Senior Division

Questions 1 to 10, 3 marks each

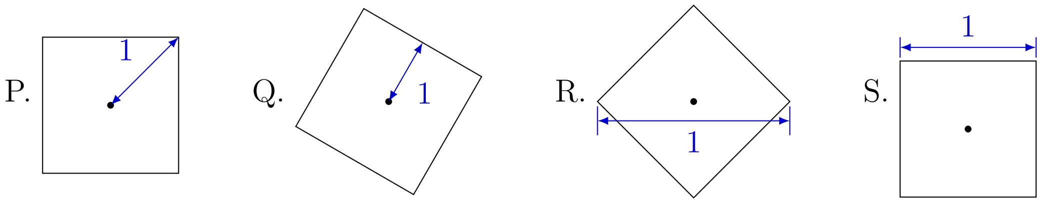
1. How many 3-digit numbers have the hundreds digit equal to 7 and the units digit equal to 8?

(A) 10 (B) 100 (C) 20 (D) 19 (E) 90

2. If $p = 7$ and $q = -4$, then $p^2 - 3q^2 =$

(A) 49 (B) 48 (C) 0 (D) 97 (E) 1

3. In each of these squares, the marked length is 1 unit. Which of the squares would have the greatest perimeter?



(A) P (B) Q (C) R (D) S (E) all are the same

4. Given that n is an integer and $7n + 6 \geq 200$, then n must be

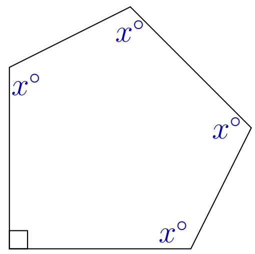
(A) even (B) odd (C) 28 or more (D) either 27 or 28 (E) 27 or less

5. King Arthur's round table has a radius of three metres. The area of the table top in square metres is closest to which of the following?

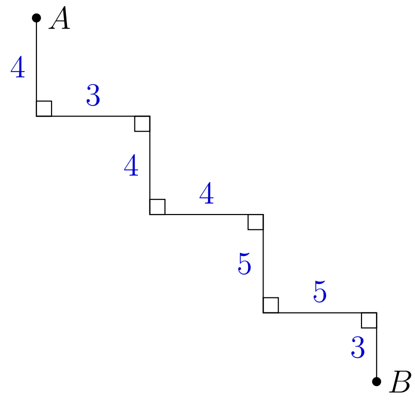
(A) 20 (B) 30 (C) 40 (D) 50 (E) 60

6. In the diagram, the value of x is

(A) 120 (B) 108 (C) 105 (D) 135 (E) 112.5



7. How far is it from A to B measured in a straight line?



(A) 20 (B) 28 (C) $10 + 9\sqrt{2}$

(D) $8 + 9\sqrt{2}$ (E) 16

---

8. The solution to the equation $\sqrt{x^2 + 1} = x + 2$ is

(A) $x = \frac{22}{7}$ (B) $x = -\frac{3}{4}$ (C) $x = -\frac{3}{2}$

(D) $x = 3$ (E) no number x satisfies the equation

---

9. A large tray in the bakery's display case has an equal number of three types of croissants: plain, chocolate and cheese. Norm rushes into the bakery and buys two croissants at random without looking. The probability that Norm does not have a chocolate croissant is closest to

(A) $\frac{1}{3}$ (B) $\frac{4}{9}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{1}{27}$

---

10. Which of these expressions is always a multiple of 3, whenever n is a whole number?

(A) n^3 (B) $n^3 + 2n$ (C) $3n^3 + 1$ (D) $n^3 + 3n^2$ (E) $n^2 + 2$

Questions 11 to 20, 4 marks each

11. The value of $2^{2016} - 2^{2015}$ is

(A) 2 (B) $2^{\frac{2016}{2015}}$ (C) 2^{2015} (D) -2^{2016} (E) 0

---

12. Oaklands and Brighton are two busy train stations on the same train line. On one particular day:

- One-fifth of the trains do not stop at Oaklands.
- 45 trains do not stop at Brighton.
- 60 trains stop at both Brighton and Oaklands.
- 60 trains stop at only Brighton or Oaklands (not both).

