Senior Division

Questions 1 to 10, 3 marks each

1. The value of $(4 \times 5) \div (2 \times 10)$ is

(A) 4 (B) $\frac{1}{4}$ (C) 2 (D) $\frac{1}{2}$ (E) 1

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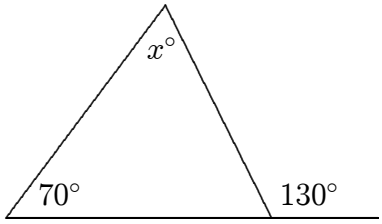
2. In the diagram, the value of x is

(A) 20 (B) 90 (C) 30 (D) 80 (E) 60



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3. $1 + \dfrac{1}{3 + \frac{1}{2}}$ equals

(A) $\frac{6}{5}$ (B) $\frac{7}{6}$ (C) $\frac{9}{2}$ (D) $\frac{3}{2}$ (E) $\frac{9}{7}$

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4. The straight line $y = x + g$ passes throught the point (2,3). The value of g is

(A) 0 (B) 1 (C) 2 (D) 3 (E) -1

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5. A two-digit number has tens digit t and its units digit u. If the digit 8 is placed between these digits, the value of the three-digit number is

(A) $t + u + 8$ (B) $10t + 80 + u$ (C) $10t + u + 8$ (D) $100t + 10u + 8$ (E) $100t + 80 + u$

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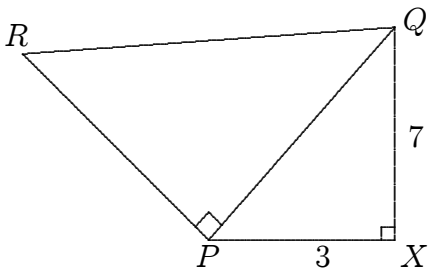
6. $\triangle PXQ$ is a right angled triangle with sides of length 3 and 7 as shown. At P, PR is drawn so that $\angle RPQ = 90°$ and $PR = PQ$. The area of $\triangle PRQ$ is

(A) $\frac{21}{2}$ (B) 29 (C) $\sqrt{58}$ (D) 58 (E) 100



7. In our school the average mark in Year 11 for a test was 70 and in Year 12 it was 80 for the same test. There were 20 students in Year 11 and 30 students in Year 12 who sat the test. The average mark for the two groups was

(A) 72 (B) 75 (C) 76 (D) 78 (E) 74

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8. Different tyres were fitted to a car, increasing the circumference of the wheels from 200 cm to 225 cm. On a journey of 1800 km, the number of revolutions of each wheel was reduced by

(A) 50 000 (B) 1000 (C) 2000 (D) 100 000 (E) 7 200 000

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9. The sum of all but one of the internal angles of a pentagon is 400°. The number of degrees in the remaining angle is

(A) 40 (B) 120 (C) 140 (D) 160 (E) 400

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10. The value of $\sqrt[4]{2} \times \sqrt{32\sqrt{2}}$ is

(A) 8 (B) 4 (C) $4\sqrt{2}$ (D) $4\sqrt[4]{2}$ (E) $16\sqrt[4]{2}$

Questions 11 to 20, 4 marks each

11. The difference between a positive fraction and its reciprocal is $\frac{9}{20}$. The sum of the fraction and its reciprocal is

(A) $\frac{20}{9}$ (B) $\frac{41}{20}$ (C) $\frac{25}{16}$ (D) 5 (E) not uniquely determined

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12. At time $t = 0$ a split forms in a balloon and the quantity Q of gas left in the balloon at time t is given by

$$Q = \frac{100}{(1+2t)^2}\,.$$

The time taken for half the gas to escape is

(A) $\frac{\sqrt{2}-1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{1+\sqrt{2}}{2}$ (D) $\sqrt{2}$ (E) $\frac{10\sqrt{2}-1}{10}$

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13. Two dice are thrown at random. The probability that the two numbers obtained are the two digits of a perfect square is

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{7}{36}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

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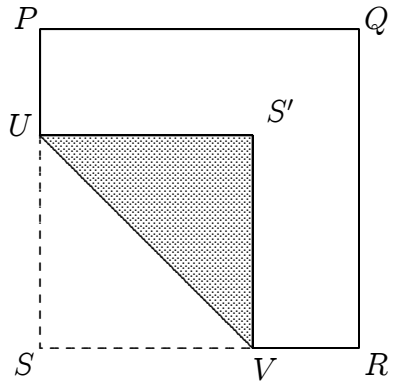
14. A square piece of paper has area $12\,\text{cm}^2$. It is coloured white on one side and shaded on the other. One corner of the paper has been folded over so that the sides of the triangle formed are parallel to the sides of the square as shown. The total visible area of the paper is half shaded and half white. What is the length, in centimetres, of the fold line UV?

