The Eighth International Young Mathematicians' Convention

IYMC-Mathematica 2018

2nd to 5th December 2018

Organised by: City Montessori School, Gomti Nagar Campus-1, Lucknow, India

International Young Mathematicians' Convention

Senior level

Individual Contest

Time limit: 90 minutes

Information:

- You are allowed 90 minutes for this paper, consisting of 8 questions to which only numerical answers are required.
- Each question is worth 10 points. No partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your name, your contestant number and your team's name on the answer sheet.
- Enter your answers in the spaces provided on the answer sheet.
- You must use either a pencil or a ball-point pen which is either black or blue.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing the question paper, your answer sheet and all scrap papers.

Team*:**__________ ***Name*:**____________ ***No.*:** __________ ***Score:________

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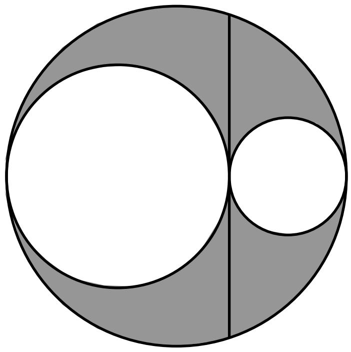
No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Total**
Score									
Score									

1. If a and b are real numbers such that $a+6b=0$, then what is the remainder when $x^3+\dfrac{a-b}{a+b}x^2+\dfrac{2b}{a+b}x+1$ is divided by $x+1$?

2. Find the remainder when $2017\times2015\times2013\times...\times1+2018\times2016\times2014\times...\times2$ is divided by 2019.

3. The length of the chord of a circle is 2 cm. The two smaller circles are tangent to the large circle and also to each other at the midpoint of the chord. Find the area, in cm^2, of the shaded region.

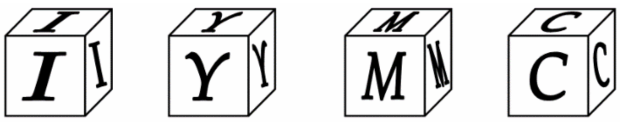


4. Find the sum of the roots of the equation :

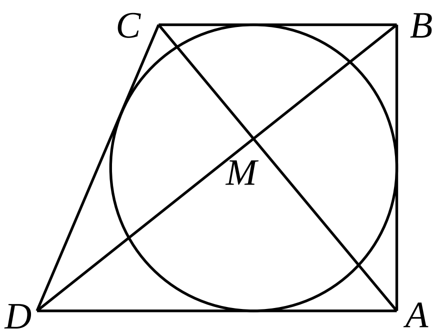
$$\sqrt{3x^2+x-1}+\sqrt{x^2-2x-3}=\sqrt{3x^2+3x+5}+\sqrt{x^2+3}.$$

5. There are seven tokens having different weights namely 1g, 2g, 4g, 8g, 16g, 32g and 64g. How many different ways can we get a weight of 21g using a regular two-sided weighing scale? (Note: Each token may be placed on either pan of the balance, and it is not necessary to use all the tokens in each weigh).

6. There are 8 wooden blocks, two of these wooden blocks have the letter "I" written on each face, another two wooden blocks have the letter "Y" written on each face, another two wooden blocks have the letter "M" written on each face and then last two wooden blocks have the letter "C" written on each face. What is the probability that when we take four out of the eight wooden blocks we can spell out the word "IYMC"?



7. In the figure, *ABCD* is a quadrilateral that has an incircle with radius 10 cm. Side *AD* is parallel to *BC* and perpendicular to *AB* and point *M* is intersection between *AC* and *BD*. Determine the area, in cm^2, of triangle *DCM*.



8. If $a^6+b^6+c^6+d^6+e^6+f^6-1=6abcdef$, where *a, b, c, d, e* and *f* are integers. How many possible values are there for $a+b+c+d+e+f$?