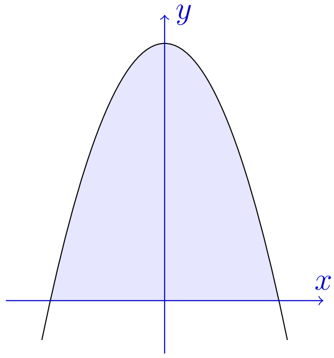
How many trains do not stop at either of these stations on that day?

(A) 60 (B) 20 (C) 45 (D) 5 (E) 40

---

13. Consider the region bounded by the parabola $y = 6 - x^2$ and the x-axis.
How many points with integer coordinates lie within that region, but not on the boundary?

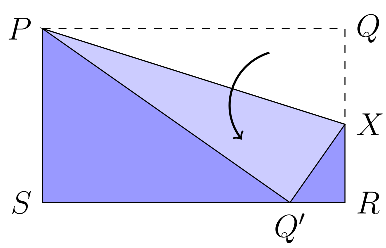
(A) 11 (B) 12 (C) 14 (D) 15 (E) 17



---

14. A rectangular sheet of paper $PQRS$ has width PQ twice the height QR. The paper is folded along a line PX such that Q lies on top of Q', a point on RS. Angle SPX is

(A) 72° (B) 45° (C) 60° (D) 67.5° (E) 75°



---

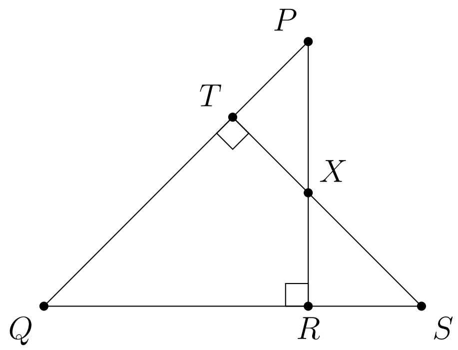
15. Ten students sit a test consisting of 20 questions. Two students get 8 questions correct and one student gets 9 questions correct. The remaining seven students all get at least 10 questions correct and the average number of questions answered correctly by these seven students is an integer. If the average number of questions answered correctly by all ten students is also an integer, then that integer is

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

---

16. In the diagram, $QR = QT = 4\,\text{cm}$ and PQR and QST are both isosceles right-angled triangles. In square centimetres, what is the area of the quadrilateral $QRXT$?

(A) $16(\sqrt{2}-1)$ (B) $4\sqrt{2}$ (C) $16\sqrt{2}-10$ (D) $8(\sqrt{2}-1)$ (E) $8-\sqrt{2}$



17. What is the smallest positive integer x for which the sum $x + 2x + 3x + 4x + \cdots + 100x$ is a perfect square?

(A) 202 (B) 5050 (C) 1010 (D) 100 (E) 101

18. Ten identical solid gold spheres will be melted down and recast into a number of smaller identical spheres whose diameter is 80% of the original ones. Ignoring any excess gold which may be left over, how many smaller spheres will be made?

(A) 12 (B) 20 (C) 8 (D) 15 (E) 19

19. A regular decagon has 10 sides of length 1. Four vertices are chosen to make a convex quadrilateral that has all side lengths greater than 1. How many non-congruent quadrilaterals can be formed this way?

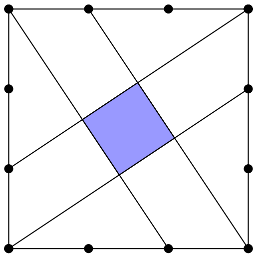
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

20. The increasing sequence of positive integers $1, 2, 4, \ldots$ has the property that each term from the third onwards is the next positive integer which is not equal to the sum of any two previous terms of the sequence. How many terms of the sequence are less than 2016?

