Solution】

Since 3 carton boxes have the same volume as one wood box, 9 carton boxes are equivalent to $9 \div 3 = 3$ wood boxes. 300 pair of shoes were equivalently put into $2 + 3 = 5$ wood boxes, each one contains $300 \div 5 = 60$ pairs. Hence (D).

Answer：（D）

8. Which of the following statements below is true:

（A）A proper fraction is always less than 1.

（B）An improper fraction is always larger than 1.

（C）A mixed fraction is always larger than an improper fraction.

（D）The maximum proper fraction with fractional unit $\frac{1}{4}$ is $\frac{4}{4}$.

（E）There are only 4 proper fractions less than $\frac{5}{6}$.

【Solution】

The numerator of a proper fraction is always less than the denominator, (A) is true;

An improper fraction can be equal to 1, for example $\frac{4}{4}$, (B) is false;

A mixed fraction can be equal to an improper fraction, for example $5\frac{1}{4}=\frac{21}{4}$, (C) is false;

$\frac{4}{4}$ is not a proper fraction, (D) is false;

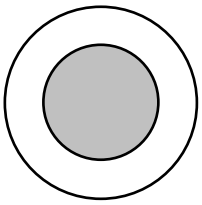
There are infinitely many proper fractions less than $\frac{5}{6}$, (E) is false.

Hence (A).

Answer：（A）

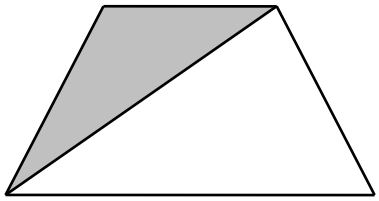
9. The fraction below each figure indicates the ratio of the area of the shaded region compared to the area of the whole figure. Which of the following options is always correct?

（A）Circle



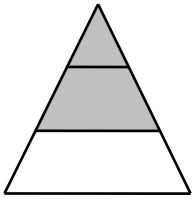
$\frac{1}{2}$

（B）Trapezoid



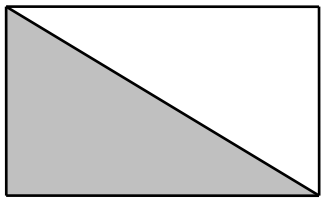
$\frac{1}{2}$

（C）Triangle



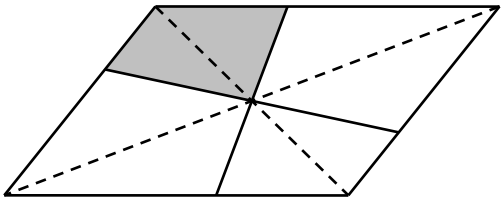
$\frac{2}{3}$

（D）Rectangle



$\frac{1}{2}$

（E）Parallelogram



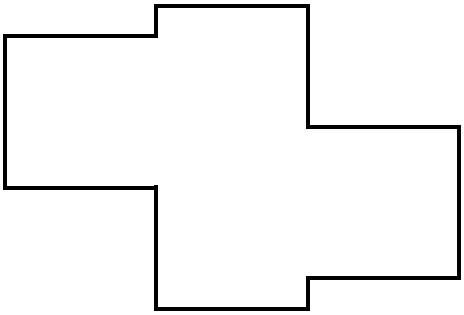
$\frac{1}{4}$

【Solution】

The diagonal of a rectangle equally divides its area, only (D) is guaranteed corrected. Conditions in other options are insufficient. Hence (D).

Answer：（D）

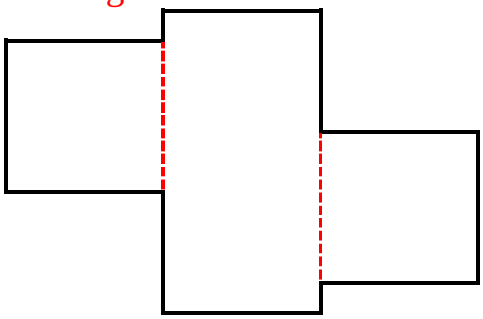
10. Mike placed 4 identical squares, each with side length 5 cm and are non-overlapping, to form a new figure as shown below. Find the perimeter, in cm, of this new figure.



