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## Intermediate Division

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### Questions 1 to 10, 3 marks each

1. The remainder when 2017 is divided by 5 is

(A) 0                      (B) 1                      (C) 2                      (D) 3                      (E) 4

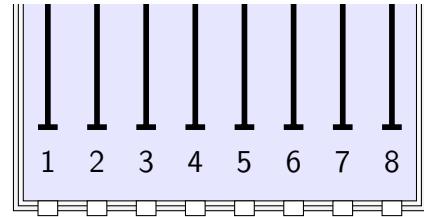
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2. Eight children in a swimming race are John, Iain, Hans, Ivan, Giovanni, Beth, Liz and Elisa.

They are put in lanes 1 to 8 randomly.

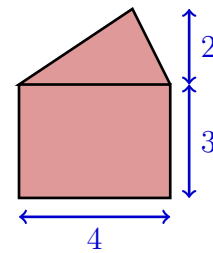
What is the probability that Beth, Liz or Elisa is in lane 1?

(A)  $\frac{3}{8}$       (B)  $\frac{1}{2}$       (C)  $\frac{3}{5}$       (D)  $\frac{5}{3}$       (E)  $\frac{1}{3}$



3. What is the total shaded area of this diagram, in square units?

(A) 14                      (B) 15                      (C) 16  
(D) 17                      (E) 18



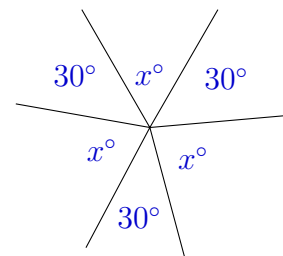
4. 1000% of 1 is

(A) 0.1                      (B) 1                      (C) 10                      (D) 100                      (E) 1000

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5. The diagram shows six angles, with three of them equal to  $30^\circ$ . The remaining three angles in the diagram are all equal to  $x^\circ$ . What is the value of  $x^\circ$ ?

(A)  $70^\circ$                       (B)  $60^\circ$                       (C)  $90^\circ$   
(D)  $120^\circ$                       (E)  $100^\circ$



6. Which of the following fractions is largest?

(A)  $\frac{1}{2}$                       (B)  $\frac{13}{42}$                       (C)  $\frac{21}{43}$                       (D)  $\frac{4}{123}$                       (E)  $\frac{14}{23}$

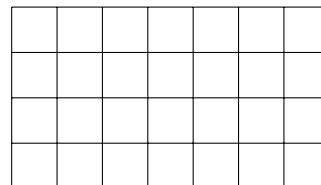
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7. Alice is playing with words. At each tick of her grandfather's clock she swaps two letters. What is the smallest number of clock ticks during which she can change WORDS to SWORD?

- (A) 3                      (B) 4                      (C) 6                      (D) 7                      (E) 8
- 

8. How many ways are there of placing a single  $3 \times 1$  rectangle on this grid so that it completely covers three grid squares?

- (A) 34                      (B) 28                      (C) 56  
 (D) 40                      (E) 10



9. Suppose  $3a = 4$  and  $9b = 7$ . Then  $18(a + b)$  is equal to

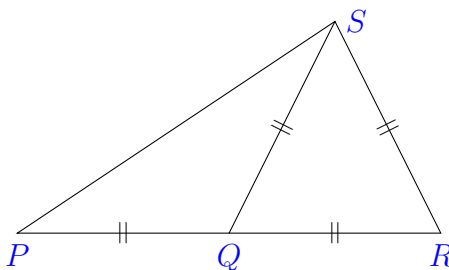
- (A) 38                      (B) 75                      (C) 198                      (D) 132                      (E)  $\frac{22}{3}$
- 

10. Mike receives a number of emails each day. One Friday, he notices that for five days in a row, the number of emails he has received is a different prime number over 20. What is the least number of emails he could have received in these five days?

- (A) 125                      (B) 139                      (C) 157                      (D) 161                      (E) 175
- 

**Questions 11 to 20, 4 marks each**

11. In the triangle below,  $PQ = SQ = SR = QR$ .



The ratio  $\angle PSR : \angle PQS$  is equal to

- (A) 1:1                      (B) 1:2                      (C) 1:3                      (D) 2:3                      (E) 3:4
-

12. A rectangular swimming pool is 50 metres long and 20 metres wide. It is divided into ten lanes, each 50 metres long and 2 metres wide, numbered in order from 1 to 10. I notice that if I swim one lap down the middle of the lane, then walk back around the edges of the pool, I have to walk 20% further than I swim.

Which of the following could be my lane number?

- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

13. A square table top is covered with identical square tiles. A total of 25 tiles are used to form the two diagonals.

How many tiles are used on the table top?

