The Seventh International Young Mathematicians' Convention
IYMC-Mathematica 2016
2nd to 5th December 2016
Organised by: City Montessori School, Gomti Nagar Campus-I, Lucknow, India

International Young Mathematicians' Convention
Senior Level

Individual Contest
Time limit: 90 minutes

Information:

- You are allowed 90 minutes for this paper, consisting of 8 questions to which only numerical answers are required.
- Each question is worth 10 points. No partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers asked for. For questions asking for several answers, full credit will only be given if all correct answers are found.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your name, your contestant number and your team's name on the answer sheet.
- Enter your answers in the spaces provided on the answer sheet.
- You must use either a pencil or a ball-point pen which is either black or blue.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing the question paper, your answer sheet and all scratch papers.

Team: ______________ ***Name:*** __________________ ***No.:*** _____________ ***Score:*** _________

1. Find the number of all real solutions of the system of equations

$$(x_3+x_4+x_5)^5=3888x_1$$
$$(x_4+x_5+x_1)^5=3888x_2$$
$$(x_5+x_1+x_2)^5=3888x_3$$
$$(x_1+x_2+x_3)^5=3888x_4$$
$$(x_2+x_3+x_4)^5=3888x_5$$

2. What is the simplified value of $\sqrt{5+\sqrt{5^2+\sqrt{5^4+\sqrt{5^8+\cdots}}}}$?

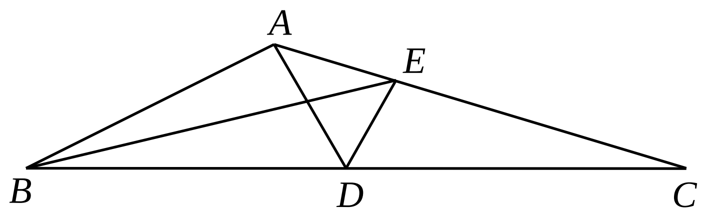
3. If a is a positive integer so that a^2+2016^2 is divisible by $2016a$, find the number of the possible values of a.

4. Let $f(x)=\dfrac{x+20}{x}$ and $f_n(x)=f(f(\cdots(f(x))\cdots))$ be the n-fold composite off.

For example, $f_2(x)=\dfrac{\frac{x+20}{x}+20}{\frac{x+20}{x}}=\dfrac{21x+20}{x+20}$ and $f_3(x)=\dfrac{\frac{21x+20}{x+20}+20}{\frac{21x+20}{x+20}}=\dfrac{41x+420}{21x+20}$.

Let S be the complete set of real solutions of $f_n(x)=x$. What is the maximal number of the elements in S ?

5. Given D and E are points on the sides BC and CA, respectively, of triangle ABC. If $\angle ADC=130°$, $\angle BEA=25°$ and BE bisects $\angle ABC$, as shown in the diagram below. Find the measure of $\angle EDC$, in degrees.



6. The sum of ten numbers on a circle is 2016. The sum of any three numbers in a row is at least 585. Determine the minimal number n such that for any such set of ten, none of them is greater than n.

7. Anna tosses 2016 coins and Boris tosses 2017 coins. Whoever has more heads wins. If they have the same number of heads, then Anna wins. What is the probability of Anna winning?

8. In triangle ABC, $AC = BC$. D is a point on AB such that the inradius of triangle CAD is equal to the exradius of triangle BCD opposite C, as shown in the right diagram. If the length of the altitude AH is 36 cm, find the length of this common radius.