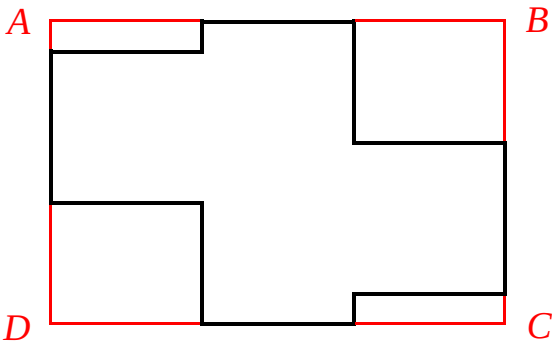
（A）15　　（B）20　　（C）30　　（D）45　　（E）50

【Solution】

The figure can be cut into three regions as below.



The left and right part must be covered by one square each and the middle region is covered by two. The region has the same length of circumference as the minimum rectangles covering it, which is $5\times3=15$ cm wide and $5\times2=10$ cm high. The circumference is $(15+10)\times2=50$ cm. Hence (E).



Answer：（E）

11. When dividing, Mike mistakenly wrote 54 instead of 45. The resulting quotient is now 18 with remainder 18. What should be the correct quotient?

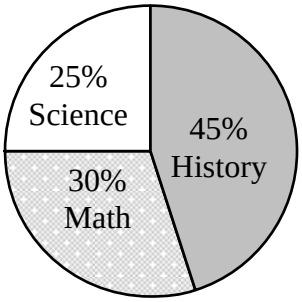
（A）15　　（B）18　　（C）22　　（D）24　　（E）28

【Solution】

The dividend is $54\times18+18=990$, the correct quotient is then $990\div45=22$.
Hence (C).

Answer：（C）

12. The figure below shows a statistical pie chart of the number of History, Math and Science books in the library of Sun Light Elementary School. It is known the total number of books of these three subjects is 1200. How many more History Books are there than Science Books?



（A）300　　（B）240　　（C）180　　（D）120　　（E）60

【Solution】

There are $1200\times(45\%-25\%)=240$ more history books. Hence (B).

Answer：（B）

13. Cut a right cylinder starting from the diameter of its top face along an up and down direction, such that it is divided into two identical pieces. The cross section is a square. How many times of height of the cylinder is the circumference of its top face?（Use $\pi=3.14$）

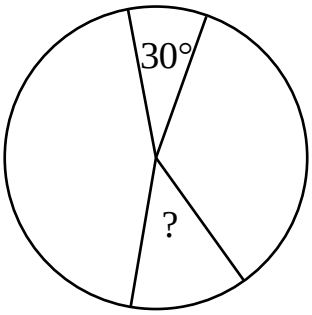
（A）1　　（B）1.5　　（C）1.57　　（D）3.14　　（E）6.28

【Solution】

Diameter of the top face is equal to the height. The multiple is $\pi=3.14$. Hence (D).

Answer：（D）

14. Two sectors are located in a circle as shown in the figure below. The first sector has a central angle $30°$ and has an area of 37.68 cm^2, while the second sector has an area of 56.52 cm^2. Find the measure, in degrees, of the central angle of the second sector.（Use $\pi=3.14$）



（A）36　　（B）40　　（C）45　　（D）50　　（E）60

【Solution 1】

Assume the radius of the circle is r. Then the area of the first sector is $\frac{30\pi r^2}{360}=37.68\,cm^2$, i.e. $r=12$. So the central angle of the second sector is $\frac{56.52}{12^2\pi}\times360°=45°$ from its area. Hence (C).

