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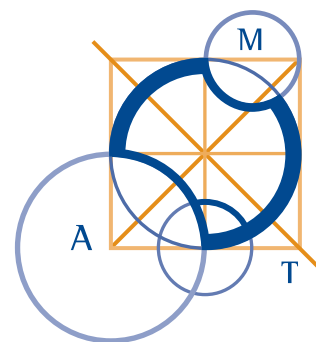
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AUSTRALIAN MATHEMATICS COMPETITION

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST

THURSDAY 2 AUGUST 2012

NAME _____



SENIOR DIVISION COMPETITION PAPER

AUSTRALIAN SCHOOL YEARS 11 AND 12
TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each with 5 possible answers given and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own State or Region so different years doing the same paper are not compared.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

Senior Division

Questions 1 to 10, 3 marks each

1. The expression $2012 - 2.012$ equals

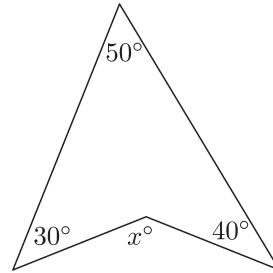
- (A) 2010.012 (B) 2010.998 (C) 2009.998 (D) 2012.012 (E) 2009.988
-

2. By what number must 6 be divided by to obtain $\frac{1}{3}$ as a result?

- (A) 18 (B) $\frac{1}{2}$ (C) $\frac{1}{18}$ (D) 2 (E) 9
-

3. In the diagram, the size in degrees of three angles are given. Find the value of x .

- (A) 90 (B) 95 (C) 100
(D) 110 (E) 120



4. The straight line joining the points (0,2) and (2,8) also passes through

- (A) (4,14) (B) (3,14) (C) (3,10) (D) (4,10) (E) (2,0)
-

5. What is the number that is halfway between $\frac{a}{b}$ and $\frac{c}{b}$?

- (A) 1 (B) $\frac{a-c}{b}$ (C) $\frac{a+c}{b}$ (D) $\frac{a-c}{2b}$ (E) $\frac{a+c}{2b}$
-

6. If $3^{16} \times 27^{10} = 9^x$, then x equals

- (A) 18 (B) 21 (C) 23 (D) 26 (E) 27
-

7. If $\frac{p}{p-2q} = 3$ then $\frac{p}{q}$ equals

- (A) 3 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 2 (E) -3
-

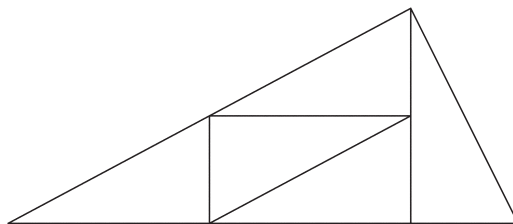
8. Two perfume bottles are similar in shape but one has twice the height of the other. Together they contain 270 mL of perfume. How much perfume, in millilitres, is in the smaller bottle?

(A) 27 (B) 30 (C) 50 (D) 54 (E) 90

9. If x and y are positive integers, how many integers are greater than xy but less than $x(y + 1)$?

(A) 1 (B) $x - 2$ (C) $y - 1$ (D) x (E) $x - 1$

10. The architecture of Federation Square in Melbourne is based on frames as shown in which a large triangle is subdivided into 5 identical triangles, each similar to the large triangle.



If the shortest side of one of the smallest triangles is 1 m, how many metres of framing are required to construct the whole shape?

(A) 20 (B) $8 + 4\sqrt{5}$ (C) $10 + 4\sqrt{5}$ (D) $12 + 4\sqrt{5}$ (E) $15 + 5\sqrt{5}$

Questions 11 to 20, 4 marks each

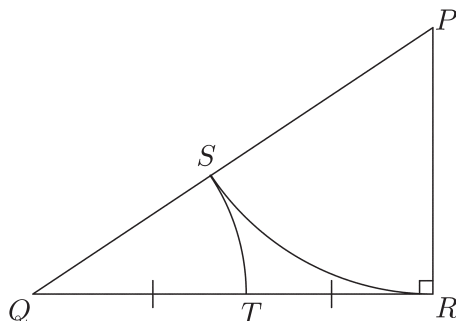
11. Five distinct integers are arranged in order, with the smallest being 5 and the largest being 16. The mean of the five integers is prime and is also equal to the median. The number of possibilities for the second largest number is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

12. The average of $3^{\frac{3}{2}}$, $3^{\frac{5}{2}}$ and $3^{\frac{7}{2}}$ is

(A) $9\sqrt{3}$ (B) $10\sqrt{3}$ (C) $11\sqrt{3}$ (D) $12\sqrt{3}$ (E) $13\sqrt{3}$

13. Triangle PQR is right-angled at R . The circle with centre P and radius PR cuts PQ at S and the circle with centre Q and radius QS cuts QR at T .



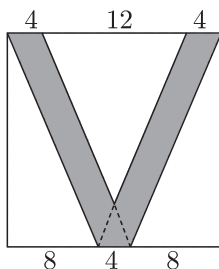
If T bisects QR , then the ratio $QS : SP$ equals

- (A) 7 : 12 (B) 5 : 12 (C) 5 : 8 (D) 3 : 4 (E) 2 : 3

14. In a three-game series of basketball with just two teams, Wollongong and Townsville, the probability that Wollongong will win each game is twice the probability that Townsville will win. What is the probability that Townsville will win the series?

