Senior Division

1. The expression that has the same meaning as $9x^{-3}$ is

(A) $\dfrac{-9}{x^3}$ (B) $\dfrac{3}{x}$ (C) $\dfrac{1}{9x^3}$ (D) $\dfrac{3}{x^3}$ (E) $\dfrac{9}{x^3}$

2. The value of $\dfrac{1}{0.04}$ is

(A) 15   (B) 20   (C) 25   (D) 40   (E) 60

3. If $p = 9$ and $q = -3$ then $p^2 - q^2$ is equal to

(A) 64   (B) 72   (C) 84   (D) 90   (E) 96

4. A circle has circumference π units. In square units, its area is

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{2}$ (C) π (D) 2π (E) 4π
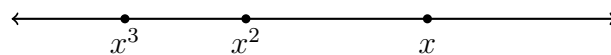
5. If $K = L + \dfrac{6}{R}$ and $L = 4$ and $K = 7$, then R equals

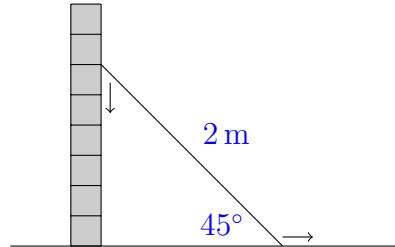(A) -18 (B) 1   (C) 12   (D) 8   (E) 2

6. If x, x^2 and x^3 lie on a number line in the order shown below, which of the following could be the value of x?

(A) -2 (B) $-\dfrac{1}{2}$ (C) $\dfrac{3}{4}$ (D) 1   (E) $\dfrac{3}{2}$

7. A 2 metre broom is leaning against a wall, with the bottom of the broom making an angle of 45° with the ground. The broom slowly slides down the wall until the bottom of the broom makes an angle of 30° with the ground.

How far, in metres, has the top of the broom slid down the wall?

(A) $\sqrt{2} - 1$ (B) $2 - \sqrt{3}$ (C) $\sqrt{3} - 1$ (D) $\sqrt{3} - \sqrt{2}$ (E) $2 - \sqrt{2}$

8. The base of a triangle is increased by 25% and its height is increased by 50%. Its area has increased by

(A) 25%   (B) 50%   (C) 66.$\dot{6}$%   (D) 75%   (E) 87.5%

9. On a section of the number line five intervals are marked as shown.
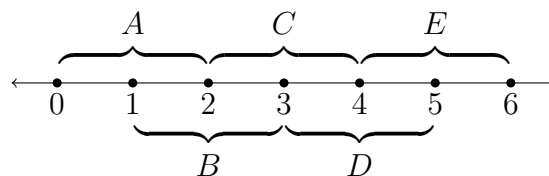
If a number x falls in the interval A and a number y falls in the interval D, then the number $\frac{1}{2}(x + y + 1)$ must fall in which interval?

(A) A (B) B (C) C (D) D (E) E

10. If $\dfrac{p}{p - 2q} = 3$ then $\dfrac{p}{q}$ equals

(A) -3 (B) 3 (C) $\dfrac{1}{3}$ (D) $\dfrac{2}{3}$ (E) 2
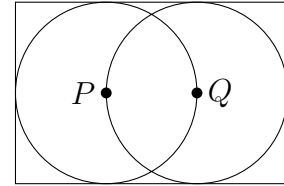
Questions 11 to 20, 4 marks each

11. In a car park there are 3 Fords, 4 Holdens and 2 Hondas. If a parking inspector chooses 2 cars at random, the probability that both are Holdens is

(A) $\dfrac{1}{4}$ (B) $\dfrac{4}{27}$ (C) $\dfrac{1}{6}$ (D) $\dfrac{4}{9}$ (E) $\dfrac{16}{81}$

12. In this figure, P and Q are the centres of two circles. Each circle has an area of $10\,\text{m}^2$. The area, in square metres, of the rectangle is

(A) 20   (B) $20 - \dfrac{10}{\pi}$ (C) $\dfrac{40}{\pi}$ (D) $\dfrac{50}{\pi}$ (E) $\dfrac{60}{\pi}$