【Solution 2】

Area of the second sector is $\frac{56.52}{37.68}=1.5$ times of the first, so the central angle is also 1.5 times of the first, which is $1.5\times30°=45°$. Hence (C).

Answer：（C）

15. A palindrome number is a positive integer that is the same when read forwards or backwards. The numbers 909 and 1221 are examples of palindromes. How many three-digit palindrome numbers are divisible by 9?

（A）10　　（B）12　　（C）15　　（D）20　　（E）24

【Solution 1】

Suppose the palindrome number divisible by 9 is $\overline{aba}$, where $1\le a\le9$, $0\le b\le9$. A number is divisible by 9 if and only if sum of its digits is divisible by 9, vice versa.

Then $a+b+a=2a+b$ is divisible by 9.

When $2a+b=27$, we have only 999.

When $2a+b=18$, we have 585, 666, 747, 828, 909.

When $2a+b=9$, we have 171, 252, 333, 414.

There are totally $1+5+4=10$ of them. Hence (A).

【Solution 2】

Suppose the palindrome number divisible by 9 is $\overline{aba}$, where $1\le a\le9$, $0\le b\le9$. A number is divisible by 9 if and only if sum of its digits is divisible by 9, vice versa.

When $a=1$, we have only 171.

When $a=2$, we have only 252.

When $a=3$, we have only 333.

When $a=4$, we have only 414.

When $a=5$, we have only 585.

When $a=6$, we have only 666.

When $a=7$, we have only 747.

When $a=8$, we have only 828.

When $a=9$, we have 909、999.

There are totally 10 of them. Hence (A).

Answer：（A）

16. The table below shows an attendance sheet (which is incompletely filled-out) of a company on Oct. 30.

Employees / Department	Total number of Employees	Employees on Duty	Percentage
Department 1	150		96%
Department 2			
Total		234	97.5%

What is the total number of employees in Department 2 on Oct. 30?

（A）90　　（B）100　　（C）144　　（D）150　　（E）160

【Solution】
Gross number of two departments is $234 \div 97.5\% = 240$, so Department 2 has $240 - 150 = 90$ employees. Hence (A).

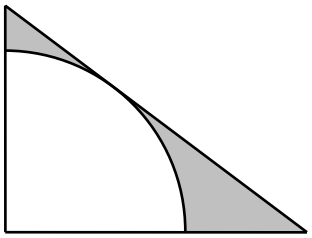
Answer：（A）

17. When some three-digit number is divided by 37, it gives a result of quotient a with remainder b, where a and b are non-negative integers. What is the maximum possible value of $a + b$?
（A）60　（B）62　（C）64　（D）66　（E）68

【Solution】
Maximum of b is 36. Since $999 = 37 \times 27$, $998 = 37 \times 26 + 36$, maximum of $a + b$ is $26 + 36 = 62$. Hence (B).

Answer：（B）

18. The side lengths of a right triangle are 3 cm, 4 cm and 5 cm, respectively. A quarter-circle is placed inside this triangle and touches the hypotenuse, as shown in the figure below. What is the area, in cm^2, of the shaded region?
($\pi = 3.14$, round-off to one decimal place)



（A）0.5　（B）1.5　（C）2.5　（D）3　（E）4.5

【Solution】
Radius of the sector is equal to the altitude on the hypotenuse, which is $\frac{3 \times 4}{5} = \frac{12}{5}$ cm.

The shaded area is then $\frac{1}{2} \times 3 \times 4 - \frac{1}{4} \times \pi \times 2.4^2 \approx 1.5\,\text{cm}^2$. Hence (B).

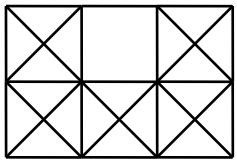
Answer：（B）

19. The greatest common divisor of n and 24 is 2, while the greatest common divisor of $n + 1$ and 24 is 3. Which of the following numbers cannot be n?
（A）2　（B）14　（C）20　（D）38　（E）50

【Solution】
It is known that n is divisible by 2 but not by 4, only (C) satisfies. Plug in other values to check that the conditions are satisfied. Hence (C).

