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Team Contest

Time limit: 60 minutes 2011/11/15

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on the first page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty, each problem is worth 40 points and the total is 400 points. Each question is printed on a separate sheet of paper. Complete solutions of problem 1, 2, 3, 4, 5, 8 and 9 are required for full credits. Partial credits may be awarded. Only Arabic Numerical answer or drawing in Problem number 6, 7 and 10 are needed.
- Diagrams are NOT drawn to scale. They are intended only as aids.
- The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each student must solve at least one problem by themselves. Each will then have 35 minutes to write the solutions of their allotted problem independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together.
- No calculator or calculating device or electronic devices are allowed.
- Answer the problems with pencil, blue or black ball pen.
- All papers shall be collected at the end of this test.

Team Name:

<table>
<thead>
<tr>
<th>Problem 1</th>
<th>Problem 2</th>
<th>Problem 3</th>
<th>Problem 4</th>
<th>Problem 5</th>
<th>Total Score</th>
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| Problem 6 | Problem 7 | Problem 8 | Problem 9 | Problem 10 |
|-----------|-----------|-----------|-----------|------------|-------------|

Malpi International School
Panauti, Kavre, Nepal.
1. Find all possible three-digit numbers $abc$ such that when the tens digit of the given three-digit number was deleted, a two-digit number $ac$ will be formed so that $abc = 9 \times ac + 4 \times c$. For example, $155 = 9 \times 15 + 4 \times 5$. 

**ANSWER:**
2. Find all positive real solutions of the simultaneous equations:

\[ x + y^2 + z^3 = 3 \quad (1) \]
\[ y + z^2 + x^3 = 3 \quad (2) \]
\[ z + x^2 + y^3 = 3 \quad (3) \]

**ANSWER:** \((x, y, z) = \)
3. Prove that $a^2 + b^2 + c^2 + d^2 + ab + ac + ad + bc + bd + cd \geq 20$, where $a, b, c$ and $d$ are positive real numbers such that $abcd = 4$. 
4. Consider a $10 \times 15$ chessboard. We wish to travel the shortest distance along the grid lines from the bottom left corner to the top right corner. What is the ratio of the number of paths which start with a move to the right to the number of paths which start a move up?

**Answer:**
5. The figure below shows a triangle $ABC$. $D$ is the midpoint of $BC$ and $E$ lies on $AC$ such that $AE : EC = 2 : 3$. If $F$ is a point of $AB$ such that the area of triangle $DEF$ is twice the area of triangle $BDF$, find the ratio of $AF : FB$.

\[ \frac{AF}{FB} = \frac{2}{3} \]

**Answer: \( \frac{AF}{FB} = \frac{2}{3} \)**
6. The 25 squares of a 5x5 chessboard are labeled as shown in the diagram below. The chessboard is cut up into a unit square and eight copies the shape shown in the diagram below on the right. These copies may be rotated. Of the 25 squares, identify by their labels those which may be the unit square.

\[
\begin{array}{cccc}
A & B & C & D & E \\
F & G & H & I & J \\
K & M & N & O & P \\
Q & R & S & T & U \\
V & W & X & Y & Z \\
\end{array}
\]

\[
\begin{array}{c}
M \\
C \\
K \\
Q \\
A \\
\end{array}
\]

**Answer:**

A

B

C

D

E

F

G

H

I

J

K

M

N

O

P

Q

R

S

T

U

V

W

X

Y

Z
7. Find 38 consecutive positive integers, such that the sum of the digits of each of them is not divisible by 11.

**Answer:**
8. Circle with center $I$ is inscribed in triangle $ABC$ and touches the sides $AC$ and $BC$ in points $M$ and $N$. The line $MN$ intersect the line $AB$ at $P$, as $B$ is between $A$ and $P$. If $BP = CM$, find $\angle AIC$, in degree.

**Answer:** $\boxed{90}$
TEAM CONTEST

Team: __________________________  Score: __________

9. Nikolai and Peter share 21 peanuts as follows:
   Peter will divide the whole piles into two, with at least two peanuts in each, after
   which Nikolai will subdivide each pile into two, with at least one peanut in each.
   Nikolai agrees in advance to take the middle two piles.
   No matter how Peter divided, what is the maximum number of peanuts that
   Nikolai is guaranteed to get? What is Nikolai’s strategy?

ANSWER: __________________________
In a soccer tournament, there are 10 participating teams. Each team plays with each of the others once. A win used to be worth 2 points, but is now worth 3 points. A draw is worth 1 point, and a loss is worth no points. It happens that there is a winning team in the tournament under the new system, and yet finishes last under the old system. Reconstruct the possible result of each game in the below table. Using W represent team A wins team B, using 0 represent team A loss to team B and 1 represent team A and team B are tied.

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<th>Total points under older system</th>
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