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第七屆東南數學競賽

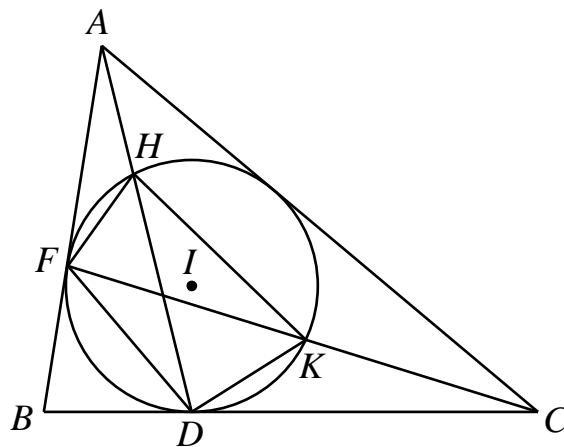
Southeast Mathematical Olympiad 2010

First Day 2010/08/17 08:00-12:00
Lukang Senior High School, Lukang, Changhua, Taiwan

1. Let $a, b, c \in \{0, 1, 2, \dots, 9\}$. The quadratic equation $ax^2 + bx + c = 0$ has a rational root. Prove that the three-digit number \overline{abc} is not a prime number.

2. For any set $A = \{a_1, a_2, \dots, a_m\}$, let $P(A) = a_1 a_2 \cdots a_m$. Let $n = C_{2010}^{99}$ and let A_1, A_2, \dots, A_n be all 99-element subsets of $\{1, 2, \dots, 2010\}$. Prove that $2011 \mid \sum_{i=1}^n P(A_i)$.

3. The incircle of triangle ABC touches BC at D and AB at F , intersects the line AD again at H and the line CF again at K . Prove that $\frac{FD \times HK}{FH \times DK} = 3$.



4. Let a and b be positive integers such that $1 \leq a < b \leq 100$. If there exists a positive integer k such that $ab \mid (a^k + b^k)$, we say that the pair (a, b) is good. Determine the number of good pairs.



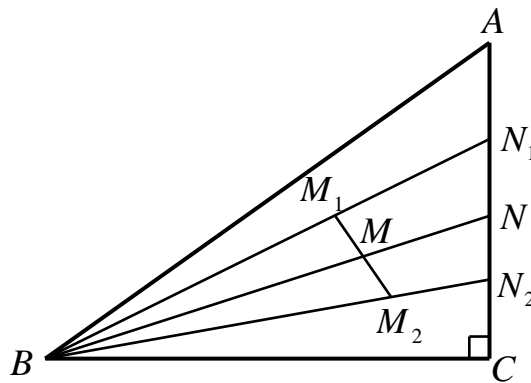
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5. ABC is a triangle with a right angle at C . M_1 and M_2 are two arbitrary points inside ABC , and M is the midpoint of M_1M_2 . The extensions of BM_1 , BM and BM_2 intersect AC at N_1, N and N_2 respectively.

Prove that
$$\frac{M_1N_1}{BM_1} + \frac{M_2N_2}{BM_2} \geq 2 \frac{MN}{BM}.$$



6. Let \mathbb{N}^* be the set of positive integers. Define $a_1 = 2$, and for $n=1, 2, \dots$,

$$a_{n+1} = \min \left\{ \lambda \mid \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} + \frac{1}{\lambda} < 1, \lambda \in \mathbb{N}^* \right\}.$$

Prove that $a_{n+1} = a_n^2 - a_n + 1$ for $n=1, 2, \dots$.

7. Let n be a positive integer. The real numbers a_1, a_2, \dots, a_n and r_1, r_2, \dots, r_n are such that $a_1 \leq a_2 \leq \dots \leq a_n$ and $0 \leq r_1 \leq r_2 \leq \dots \leq r_n$.

Prove that
$$\sum_{i=1}^n \sum_{j=1}^n a_i a_j \min(r_i, r_j) \geq 0.$$

8. A_1, A_2, \dots, A_8 are fixed points on a circle. Determine the smallest positive integer n such that among any n triangles with these eight points as vertices, two of them will have a common side.