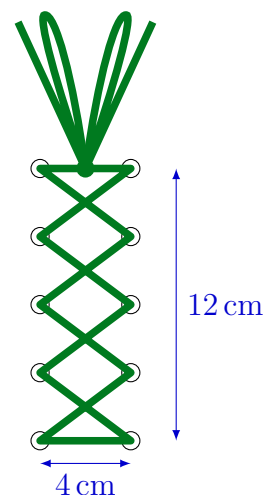
- (A) 625                      (B) 269                      (C) 425                      (D) 225                      (E) 169

14. Gwen's boots have 5 pairs of holes for the laces. Opposite holes are 4 cm apart and the first and last hole on each side are 12 cm apart.

Gwen laces and ties each boot as shown, where the knot and bow use a total of 40 cm of bootlace.

Approximately how long is the lace?

- (A) 60 cm                      (B) 70 cm                      (C) 80 cm  
(D) 90 cm                      (E) 100 cm



15. The number

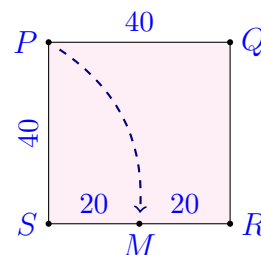
$$\sqrt{20 - \sqrt{14 + \sqrt{5 - \sqrt{1}}}}$$

is closest to

- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

16. This square sheet of paper  $PQRS$  measures  $40\text{ cm} \times 40\text{ cm}$ . The top-left corner  $P$  is folded down to meet  $M$ , the midpoint of the bottom side  $RS$ , making a single straight crease. Where does the crease cross  $QR$ , the right-hand side of the sheet of paper?

- (A) at  $Q$                       (B) 5 cm from  $Q$                       (C) 10 cm from  $Q$   
(D) 20 cm from  $Q$                       (E) not at all

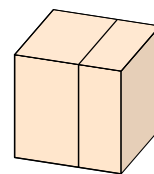


17. The numbers 18,  $p$ , 13,  $q$ , 15,  $r$ , 7 have a mean of 11.  
The mean of  $p$ ,  $q$  and  $r$  is:
- (A) 7                      (B) 8                      (C) 9                      (D) 11                      (E) 12
- 

18. The identity  $a(a^9 - a^8) + a^9 = a^x$  is true for all  $a$ . The value of  $x$  is
- (A) 0                      (B) 1                      (C) 8                      (D) 9                      (E) 10
- 

19. All of the digits from 0 to 9 are used to form two 5-digit numbers. What is the smallest possible difference between these two numbers?
- (A) 1                      (B) 9                      (C) 99                      (D) 247                      (E) 315
- 

20. A cube of surface area  $X$  is sliced into two rectangular prisms. One of the prisms has surface area  $\frac{1}{2}X$ . What is the surface area of the other prism?



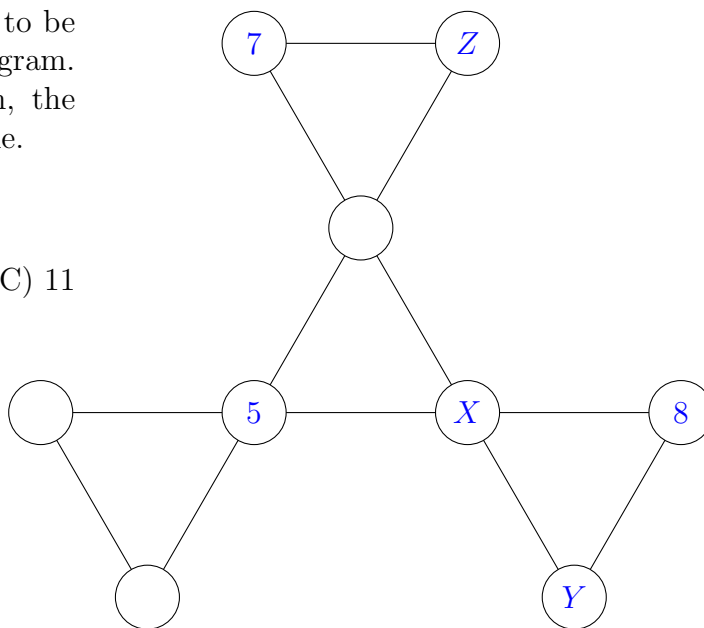
- (A)  $\frac{1}{4}X$                       (B)  $\frac{1}{2}X$                       (C)  $\frac{2}{3}X$                       (D)  $\frac{3}{4}X$                       (E)  $\frac{5}{6}X$
- 

**Questions 21 to 25, 5 marks each**

21. A quadrilateral has two parallel sides measuring 25 cm and 37 cm. What is the distance, in centimetres, between the midpoints of the diagonals?
- (A) 3                      (B) 5                      (C) 6                      (D) 7                      (E) 12
- 

22. The whole numbers from 1 to 9 are to be placed in the nine circles in the diagram. In each of the four triangles drawn, the sum of the three numbers is the same. Three of the numbers are given. What is  $X + Y + Z$ ?