Answer：（C）

20. In the figure below, six identical squares are used to form a 3×2 rectangle, wherein the diagonals of five of these squares are drawn. How many right isosceles triangles are there in the figure?

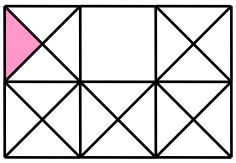


（A）20　（B）48　（C）51　（D）52　（E）53

【Solution】

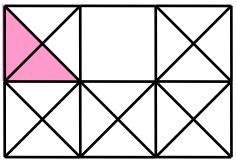
Enumerate the number of right isosceles triangles of different 5 sizes:

(i) The smallest one



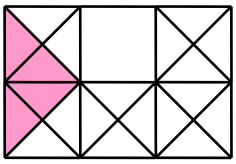
Such triangles appear 4 times in each of the five non-empty squares, totally 20;

(ii) Triangles combined by two smallest ones.



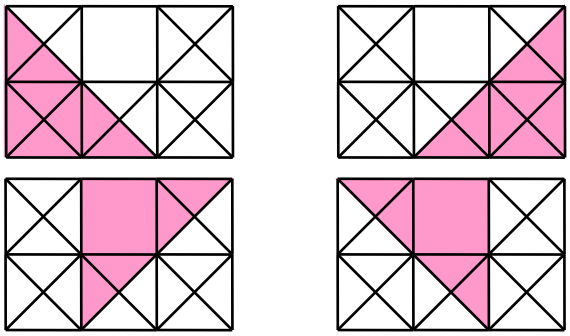
Such triangles appear 4 times in each of the five non-empty squares, totally 20;

(iii) Triangles combined by 4 smallest ones.



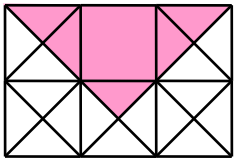
Such triangles appear in the rectangle combined by adjacent non-empty squares, each such rectangle contains 4. There totally 4 pairs of adjacent non-empty squares. There are $4\times 2=8$ triangles of this size.

(iv) Triangle of area 8 times that of the smallest one.



There are four such triangles as above.

(iv) Triangle of area 9 times that of the smallest one.



There is only one of such.

Totally there are $20+20+8+4+1=53$ isosceles right triangles. Hence (E).

Answer：（E）

21. Mike constructs a sequence in the following way: the first two terms are 1 and 2. Starting from the third term, each term is the smallest possible integer that is not relatively prime to the previous term and has not yet appeared in any of the previous terms. Find the 20^{th} term of this sequence.

【Solution】

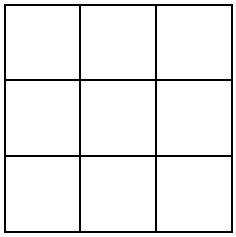
Write down the terms one by one as:

1, 2, 4, 6, 3, 9, 12, 8, 10, 5,
15, 18, 14, 7, 21, 24, 16, 20, 22, 11,
33, ⋯.

The 20^{th} term is 11.

Answer：011

22. Shade 3 unit squares on the 3×3 grid below, such that there must be two shaded squares in some row and two shaded squares in some column but it must not have three shaded squares in any row or column. Find the total number of ways in shading the figure.



【Solution】

There is one black square (say A); one black square (say B) on the same row as A; one black square (say C) on the same column as A. All other squares are white. A is special since it is the only black square with black square one its same row and with black square on its same column. B is special since it is the only square on the same row as A. C is special since it is the only square on the same column as A. There are 9 ways of choosing A; 2 ways of choosing B; 2 ways of choosing C. By the principle of multiplication, there are $9\times2\times2=36$ ways of coloring.