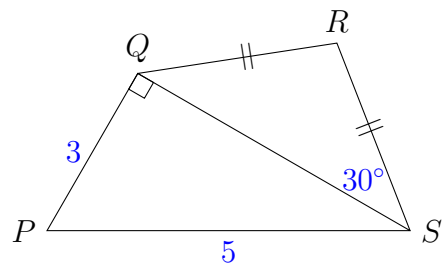**13.** The value of $\sqrt{1 + 2 + 3 + 4 + \cdots + 99 + 100}$ lies between

(A) 14 and 15   (B) 25 and 26   (C) 30 and 31   (D) 71 and 72   (E) 100 and 101

14. If $\dfrac{x-a}{x-b} = \dfrac{x-b}{x-a}$ and $a \neq b$, what is the value of x?

(A) $\dfrac{a-b}{2}$ (B) $\dfrac{a^2+b^2}{a+b}$ (C) $\dfrac{a^2+b^2}{2(a+b)}$ (D) $a+b$ (E) $\dfrac{a+b}{2}$

15. In the diagram, $PS = 5$, $PQ = 3$, $\triangle PQS$ is right-angled at Q, $\angle QSR = 30°$ and $QR = RS$. The length of RS is

(A) $\dfrac{\sqrt{3}}{2}$ (B) $\sqrt{3}$ (C) 2

(D) $\dfrac{4\sqrt{3}}{3}$ (E) 4

16. Billy, a seasonal worker in the town of Cowra, collected an even number of buckets of cherries on his first day. Each day after that he increased the number of buckets he picked by 2 buckets per day. In the first 50 days he collected 3250 buckets. The number of buckets Billy collected on the 50th day was

(A) 66   (B) 110   (C) 114   (D) 116   (E) 120

17. A farmer walks 1 km east across his paddock, then 1 km north-east and then another 1 km east. Find the distance, in kilometres, between the farmer's initial position and his final position.

(A) $\sqrt{5 + 2\sqrt{2}}$ (B) $\sqrt{10}$ (C) $\sqrt{5}$ (D) $2 + \sqrt{2}$ (E) $\sqrt{\frac{11}{2} + 2\sqrt{10}}$

18. Two machines move at constant speeds around a circle of circumference 600 cm, starting together from the same point. If they travel in the same direction then they next meet after 20 seconds, but if they travel in opposite directions then they next meet after 5 seconds. At what speed, in centimetres per second, is the faster one travelling?

(A) 60   (B) 65   (C) 70   (D) 75   (E) 85

19. The equation $x^2 - kx + 374 = 0$ has two integer solutions. How many distinct values of k are possible?

(A) 2   (B) 4   (C) 6   (D) 8   (E) 10

20. Given that $f_1(x) = \dfrac{x}{x+1}$ and $f_{n+1}(x) = f_1(f_n(x))$, then $f_{2014}(x)$ equals

(A) $\dfrac{x}{2014x + 1}$ (B) $\dfrac{2014x}{2014x + 1}$ (C) $\dfrac{x}{x + 2014}$ (D) $\dfrac{2014x}{x + 1}$ (E) $\dfrac{x}{2014(x + 1)}$

Questions 21 to 25, 5 marks each

21. Starting with $\dfrac{2}{3}$ of a tank of fuel, I set out to drive the 550 km from Scone to Canberra. At Morisset, 165 km from Scone, I have $\dfrac{1}{2}$ of a tank remaining. If I continue with the same fuel consumption per kilometre and without refuelling, what happens?

(A) I will arrive in Canberra with $\dfrac{1}{9}$ of a tank to spare.

(B) I will arrive in Canberra with $\dfrac{1}{20}$ of a tank to spare.

(C) I will run out of fuel precisely when I reach Canberra.

(D) I will run out of fuel 110 km from Canberra.

(E) I will run out of fuel 220 km from Canberra.

22. Thanom has a roll of paper consisting of a very long sheet of thin paper tightly rolled around a cylindrical tube, forming the shape indicated in the diagram.

