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# Individual Contest

Time limit: 120 minutes

2011/11/15

## Section A.

In this section, there are 12 questions. Fill in the correct answer on the space provided at the end of each question. Each correct answer is worth 5 points.

1. Let  $6! = a! \times b!$  where  $a > 1$  and  $b > 1$ . What is the value of  $a \times b$ ?

Answer : \_\_\_\_\_

2. If  $3^{2011} + 3^{2011} + 3^{2011} + 3^{2011} + 3^{2011} + 3^{2011} + 3^{2011} + 3^{2011} + 3^{2011} = 3x$ , then what is the value(s) of  $x$ ?

Answer : \_\_\_\_\_

3. The perimeter of a square lawn consists of four straight paths. Anuma and Gopal started at the same corner at the same time, running clockwise at constant speeds of 12 and 10 kilometres per hour respectively. Anuma finished one lap around the lawn in 60 seconds. For how many seconds were Anuma and Gopal together on the same path in one lap?

Answer : \_\_\_\_\_ seconds

4. If  $a_1 = 12 \times 8$ ,  $a_2 = 102 \times 98$ ,  $a_3 = 1002 \times 998$ ,  $a_4 = 10002 \times 9998$ ,  $\dots$  and  $S = a_1 + a_2 + a_3 + a_4 + \dots + a_{20}$ , then what is the sum of all the digits of  $S$ ?

Answer : \_\_\_\_\_

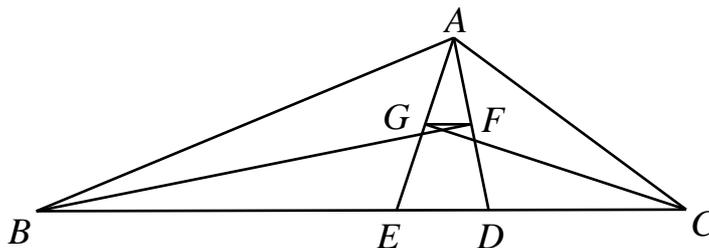
5. How many different bags, each containing 10 marbles of four different colours, can be bought if each bag must contain at least one marble of each colour?

Answer : \_\_\_\_\_ bags

6. Find the value of  $n$ , if the sum of even positive integers between  $n^2 - n + 1$  and  $n^2 + n + 1$  is a number between 2500 and 3000.

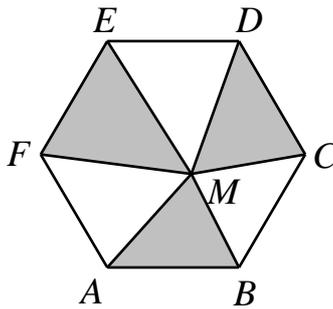
Answer : \_\_\_\_\_

7. In triangle  $ABC$ ,  $BC = 56$ ,  $CA = 25$  and  $AB = 39$ .  $D$  and  $E$  are points on  $BC$  such that  $BD = BA$  and  $CE = CA$ . The bisector of  $\angle B$  meets  $AD$  at  $F$ , and the bisector of  $\angle C$  meets  $AE$  at  $G$ . Determine the length of  $FG$ .



Answer : \_\_\_\_\_

8. The length of each side of a regular hexagon is 10.  $M$  is a point in it and  $BM = 8$ . What is the sum of the areas of triangles  $ABM$ ,  $CDM$  and  $EFM$ ?



Answer : \_\_\_\_\_

9. Find the product  $xyz$  where  $x, y$  and  $z$  are positive integers and  $2^x + 7^y = z^4$ .

Answer : \_\_\_\_\_

10. To determine the source of news in the employees of a company,  $N$  people have been interviewed and the following results have been obtained :

- A) 50 people use TV as well as other sources
- B) 61 people do not use radio
- C) 13 people do not use newspaper
- D) 74 people use at least two sources

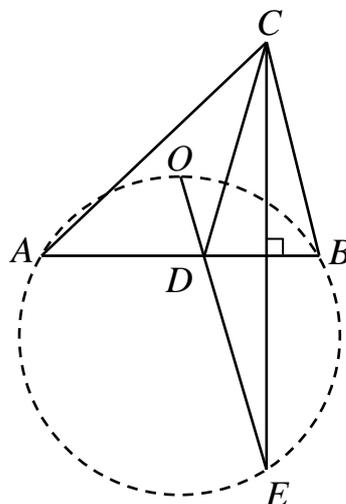
If the sources contain only newspaper, TV and radio, what is the least value of  $N$ ?

Answer : \_\_\_\_\_

11. Each of 2011 numbers is 1, 0 or  $-1$ . Determine the minimum value of the sum of all products of these numbers, taken two at a time.

Answer : \_\_\_\_\_

12. In acute triangle  $ABC$ ,  $CD$  is bisector of  $\angle C$ ,  $O$  is the circumcenter. The perpendicular from  $C$  to  $AB$  meets line  $OD$  in a point lying on the circumcircle of  $AOB$ . Find  $\angle C$ , in degree.



Answer : \_\_\_\_\_

**Section B.**

Answer the following 3 questions. Show your detailed solution on the space provided after each question. Each question is worth 20 points.

1. Let  $n$  be a positive integer such that  $n! = 1 \times 2 \times 3 \times \cdots \times n$ . What is the result of  $\frac{3}{1!+2!+3!} + \frac{4}{2!+3!+4!} + \frac{5}{3!+4!+5!} + \frac{6}{4!+5!+6!} + \frac{7}{5!+6!+7!} + \frac{8}{6!+7!+8!}$ ?

*Answer :* \_\_\_\_\_

2. There is an integral number in each cell of an  $n \times n$  table. In each move, we may change the signs of all the numbers in any row or column. Prove that after a finite number of such moves, it is possible to have the sum of the numbers in each row and column to be non-negative.

3. In an acute triangle  $ABC$ , the longest altitude  $AH$  has the same length as the median  $BM$ . Prove that  $\angle ABC \leq 60^\circ$ .