Answer：036

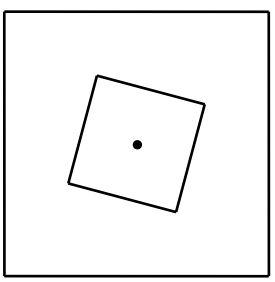
23. Three robots are programmed to count numbers. Robot A starts with the number 20 and it counts by increasing the number by 11 every second. Robot B starts with the number 2018 and it counts by decreasing the number by 100 every second. Robot C starts with some number and it counts by decreasing the number by 1 for the 1^{st} second, 2 for the 2^{nd} second, 3 for the 3^{rd} second and so on. If all three Robots started counting at the same time, and after some time, all of them got the same number, then what is the number that Robot C starts with?

【Solution】

It takes $(2018-20)\div(100+11)=18$ seconds for robot A and B to get the same number. This number is $2018-18\times100=218$. In 18 seconds, robot C decreases its number by $1+2+3+\cdots+18=171$. Robot C starts with $218+171=389$.

Answer：389

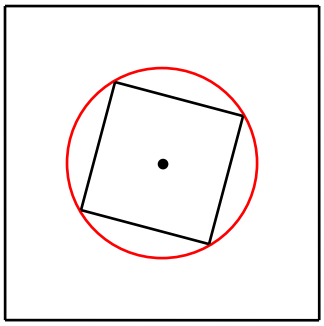
24. Two squares having the same center are shown in the figure below. The larger square has a side length of 20 cm. The smaller square rotates around its center. During the rotation, it is known that the minimum distance between the vertices of the smaller square and the sides of the larger square is 4 cm. What is the area, in cm^2, of the smaller square?



【Solution】

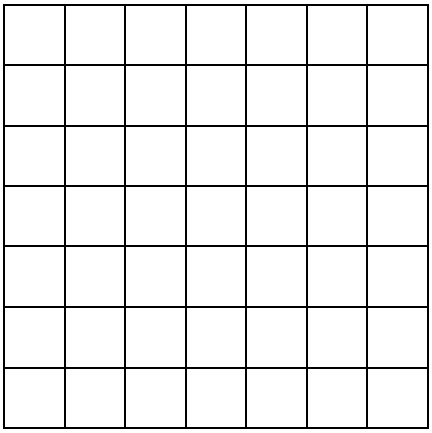
The locus of the vertices of the small square is on a circle of diameter the same as diagonal of the small square, as in the figure. Diagonal of the small square is then $20-4\times 2=12\,\text{cm}$.

Area of the small square is $\dfrac{12\times 12}{2}=72\,\text{cm}^2$.



Answer：072

25. Cut the 7×7 square table below into rectangles along grid lines such that no two rectangles are identical. What is the maximum number of rectangles one can get? (Note: A square is considered a rectangle.)



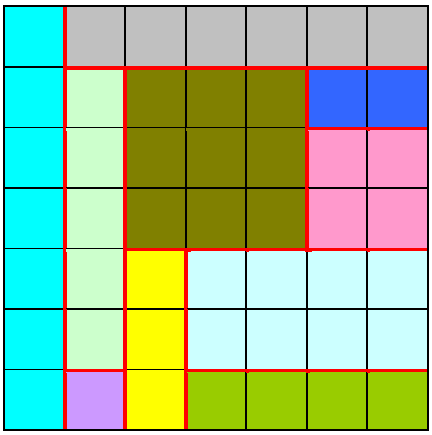
【Solution】

For more rectangles, the rectangles we get need to be as small in area as possible. There one rectangle of area 1, 2 or 3; two rectangles of area 4(1×4 and 2×2); one of area 5; two of area 6(1×6 and 2×3); one of area 7 (1×7); two of area 8 (1×8 and 2×4). These are the first 11 rectangles of smallest area. Since

$$1+2+3+4+4+5+6+6+7+8=46<7\times 7=49,$$

$$7\times 7=49<1+2+3+4+4+5+6+6+7+8+8=54,$$

It shows that there are at most 10 different rectangles. The figure below shows an example:



Answer：010