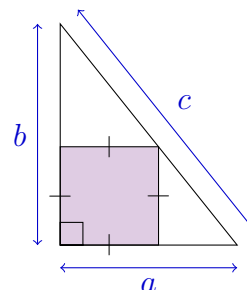
- (A) 9                      (B) 10                      (C) 11  
(D) 12                      (E) 13



- 23.** How many three-digit numbers are thirteen times the sum of their digits?  
 (A) 0                      (B) 1                      (C) 2                      (D) 3                      (E) 4
- 

- 24.** A group of people was surveyed about whether the council should collect recycling once a week instead of once a fortnight. Two-thirds of the people said ‘Yes’, and one-third said ‘No’.  
 A year later the same group of people was surveyed again and this time one-quarter of them had changed their minds, resulting in a tie between the ‘Yes’ and ‘No’ votes. Of the people who originally voted ‘Yes’, what fraction changed their minds?  
 (A)  $\frac{1}{8}$                       (B)  $\frac{1}{6}$                       (C)  $\frac{1}{2}$                       (D)  $\frac{1}{4}$                       (E)  $\frac{5}{16}$
- 

- 25.** A square is drawn in the corner of a right-angled triangle with side lengths  $a$ ,  $b$  and  $c$ , as shown.  
 Which expression gives the ratio of the unshaded area to the shaded area in all cases?  
 (A)  $1 : 1$                       (B)  $c : (a + b)$                       (C)  $ab : c^2$   
 (D)  $(a + b)^2 : 2c^2$                       (E)  $c^2 : 2ab$




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**For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.**

**Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.**

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- 26.** Adding 1 to the product of four consecutive positive integers always results in a perfect square. The first 2017 such square numbers can be found:

$$\begin{aligned}
 1 \times 2 \times 3 \times 4 + 1 &= 25 = 5^2 \\
 2 \times 3 \times 4 \times 5 + 1 &= 121 = 11^2 \\
 3 \times 4 \times 5 \times 6 + 1 &= 361 = 19^2 \\
 &\vdots \\
 2017 \times 2018 \times 2019 \times 2020 + 1 &= 16\,600\,254\,584\,281 = 4\,074\,341^2
 \end{aligned}$$

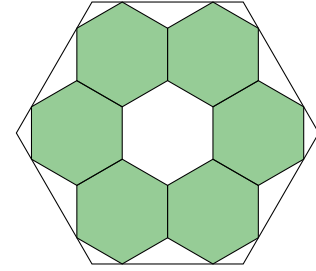
In the list of 2017 numbers

$$5, 11, 19, \dots, 4\,074\,341$$

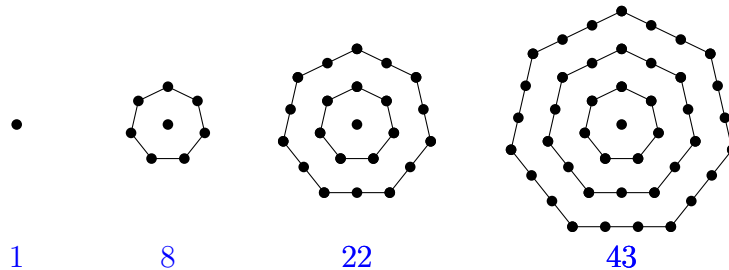
whose squares are found in this way, how many have last digit equal to 1?

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- 27.** Six identical regular hexagons are arranged inside a larger regular hexagon, as shown. The outer hexagon has area 900. What is the shaded area?



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- 28.** For  $n \geq 3$ , the sequence of *centred  $n$ -gon numbers* is found by starting with a central dot, then adding layers consisting of  $n$ -gons of dots around this centre, where the number of dots on each side increases by 1 for each layer. For instance, the sequence of centred 7-gon numbers starts 1, 8, 22, 43, ... as shown.



What is the smallest  $n$  for which 2017 is in the sequence of centred  $n$ -gon numbers?

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- 29.** I have a large number of toy soldiers, which I can arrange into a rectangular array consisting of a number of rows and a number of columns. I notice that if I remove 100 toy soldiers, then I can arrange the remaining ones into a rectangular array with 5 fewer rows and 5 more columns. How many toy soldiers would I have to remove from the original configuration to be able to arrange the remaining ones into a rectangular array with 11 fewer rows and 11 more columns?
- 

- 30.** One googol is the number  $G = 10^{100}$  and one googolplex is the number  $10^G$ . Let  $n$  be the largest whole number for which  $n^n < 10^G$ . How many digits does  $n$  have?
-