Senior Division

Questions 1 to 10, 3 marks each

1. The value of $\dfrac{6 \times 25}{3 \times 5 \times 2}$ is

(A) 1 (B) 2 (C) 3 (D) 5 (E) 6

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2. If $a = 2b - 5$, then b equals

(A) $\dfrac{a}{2}$ (B) $\dfrac{a}{2} + 5$ (C) $\dfrac{a-5}{2}$ (D) $\dfrac{a+5}{2}$ (E) $2a + 5$

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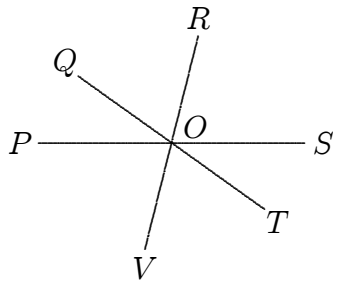
3. In the diagram, $\angle POR = 120°$ and $\angle QOS = 145°$. The size of $\angle TOV$ is

(A) 45° (B) 60° (C) 85° (D) 90° (E) 95°



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4. Which of the following is equal to $\dfrac{7}{x^2}$?

(A) $(7x)^{-2}$ (B) $\dfrac{1}{7x}$ (C) $\dfrac{1}{7x^2}$ (D) $\dfrac{x^2}{7}$ (E) $7x^{-2}$

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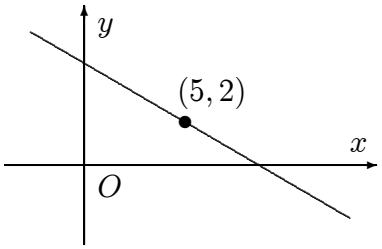
5. In the figure, if the line has gradient -1, what is the y-intercept?

(A) 4 (B) 2 (C) 6 (D) 7 (E) 5



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6. The pages of a book are consecutive whole numbers. If you begin reading at the top of page x and stop reading at the bottom of page y, the number of pages you have read is

(A) $x - y$ (B) $y - x$ (C) $x + y$ (D) $y - x + 1$ (E) $y - x - 1$

7. A rectangular box has faces with areas of 35, 60 and 84 square centimetres. The volume of the box, in cubic centimetres, is

(A) 420 (B) 480 (C) 512 (D) 563 (E) 635

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8. If $x = 3^n + 3^n + 3^n$, which of the following is equal to x^2?

(A) 9^{3n} (B) 3^{2n+2} (C) 27^{2n} (D) 3^{2n} (E) 3^{n^2+6n+9}

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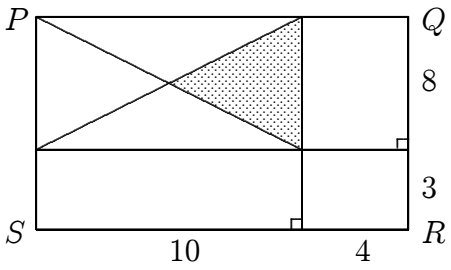
9. What fraction of the rectangle $PQRS$ in the diagram is shaded?

(A) $\frac{1}{16}$ (B) $\frac{3}{5}$ (C) $\frac{1}{8}$ (D) $\frac{1}{10}$ (E) $\frac{10}{77}$



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10. A train travelling at constant speed takes a quarter of a minute to pass a signpost and takes three-quarters of a minute to pass completely through a tunnel which is 600 m in length. The speed of the train, in kilometres per hour, is

(A) 50 (B) 56 (C) 64 (D) 72 (E) 80

Questions 11 to 20, 4 marks each

11. In a container are 8 red, 3 white and 9 blue balls. If 3 balls are selected at random, the probability of getting 2 red balls and 1 white ball is

(A) $\frac{1}{12}$ (B) $\frac{1}{4}$ (C) $\frac{7}{285}$ (D) $\frac{2}{3}$ (E) $\frac{7}{95}$

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12. The number of digits in the answer to the product $16^8 \times 5^{25}$ is

(A) 24 (B) 25 (C) 26 (D) 27 (E) 28

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13. If $x < y < 0 < z$, which of the following must be true?

(A) $x + y + z > 0$ (B) $(x + y)^2 - z > 0$ (C) $x + y + z^2 > 0$ (D) $x + y - z > 0$ (E) $x + y - z < 0$

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14. In a triangle PQR, $\sin \angle P = \frac{1}{3}$ and $\sin \angle Q = \frac{1}{4}$. How many different values can the size of $\angle R$ have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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15. How many different pairs of 2-digit numbers multiply to give a 3-digit number with all digits the same?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

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16. I have 450 grams of salt and flour mix. How many grams of flour should I add to reduce the percentage of salt in the mixture to 90% of what it was?