(A) 1008 (B) 63 (C) 11 (D) 15 (E) 673

Questions 21 to 25, 5 marks each

21. A line is drawn from each corner of a large square to the point of trisection of the opposite side, as in the diagram.



If the shaded square has area 1, what is the area of the large square?

(A) 16 (B) 15 (C) 14 (D) 13 (E) 12

---

22. What is the sum of all positive integers n such that $n^2 + n + 34$ is a perfect square?

(A) 50 (B) 16 (C) 43 (D) 34 (E) 49

---

23. What is the radius of the largest sphere that will fit inside a hollow square pyramid, all of whose edges are of length 2?

(A) $\sqrt{2} - 1$ (B) $\frac{2 - \sqrt{2}}{2}$ (C) $\frac{\sqrt{6} - \sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{3}$ (E) $\frac{\sqrt{2}}{4}$

---

24. Ten positive integers are written on cards and the cards are placed around a circle. If a number is greater than the average of its two neighbours, the card is coloured green. What is the largest number of green cards there can be in the circle?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 9

---

25. What is the least value of $\sqrt{x^2 + (1-x)^2} + \sqrt{(1-x)^2 + (1+x)^2}$?

(A) 2 (B) $\frac{\sqrt{2} + \sqrt{10}}{2}$ (C) $\sqrt{5}$ (D) 0 (E) $1 + \sqrt{2}$

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

26. A high school marching band can be arranged in a rectangular formation with exactly three boys in each row and exactly five girls in each column. There are several sizes of marching band for which this is possible. What is the sum of all such possible sizes?

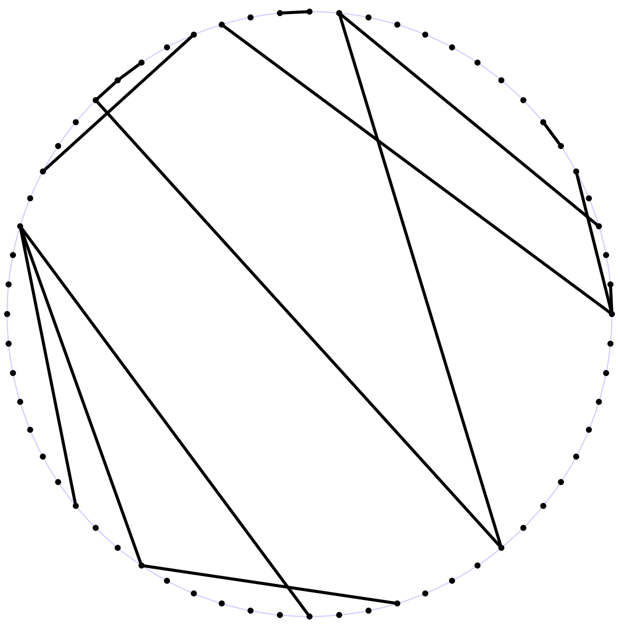
27. Let a, b, c, m, and n be integers such that $m < n$ and define the quadratic function $f(x) = ax^2 + bx + c$ where x is real. Then $f(x)$ has a graph that contains the points $(m, 0)$ and $(n, 2016^2)$. How many values of $n - m$ are possible?

28. If a and b are whole numbers from 1 to 100, how many pairs of numbers (a, b) are there which satisfy $a^{\sqrt{b}} = \sqrt{a^b}$?

29. Around a circle, I place 64 equally spaced points, so that there are $64 \times 63 \div 2 = 2016$ possible chords between these points.

I draw some of these chords, but each chord cannot cut across more than one other chord.

What is the maximum number of chords I can draw?



30. A function f defined on the set of positive integers has the properties that, for any positive integer n, $f(f(n)) = 2n$ and $f(4n + 1) = 4n + 3$. What are the last three digits of $f(2016)$?