- (A) $\frac{1}{3}$ (B) $\frac{2}{9}$ (C) $\frac{5}{27}$ (D) $\frac{7}{27}$ (E) $\frac{5}{18}$

15. The square shown has side 20 cm. A 'V' is shaded as shown.



The area shaded, in square centimetres, is

- (A) 136 (B) 150 (C) 164 (D) 188 (E) 200

16. If the number $10^{2012} - 2012$ is written out in full, the sum of its digits is

- (A) 18100 (B) 18104 (C) 18108 (D) 18112 (E) 18116

-
17. The side lengths of a right-angled triangle are in geometric progression and the shortest side has length 2. What is the length of the hypotenuse?

(A) $1 + \sqrt{5}$ (B) $\sqrt{10}$ (C) $3\sqrt{2} - 1$ (D) $\sqrt{11}$ (E) $2\sqrt{3}$

18. Jess marks four of eight identical cards with an X and the other four with a Y . She shuffles them and puts them face down in a row and challenges Mike to guess the letter on each card. Mike decides to choose $XYXYXYXY$. What is the probability of Mike correctly guessing all eight cards?

(A) $\frac{4}{8}$ (B) $\frac{1}{70}$ (C) $\frac{1}{85}$ (D) $\frac{1}{90}$ (E) $\frac{1}{256}$

19. The diagonal of a rectangular tile with integer sides is one-third of the length of the diagonal of a row of tiles identical to the original one. If the perimeter of the single tile is 24 cm, how many tiles are in the row?

(A) 3 (B) 6 (C) 9 (D) 12 (E) 15

20. In the $\triangle PQR$, $\angle R = 2\angle P$, $PR = 5$ and $QR = 4$. The length of PQ is

(A) $2\sqrt{10}$ (B) 6 (C) 7 (D) $2\sqrt{7}$ (E) $5\sqrt{2}$

Questions 21 to 25, 5 marks each

21. Let $p(x)$ be a polynomial such that

$$p(x) = (x - 2)^{2012}(x + 2012) + (x - 2)^{2011}(x + 2011) + \cdots + (x - 2)(x + 1).$$

The sum of the coefficients of $p(x)$ is

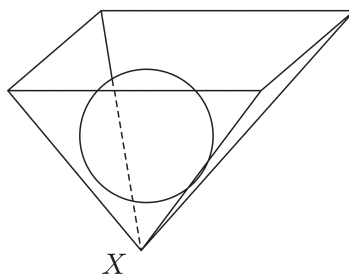
(A) 1006 (B) 2012 (C) 2027090 (D) 0 (E) 1

22. Twelve points are marked on the circumference of a circle. All possible straight lines that join any two of these points are drawn. We will call a pair of such lines *free* if the two lines in the pair do not intersect and have no common endpoints. The total number of free pairs is

(A) 132 (B) 210 (C) 495 (D) 990 (E) 1485

23. If x and y are positive integers which satisfy $x^2 - 8x - 1001y^2 = 0$, what is the smallest possible value of $x + y$?
- (A) 73 (B) 100 (C) 102 (D) 114 (E) 136

24. A spherical ball of radius 1 rests inside a holder in the shape of an inverted pyramid. The pyramid has a horizontal square top and its other faces are equilateral triangles. It is large enough to enclose the ball.



How far is the centre of the ball above the apex X of the pyramid?

- (A) $\sqrt{2}$ (B) $\frac{3}{2}$ (C) $3 - \sqrt{2}$ (D) $\sqrt{3}$ (E) 2

25. In 'base -2 ' notation, digits are 0 and 1 only and the places go up in powers of -2 . For example, 110110 stands for

$$1 \times (-2)^5 + 1 \times (-2)^4 + 0 \times (-2)^3 + 1 \times (-2)^2 + 1 \times (-2) + 0 \times 1 = -14.$$

If the decimal number 2000 is written in base -2 notation, how many non-zero digits does it contain?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

26. A courier company has motorbikes that can travel 300 km on a full tank of fuel. Two couriers, Anna and Brian, set off from the depot together to deliver a letter to Connor's house. They can transfer fuel between the bikes at any time but do not return to the depot to get more fuel. While only one bike is required to deliver the letter, both must return to the depot. What is the greatest distance, in kilometres, that Connor's house could be from the depot?

27. Twenty-eight points are equally spaced around the circumference of a circle. What is the total number of triangles whose three vertices are from those twenty-eight points and the size of one of the angles is twice the size of another?

28. In the grid shown, we need to fill in the squares with numbers so that the number in every square, except for the corner ones, is the average of its neighbours. The edge squares have three neighbours, the others four.

+1000				-1000
	x			
-1000				+1000

What is the value of the number in the square marked x ?

29. Terry has invented a new way to extend lists of numbers. To *Terryfy* a list such as $[1, 8]$ he creates two lists $[2, 9]$ and $[3, 10]$ where each term is one more than the corresponding term in the previous list, and then joins the three lists together to give $[1, 8, 2, 9, 3, 10]$. If he starts with a list containing one number $[0]$ and repeatedly *Terryfies* it he creates the list

$$[0, 1, 2, 1, 2, 3, 2, 3, 4, 1, 2, 3, 2, 3, 4, 3, 4, 5, 2, 3, 4, \dots].$$

What is the 2012th number in this *Terryfic* list?

30. If $\sin x \cos x + \sin y \cos y + \sin x \sin y + \cos x \cos y = 1$ and $\cos(x - y)$ is the smallest possible, what is the value of $2x - y$, expressed in degrees, that is closest to 360° ?