第七屆東南數學競賽

Southeast Mathematical Olympiad 2010

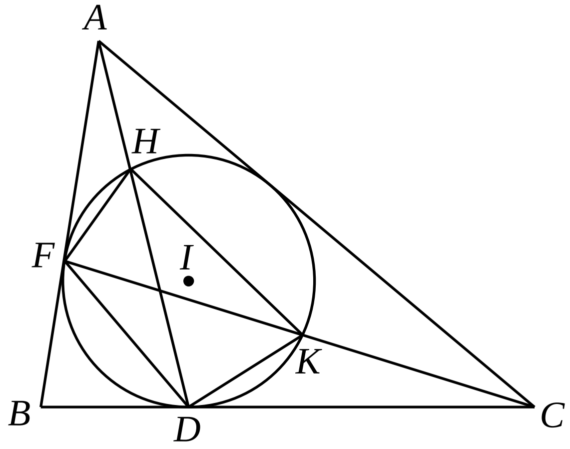
First Day 2010/08/17 08:00-12:00

Lukang Senior High School, Lukang, Changhua, Taiwan

1. Let $a, b, c \in \{0,1,2,\cdots,9\}$. The quadratic equation $ax^2+bx+c=0$ has a rational root. Prove that the three-digit number $\overline{abc}$ is not a prime number.

2. For any set $A=\{a_1,a_2,\cdots,a_m\}$, let $P(A)=a_1a_2\cdots a_m$. Let $n=C_{2010}^{99}$ and let A_1、A_2、$\cdots$、A_n be all 99-element subsets of $\{1,2,\cdots,2010\}$. Prove that $2011\left|\sum_{i=1}^{n}P(A_i)\right.$。

3. The incircle of triangle *ABC* touches *BC* at *D* and *AB* at *F*, intersects the line *AD* again at *H* and the line *CF* again at *K*. Prove that $\dfrac{FD\times HK}{FH\times DK}=3$.



4. Let *a* and *b* be positive integers such that $1\le a<b\le 100$. If there exists a positive integer *k* such that $ab\mid(a^k+b^k)$, we say that the pair (*a*, *b*) is good. Determine the number of good pairs.

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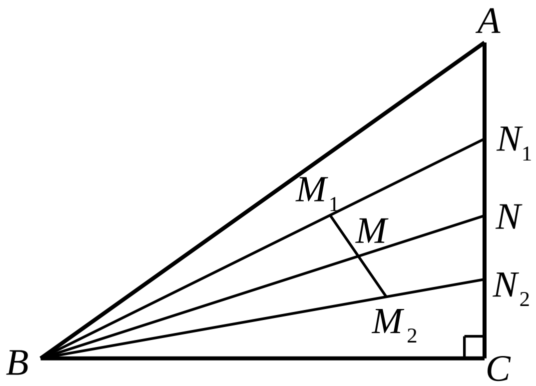
5. *ABC* is a triangle with a right angle at *C*. M_1 and M_2 are two arbitrary points inside *ABC*, and *M* is the midpoint of M_1M_2. The extensions of BM_1, BM and BM_2 intersect *AC* at N_1, *N* and N_2 respectively.

Prove that $\dfrac{M_1N_1}{BM_1}+\dfrac{M_2N_2}{BM_2}\geq 2\dfrac{MN}{BM}$.



6. Let N^* be the set of positive integers. Define $a_1=2$, and for *n*=1, 2, ⋯,

$$a_{n+1}=\min\{\lambda \mid \frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}+\frac{1}{\lambda}<1, \lambda\in \mathrm{N}^*\}.$$

Prove that $a_{n+1}=a_n^2-a_n+1$ for *n*=1, 2, ⋯.

7. Let *n* be a positive integer. The real numbers a_1、a_2、⋯、a_n and r_1、r_2、⋯、r_n are such that $a_1\leq a_2\leq\cdots\leq a_n$ and $0\leq r_1\leq r_2\leq\cdots\leq r_n$.

Prove that $\sum_{i=1}^{n}\sum_{j=1}^{n}a_ia_j\min(r_i,r_j)\geq 0$。

8. A_1, A_2, ⋯, A_8 are fixed points on a circle. Determine the smallest positive integer *n* such that among any *n* triangles with these eight points as vertices, two of them will have a common side.