(A) 4 (B) $\sqrt{12}$ (C) 3 (D) 6 (E) $\sqrt{8}$



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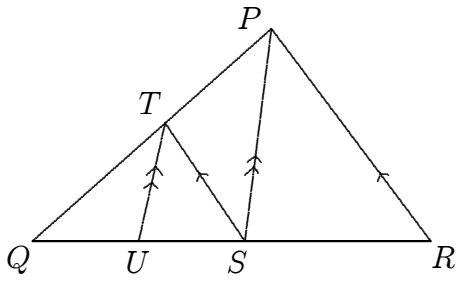
15. In the triangle PQR shown, S and U are points on QR and T is a point on PQ such that $TS \parallel PR$ and $UT \parallel SP$.
If $QS = 4\,\text{cm}$ and $SR = 2.4\,\text{cm}$, then the length of QU, in centimetres, is

(A) 2.4 (B) 2.5 (C) 3 (D) 3.2 (E) 4



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16. A train leaves Canberra for Sydney at 12 noon, and another train leaves Sydney for Canberra forty minutes later. Both trains follow the same route and travel at the same uniform speed, taking $3\frac{1}{2}$ hours to complete the journey. At what time will they pass?

(A) 1:45 pm (B) 2:00 pm (C) 2:05 pm (D) 2:15 pm (E) 2:25 pm

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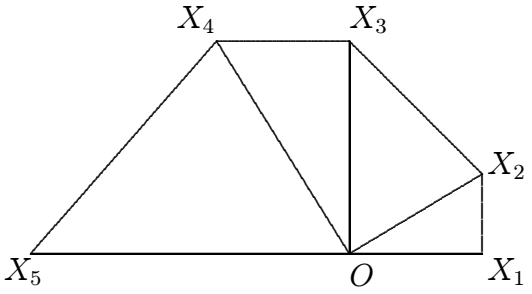
17. A spiral is formed by starting with an isosceles right-angled triangle OX_1X_2, where OX_1 is of length 1, then using the hypotenuse OX_2 as a shorter side of another isosceles right-angled triangle, and so on. The first few steps are shown in the diagram.
Eventually we will reach for the first time a situation where a side OX_k of a triangle overlaps OX_1. What is the length of X_1X_k?



(A) 8 (B) $8\sqrt{2}-1$ (C) $8\sqrt{2}$ (D) 15 (E) 14

18. The number of 5-digit numbers in which every two neighbouring digits differ by 3 is

(A) 40 (B) 41 (C) 43 (D) 45 (E) 50

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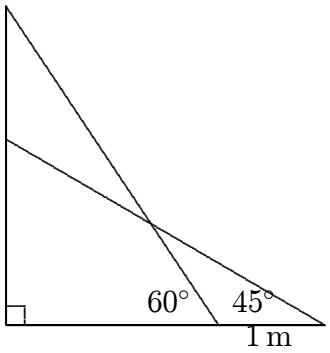
19. A ladder resting against a wall makes an angle of 60° with the wall. When the base of the ladder is moved 1 m further from the wall it makes an angle of 45° with the wall. The length of the ladder, in metres, is



(A) 2 (B) $2(\sqrt{2}+1)$ (C) $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$

(D) $\sqrt{5}$ (E) $\dfrac{2}{\sqrt{2}+1}$

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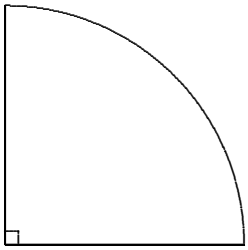
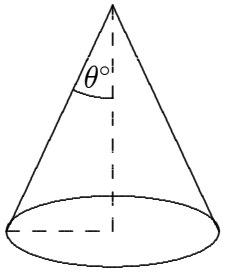
20. A quarter circle is folded to form a cone.





