8th International Mathematics Assessments for Schools (2018-2019)

Junior Division Round 2

Time: 120 minutes

Printed Name: ________________ *Code*: __________ *Score*: __________

Instructions:

- Do not open the contest booklet until you are told to do so.
- Be sure that your name and code are written on the space provided above.
- Round 2 of IMAS is composed of three parts; the total score is 100 marks.
- Questions 1 to 5 are given as a multiple-choice test. Each question has five possible options marked as A, B, C, D and E. Only one of these options is correct. After making your choice, fill in the appropriate letter in the space provided. Each correct answer is worth 4 marks. There is no penalty for an incorrect answer.
- Questions 6 to 13 are a short answer test. Only Arabic numerals are accepted; using other written text will not be honored or credited. Some questions have more than one answer, as such all answers are required to be written down in the space provided to obtain full marks. Each correct answer is worth 5 marks. There is no penalty for incorrect answers.
- Questions 14 and 15 require a detailed solution or process in which 20 marks are to be awarded to a completely written solution. Partial marks may be given to an incomplete presentation. There is no penalty for an incorrect answer.
- Use of electronic computing devices is not allowed.
- Only pencil, blue or black ball-pens may be used to write your solution or answer.
- Diagrams are not drawn to scale. They are intended as aids only.
- After the contest the invigilator will collect the contest paper.

The following area is to be filled in by the judges; the contestants are not supposed to mark anything here.

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total Score	Signature
Score																	
Score																	

Junior Division Round 2

Questions 1 to 5, 4 marks each

1. Which of the following statement is false?
   （A） If a divides b and k is an integer, then a divides kb.
   （B） If a divides b and b divides c, then a divides c.
   （C） If $a = bc$, and b, c are positive integers, then a is divisible by b or c.
   （D） If b divides a and c divides a, then bc divides a.
   （E） If $p|bc$, then $p|b$ or $p|c$, where p is a prime number, b and c are integers.

Answer：________________

2. How many positive integers from 1 to 2019 can be expressed as $n^3 - 3n^2 + 2n$, where n is an positive integer?
   （A） 11 （B） 12 （C） 13 （D） 44 （E） 45

Answer：________________

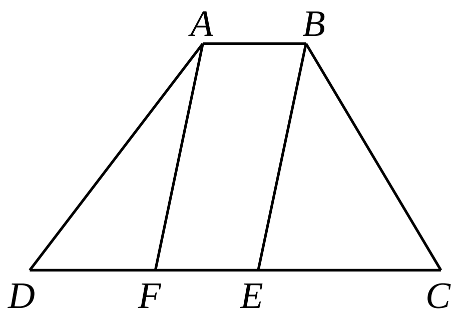
3. The perimeter of an isosceles triangle is known to be 32 cm and length of each side is an integer, in cm. How many different non-identical such triangles are there?
   （A） 5 （B） 6 （C） 7 （D） 8 （E） 9

Answer：________________

4. Given four distinct non-zero digits a, b, c and d, if $\overline{ab} + \overline{cd} = \overline{dc} + \overline{ba}$, then this expression is called a palindrome expression and the sum of the two numbers $\overline{ab} + \overline{cd}$ is called a palindrome sum. For example, $53 + 46 = 64 + 35 = 99$. What is the minimum possible value of a palindrome sum?
   （A） 22 （B） 33 （C） 44 （D） 55 （E） 99

Answer：________________

5. In the figure below, the area of the trapezoid $ABCD$ is 100 cm^2, the area of parallelogram $ABEF$ is 40 cm^2 and $CD = 10$ cm. What is the length, in cm, of AB?

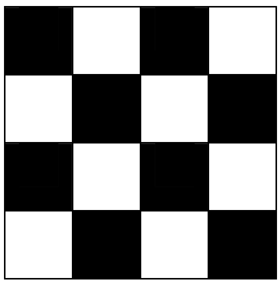


（A）2 （B）2.5 （C）4 （D）5 （E）10

Answer：______________

Questions 6 to 13, 5 marks each

6. Four identical chess pieces are to be placed into a 4×4 chess board that is colored black and white alternately, as shown in the figure below. You can place at most one chess piece on each square. All chess pieces must be placed in squares of the same color and no two pieces are on the same row or on the same column. In how many different ways can the chess pieces be placed?

