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The Seventh International Young Mathematicians' Convention
IYMC-Mathematica 2016
2nd to 5th December 2016
Organised by: City Montessori School, Gomti Nagar Campus-I, Lucknow, India

# International Young Mathematicians' Convention
# Senior Level
# Team Contest
Time limit: 60 minutes

**Information:**

- You are allowed 60 minutes for this paper, consisting of 6 questions printed on separate sheets. For questions 1, 3 and 5, only numerical answers are required. For questions 2, 4 and 6, full solutions are required.
- Each question is worth 40 points. For odd-numbered questions, no partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. For even-numbered questions, partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

**Instructions:**

- Write down your team's name on the spaces provided on every question sheet.
- Enter your answers in the spaces provided after individual questions on the question paper.
- During the first 10 minutes, the three team members examine the questions together, with discussion. Then they distribute the questions among themselves, with each team member allotted at least 1 question.
- During the next 50 minutes, the three team members write down the solutions to their allotted problems on the respective question sheets, with no further communication among themselves.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing all question sheets and all scrap papers.

***Team***: ______________________________ ***Score:*** ____________________

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| No. | **1** | **2** | **3** | **4** | **5** | **6** | **Total** |
|---|---|---|---|---|---|---|---|
| Score | | | | | | | |
| Score | | | | | | | |

# *International Young Mathematicians' Convention*

## *Senior Level*

# TEAM CONTEST

Team：________________________ Score：______________

1. If $x$ and $y$ are positive real numbers, find the smallest value of

$$\sqrt{225-15\sqrt{2}x+x^2}+\sqrt{200-20y+y^2}+\sqrt{x^2-\sqrt{2}xy+y^2}.$$

Answer: ______________________________

# International Young Mathematicians' Convention

## Senior Level

## TEAM CONTEST

Team：____________________ Score：_______________

2. The sum of 2016 real numbers is 2017 and each of them is less than $\frac{2017}{2015}$. Prove that the sum of any two of the numbers is greater than or equal to $\frac{2017}{2015}$.

# *International Young Mathematicians' Convention*
## *Senior Level*

# TEAM CONTEST

Team: ____________________ Score: ____________

3. There are 2016 unit cubes, each of which can be painted black or white. How many values of $n$ is it possible to construct an $n\times n\times n$ cube with $n^3$ unit cubes such that each cube shares a common face with exactly three cubes of the opposite colour?

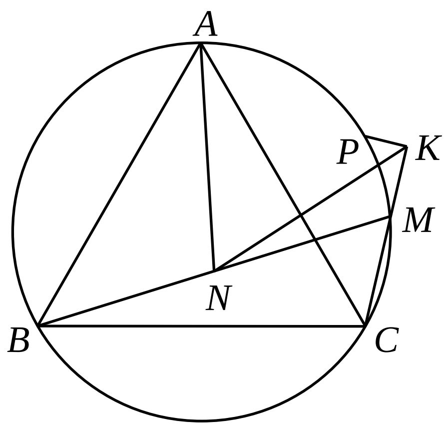
Answer: ____________________

# International Young Mathematicians' Convention
# Senior Level

# TEAM CONTEST

Team：________________________ Score：______________

4. $P$ is the midpoint of the arc $AC$ of the circumcircle of an equilateral triangle $ABC$. $M$ is another point on this arc and $N$ is the midpoint of $BM$. $K$ is the projection of $P$ on the line $MC$, as shown in the diagram below. If the length of $NA$ is 19 cm, find the length of $NK$ in cm.




Answer: ______________________________ cm

# International Young Mathematicians' Convention

## Senior Level

# TEAM CONTEST

Team: ____________________ Score: ______________

5. Let $S_n$ denote the $n$-th sequence so that every word in a sequence consists only of the letters $A$ and $B$. The first word has only one letter $A$. For $k \geq 2$, the $k$-th word is obtained from the $(k-1)$-th by simultaneously replacing every $A$ by $AAB$ and every $B$ by $A$. Then every word is an initial part of the next word. For example, $S_1 = A$, $S_2 = AAB$, $S_3 = AABAABA$ and $S_4 = AABAABAAABAABAAAB$.
Find the number of $A$s in $S_{10}$.

Answer: ______________________________

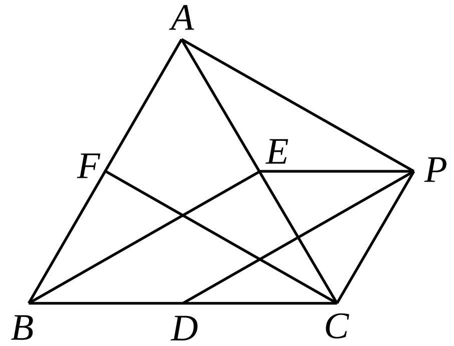
# *International Young Mathematicians' Convention*

# *Senior Level*

# TEAM CONTEST

Team：____________________ Score：____________

6. $D$, $E$ and $F$ are points on the sides $BC$, $CA$ and $AB$, respectively, of triangle $ABC$ such that $AD$, $BE$ and $CF$ are concurrent. The area of triangle $ABC$ is 2016 cm$^2$. If there exists a point $P$ such that both $BDPE$ and $AFCP$ are parallelograms, as shown in the diagram below. Find the area of triangle of $DEF$, in cm$^2$.




Answer: ____________________ cm