If θ° is the angle between the axis of symmetry and the slant height of the cone, then $\sin\theta^\circ$ equals

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{\sqrt{2}}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{\sqrt{3}}{2}$ (E) $\dfrac{1}{\sqrt{3}}$

Questions 21 to 30, 5 marks each

21. The number of real solutions of $x+\sqrt{x^2+\sqrt{x^3+1}}=1$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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22. The area of the shaded rectangle is

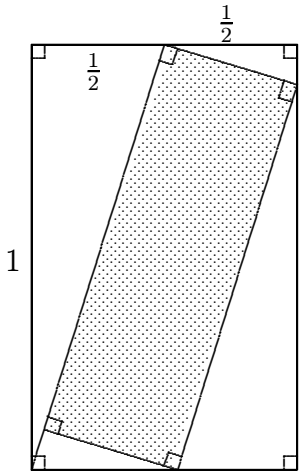
(A) between $\frac{1}{4}$ and $\frac{5}{16}$

(B) between $\frac{5}{16}$ and $\frac{3}{8}$

(C) between $\frac{3}{8}$ and $\frac{7}{16}$

(D) between $\frac{7}{16}$ and $\frac{1}{2}$

(E) more than $\frac{1}{2}$



23. When $(1-2x)^3(1+kx)^2$ is expanded, two values k_1 and k_2 of k give the coefficient of x^2 as 40. The value of $k_1 + k_2$ is

(A) -1 (B) 8 (C) 10 (D) 12 (E) 14

24. What is the area, in square units, enclosed by the figure whose boundary points satisfy

$$|x| + |y| = 4?$$

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

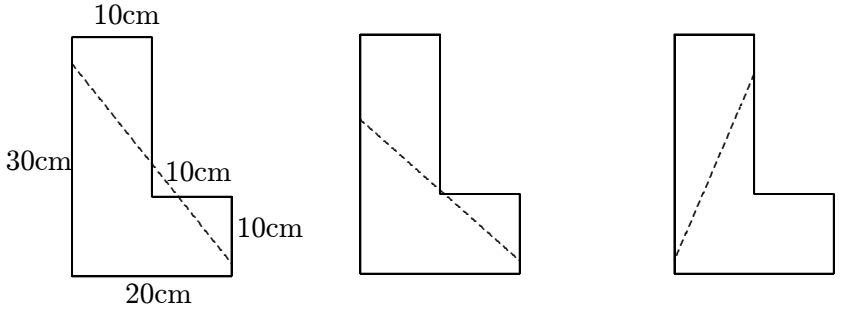
25. The number of digits in the decimal expansion of 2^{2005} is closest to

(A) 400 (B) 500 (C) 600 (D) 700 (E) 800

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

26. My name is Louis and my father has cooked me an L-shaped cake for my birthday. He says that I must cut it into three pieces with a single cut, so that my brother and sister can have a piece too. So, I have to cut it

like this or this but not like this.



He says that I have to be polite and let them have the first choice of the pieces, but I just know they'll be greedy and leave the smallest possible piece for me. So I want to cut the cake so that my little piece will be as big as possible. If I do this, how big, in square centimetres, will my piece be?

27. The function $y = f(x)$ is a function such that $f(f(x)) = 6x - 2005$ for every real number x. An integer t satisfies the equation $f(t) = 6t - 2005$. What is this value of t?

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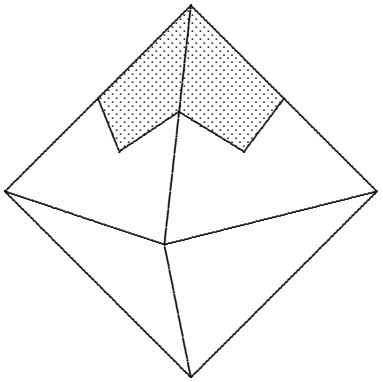
28. A regular octahedron has eight triangular faces and all sides the same length. A portion of a regular octahedron of volume 120 cm^3 consists of that part of it which is closer to the top vertex than to any other one. In the diagram, the outside part of this volume is shown shaded, and it extends down to the centre of the octahedron. What is the volume, in cubic centimetres, of this unusually shaped portion?



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29. If x, y and z satisfy the system of equations

$$\begin{aligned} x + y + z &= 5 \\ x^2 + y^2 + z^2 &= 15 \\ xy &= z^2, \end{aligned}$$

determine the value of $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}$.

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30. A positive integer is equal to the sum of the squares of its four smallest positive divisors. What is the largest prime that divides this positive integer?
