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Individual Contest

Time limit: 120 minutes

2013/12/28

Name: _____ No. _____ Team: _____ Score: _____

Section A.

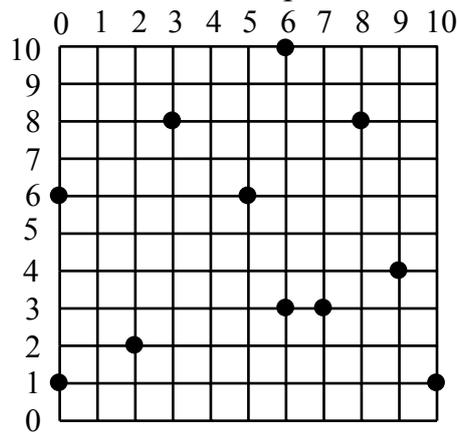
In this section, there are 12 questions. Fill in the correct answer on the space provided at the end of each question. Each correct answer is worth 5 points. Be sure to read carefully exactly what the question is asking.

1. In the following addition, the same letter stands for the same digit and different letters stand for different digits. What is the maximum value of the five-digit number *AITMO*?

$$\begin{array}{r}
 A \quad I \quad T \quad M \quad O \\
 A \quad I \quad T \quad M \quad O \\
 A \quad I \quad T \quad M \quad O \\
 + \quad A \quad I \quad T \quad M \quad O \\
 \hline
 T \quad A \quad L \quad E \quad N \quad T
 \end{array}$$

Answer : _____

2. The diagram below show a city with 11 streets numbered 0 to 10 from West to East, and 11 avenues numbered 0 to 10 from South to North. Eleven students live in junctions marked by black dots. They decide to meet one another at one of the junctions which minimizes the total walking distances from their homes along the grid lines to that junction. What is the product of its street number and avenue number?



Answer : _____

3. The first digit from the left of a five-digit number is 6, the third from the left is 7 and the fourth 2. When this number is divided by 99, the remainder is 0. What is the quotient?

Answer : _____

4. Determine the order pair of real numbers (x, y) that satisfy the equation $(4x^2 + 12x + 13)(4y^2 - 20y + 33) = 32$.

Answer : _____

5. Each of A, B, C and D either always tells the truth or always lies. They are asked how many of them are truth-tellers. They make the following responses.

- A: The number is 0, 1 or 3.
- B: The number is 1, 2 or 3.
- C: Excluding me, the number is 0, 1 or 3.
- D: Excluding me, the number is 1, 2 or 3.

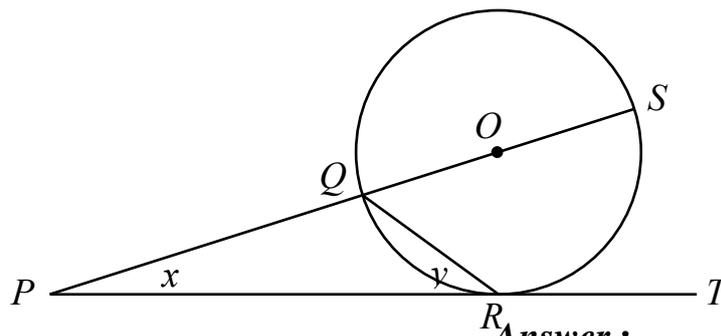
What is the correct number?

Answer : _____

6. Consider the nine numbers $1!, 2!, 3!, 4!, 5!, 6!, 7!, 8!$ and $9!$. What is the maximum number of them we can choose so that the product of the chosen numbers is the square of an integer?

Answer : _____

7. In the figure below, PT touches a circle with centre O at R . Diameter SQ when produced meets PT at P . If $\angle SPR = x^\circ$ and $\angle QRP = y^\circ$, what is the value of $x+2y$?



Answer : _____

8. What is the maximum value of n such that the number $n!$ can be represented as a product of $(n-3)$ consecutive positive integers?

Answer : _____

9. What is the largest positive integer less than $(\sqrt{6} + \sqrt{5})^6$?

Answer : _____

10. The Fibonacci sequence starts with 1, 1, 2, 3, 5, 8, 13, ... (Start from the 3rd term, each term is the sum of the two previous terms). Let F_n be the n th term of this

sequence. S is defined as $S = \frac{1}{2} + \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{5}{32} + \dots + \frac{F_n}{2^n} + \dots$.

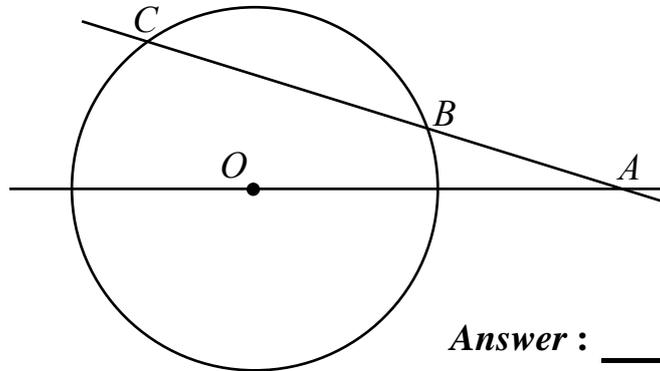
Calculate the value of S .

Answer : _____

11. There are 4 sixteen-year-old girls, 5 fifteen-year-old girls and 7 fourteen-year-old girls. In how many ways can they be formed into 8 pairs so that the 2 girls in each pair are of different ages?

Answer : _____ **ways**

12. O is the centre of a circle with radius 10 cm. A line, through a point A at the distance 20 cm from O , intersects the circle at points B and C , with B between A and C . If the distance from point O to BC is 6 cm, find the length of the segment AB , in cm.

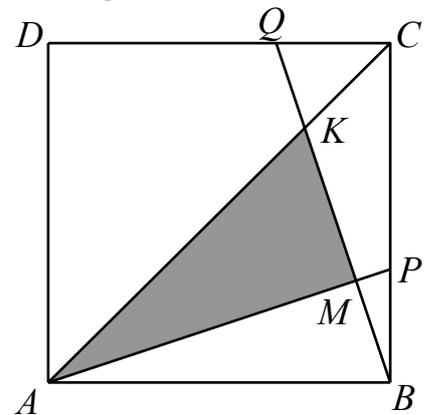


Answer : _____ cm

Section B.

Answer the following 3 questions. Show your detailed solution on the space provided after each question. Each question is worth 20 points.

1. The area of square $ABCD$ is 60 cm^2 . The point P lies on the side BC , the point Q lies on the side CD and $BP : CP = CQ : DQ = 1 : 3$. If BQ and AP intersect at point M , and BQ and AC intersect at point K , find the area of triangle AMK , in cm^2 .



Answer : _____ cm^2

2. One morning, the King sent the Queen 33 messages and the Queen sent the King 61 messages. Each was carried by a messenger. The messages left the respective Palaces one after another, all travelling at the same constant speed on the same road which connected the two Palaces. All meeting between two messengers occurred in the afternoon. Whenever two messages met, they exchanged the messages they were carrying and headed the other way. How many exchanges took place during that day?

Answer : _____

3. Determine all possible positive integers k such that all the zeroes(or roots) of the equation in terms of x which is $x^3 + (k + 12)x^2 + (43 - k)x - 56 = 0$ are integers.

Answer : _____