(A) 50 (B) 10 (C) 30 (D) 45 (E) 60

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17. Five bales of hay are weighed two at a time in all possible combinations. The weights, in kilograms, are:-

110, 112, 113, 114, 115, 116, 117, 118, 120 and 121.

What is the weight, in kilograms, of the heaviest bale?

(A) 58 (B) 59 (C) 60 (D) 61 (E) 62

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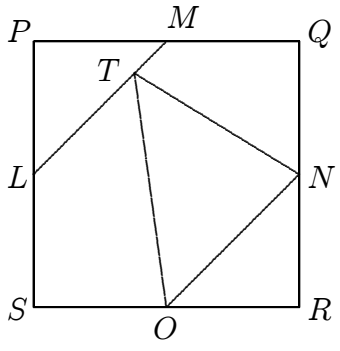
18. In the diagram, $PQRS$ is a square of side 2 units. M, N, O and L are the mid-points of PQ, QR, RS and SP respectively, and T is a point on LM.



The area, in square units, of $\triangle TNO$ is

(A) 2 (B) 1 (C) $\sqrt{2}$ (D) $\frac{4}{5}$ (E) $\frac{\sqrt{3}}{2}$

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19. If $7^{x+1} - 7^{x-1} = 336\sqrt{7}$, then the value of x is

(A) $\frac{5}{2}$ (B) $\frac{3}{2}$ (C) $\frac{-3}{2}$ (D) $\frac{7}{2}$ (E) $\frac{1}{2}$

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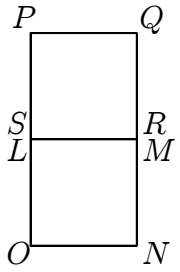
20. The nine squares of a 3×3 grid painted on a wall are to be coloured red, white and blue so that no row or column contains squares of the same colour. One such pattern is shown in the diagram. How many different patterns can be made?

(A) 15 (B) 6 (C) 9 (D) 12 (E) 24

R	W	B
B	R	W
W	B	R

Questions 21 to 30, 5 marks each

21. The squares $PQRS$ and $LMNO$ have equal sides of 1 m and are initially placed so that the side SR touches LM as shown. The square $PQRS$ is rotated about R until Q coincides with N. The square is then rotated about Q until P coincides with O.
It is then rotated about P until S coincides with L and then finally rotated about S until R coincides with M and the square is now back to its original position.
The length, in metres, of the path traced out by the point P in these rotations is



(A) $\pi(2+\sqrt{2})$ (B) 4π (C) $2\pi(2+\sqrt{2})$ (D) 2π (E) $\pi(3+\sqrt{2})$

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22. The vertices of a cube are each labelled with one of the integers 1, 2, 3, ..., 8. A *face-sum* is the sum of the labels of the four vertices on a face of the cube. What is the maximum number of equal face-sums in any of these labellings?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

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23. In a tetrahedron $PQRS$, $\angle PSR = 30°$ and $\angle QSR = 40°$. If the size of $\angle PSQ$ is an integral number of degrees, how many possible values can it have?

(A) 9 (B) 59 (C) 69 (D) 90 (E) 180

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24. For how many positive integer values of a does the equation

$$\sqrt{a+x} + \sqrt{a-x} = a$$

have a real solution for x?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

25. Eight points lie on the circumference of a circle. One of them is labelled P. Chords join some or all of the pairs of these points so that the seven points other than P lie on different numbers of chords. What is the minimum number of chords on which P lies?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

26. Each of the students in a class writes a different 2-digit number on the whiteboard. The teacher claims that no matter what the students write, there will be at least three numbers on the whiteboard whose digits have the same sum. What is the smallest number of students in the class for the teacher to be correct?

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27. The sum of three numbers is 4, the sum of their squares is 10 and the sum of their cubes is 22. What is the sum of their fourth powers?

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28. In a regular polygon there are two diagonals such that the angle between them is $50°$. What is the smallest number of sides of the polygon for which this is possible?

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29. The sum of n positive integers is 19. What is the maximum possible product of these n numbers?

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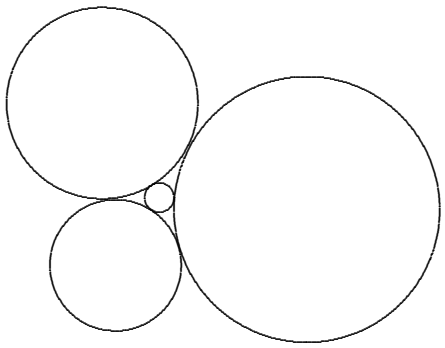
30. Three circles of radius 1, 2 and 3 centimetres just touch each other as shown. A smaller circle lies in the space between them, just touching each one.



The radius of the smallest circle is, in centimetres, $\frac{p}{q}$, where p and q are integers with no common factors. What is the value of $p+q$?