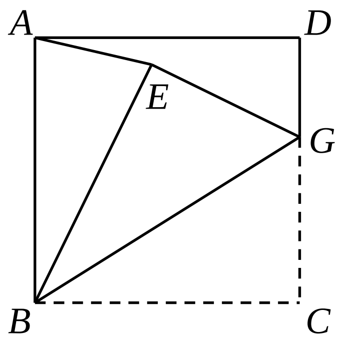


Answer：______________ ways

7. It is known that for any $x \neq \pm\frac{1}{2}$, then $\dfrac{a}{x+\frac{1}{2}} + \dfrac{b}{x-\frac{1}{2}} = \dfrac{24x+4}{4x^2-1}$. What is the value of $a+b$?

Answer：______________

8. In the figure below, $ABCD$ is a square and the point G lies on the side CD. Now, flip triangle BCG along BG to get a new triangle BEG. If $\angle CBG = 32°$, then what is the size, in degrees, of $\angle DAE$?



Answer： ________________ °

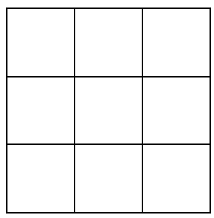
9. How many ordered triples (a, b, c) of integers that satisfy the equation $|ab|+|bc|+|ca|=9$?

Answer： ________________ triples

10. If $x+y=\sqrt{4z-1}$, $y+z=\sqrt{4x-1}$ and $z+x=\sqrt{4y-1}$, where x, y and z are real numbers, then what is value of $x+y+z$?

Answer： ________________

11. Place 9 distinct positive integers into each of the unit squares of the 3×3 square below, with one number in each unit square, such that the sum of the numbers in every 2×2 square is 50. What is the minimum possible value of sum of the 9 integers?



Answer： ________________

12. The three side lengths of an acute triangle are consecutive integers, in cm, and it is known that one altitude on one side is 12 cm. What is the area, in cm^2, of this triangle?

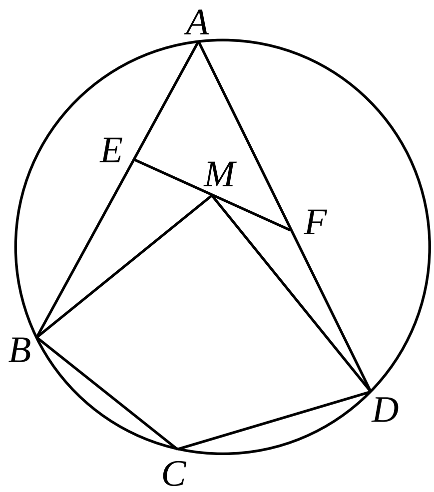
Answer： ________________ cm^2

13. Arrange all positive integers less than 30 and not divisible by 3 in an increasing order, and compute the sum of the reciprocals of product of every three consecutive numbers, that is $S=\dfrac{1}{1\times 2\times 4}+\dfrac{1}{2\times 4\times 5}+\cdots+\dfrac{1}{26\times 28\times 29}$. Now, if we reduce S into its simplest form, then what would be the value of the numerator?

Answer：________________

Questions 14 to 15, 20 marks each
(Detailed solutions are needed for these two problems)

14. In the figure below, a convex quadrilateral $ABCD$ is inscribed in circle O. Points E and F are on segments AB and AD respectively such that $BE=CD$ and $DF=BC$. If point M is the midpoint of EF, then prove that $BM\perp DM$.



15. A robot can generate a set of digit codes according to user's reasonable instructions. Wayne gives out the following commands:
    (1) Each code is a four-digit number (nonzero for the left-most digit).
    (2) Every two codes in the set have identical digits at no more than two corresponding positions.

    Find the maximum number of codes in a set the robot can generate.

Answer： ________________ codes