Initially, the diameter of the roll is $12\,\text{cm}$ and the diameter of the tube is $4\,\text{cm}$. After Thanom uses half of the paper, the diameter of the remaining roll is closest to

(A) $6\,\text{cm}$ (B) $8\,\text{cm}$ (C) $8.5\,\text{cm}$

(D) $9\,\text{cm}$ (E) $9.5\,\text{cm}$
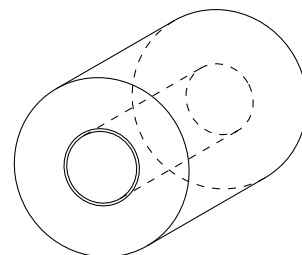
23. For every 100 people living in the town of Berracan, 50 live in two-person households, 30 live in three-person households and 20 live in four-person households. What is the average number of people living in a household?

(A) 2.0 (B) 2.5 (C) 2.7 (D) 2.8 (E) 3.0

24. In the diagram, QS, RT and SV are tangents to the circle. The length of RS is $1\,\text{m}$. What is the diameter of the circle, in metres?

(A) $3 + \sqrt{3}$ (B) 4 (C) $2\sqrt{3} + 2$

(D) $3\sqrt{3}$ (E) $\dfrac{9}{2}$
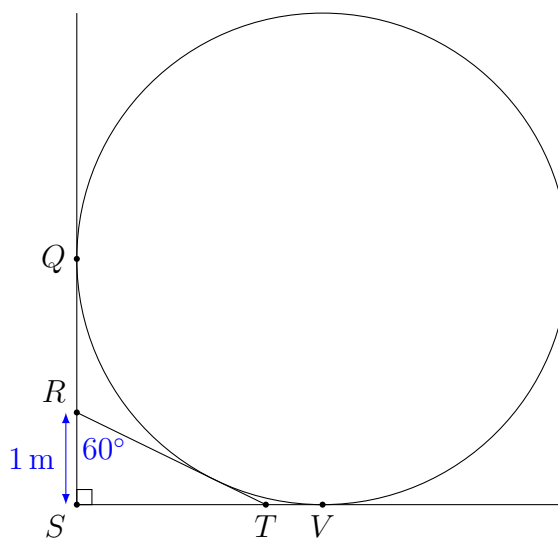
25. The sequence

$$2\ ,\ 2^2\ ,\ 2^{2^2}\ ,\ 2^{2^{2^2}}\ ,\ \ldots$$

is defined by $a_1 = 2$ and $a_{n+1} = 2^{a_n}$ for all $n \geq 1$. What is the first term in the sequence greater than 1000^{1000}?

(A) $a_4 = 2^{2^{2^2}}$ (B) $a_5 = 2^{2^{2^{2^2}}}$ (C) $a_6 = 2^{2^{2^{2^{2^2}}}}$ (D) $a_7 = 2^{2^{2^{2^{2^{2^2}}}}}$ (E) $a_8 = 2^{2^{2^{2^{2^{2^{2^2}}}}}}$

26. What is the largest three-digit number with the property that the number is equal to the sum of its hundreds digit, the square of its tens digit and the cube of its units digit?

27. Igor wants to make a secret code of five-letter words. To make them easy to say, he follows these two rules:

    (i) no more than two consonants or two vowels in succession

    (ii) no word to start or end with two consonants

He rejects the letter 'Q' as too hard, so he has 20 consonants and 5 vowels to choose from. If N is the number of code words possible, what are the first three digits of N?

28. Consider the sequence a_1, a_2, a_3, a_4, … such that $a_1 = 2$ and for every positive integer n,

$$a_{n+1} = a_n + p_n, \qquad \text{where } p_n \text{ is the largest prime factor of } a_n.$$

The first few terms of the sequence are $2, 4, 6, 9, 12, 15, 20$. What is the largest value of n such that a_n is a four-digit number?

29. A *lattice point* in the plane is a point whose coordinates are both integers. Consider a triangle whose vertices are lattice points $(0,0)$, $(a,0)$, and $(0,b)$, where $a \geq b > 0$. Suppose that the triangle contains exactly 74 lattice points in its interior, not including those lattice points on the sides of the triangle. Determine the sum of the areas of all such triangles.

30. A polynomial $p(x)$ is called *self-centered* if it has integer coefficients and $p(100) = 100$. If $p(x)$ is a self-centred polynomial, what is the maximum number of integer solutions k to the equation $p(k) = k^3$?