Senior Division

Questions 1 to 10, 3 marks each

1. The value of $2010 - 20.10$ is

(A) 1990.09 (B) 1990.9 (C) 1989.09 (D) 1989.9 (E) 1998.9

2. If $m = 3$ and $n = -\frac{3}{5}$, then $\frac{m}{n}$ equals

(A) -5 (B) 5 (C) $-\frac{9}{5}$ (D) $-\frac{5}{3}$ (E) 15

3. The midpoint of PQ is $M(-4, 6)$. The point Q has coordinates $(10, 12)$. The point P is

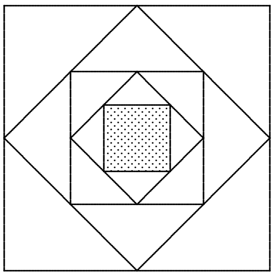
(A) $(-18, 0)$ (B) $(-18, 18)$ (C) $(-10, 0)$ (D) (3,9) (E) (3,18)

4. The number 63 is 87.5% of which number?

(A) 45 (B) 70 (C) 72 (D) 74 (E) 75

5. What percentage of the largest square is covered by the shaded square?

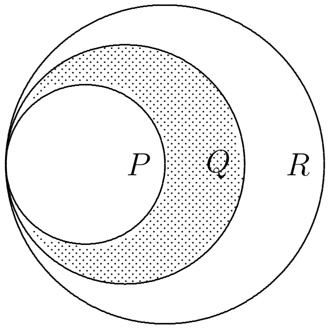
(A) 6.25% (B) 10% (C) 12.5% (D) 16% (E) 25%



6. Seven scores 8, 10, 24, 28, 23, 9 and x, have the property that the mean and median are both x. The value of x is

(A) 15 (B) 17 (C) 19 (D) 21 (E) 23

7. The radius of circle P is $\frac{2}{3}$ of the radius of circle Q and the radius of circle Q is $\frac{3}{4}$ of the radius of circle R.



What fraction of the area of the largest circle R is the shaded area?

(A) $\frac{1}{3}$ (B) $\frac{5}{9}$ (C) $\frac{1}{4}$ (D) $\frac{3}{16}$ (E) $\frac{5}{16}$

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8. A coin is tossed five times. What is the probability that the result will not be five tails in a row?

(A) $\frac{15}{16}$ (B) $\frac{27}{32}$ (C) $\frac{4}{5}$ (D) $\frac{9}{10}$ (E) $\frac{31}{32}$

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9. A rectangle is divided into x rows of y identical squares. Half of them are shaded to form the border with uniform width of 1 square as shown.

The sum of x and y could be

(A) 17 (B) 20 (C) 18 (D) 19 (E) 16

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10. When the numbers x^3, x^2, x, $-x$ and $\sqrt{x}$ are arranged in order from the largest to the smallest for any value of x where $0 < x < 1$, the middle number is

(A) x^3 (B) x^2 (C) x (D) $-x$ (E) $\sqrt{x}$

Questions 11 to 20, 4 marks each

11. For all values of x, the expression $\dfrac{7^{3x}+7^{2x}}{7^{2x}+7^{x}}$ is equal to

(A) 49 (B) 7^{2x} (C) 7 (D) 7^{x} (E) 1

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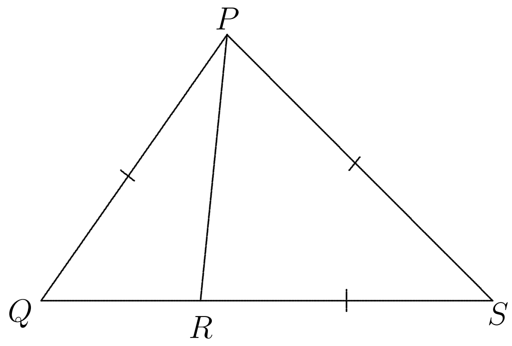
12. PQS is a triangle with R lying on QS, with $PQ = PS = SR$ and $\angle QRP = \angle QPS$.



The size of $\angle PSR$, in degrees, is

(A) 30 (B) 36 (C) 45 (D) 60 (E) 70

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13. If $\dfrac{3a+4b}{2a-2b} = 5$, then $\dfrac{a^2+2b^2}{ab}$ equals

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

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14. The value of $(123456785) \times (123456782) - (123456783) \times (123456784)$ is

(A) -2 (B) -1 (C) 0 (D) 1 (E) none of these

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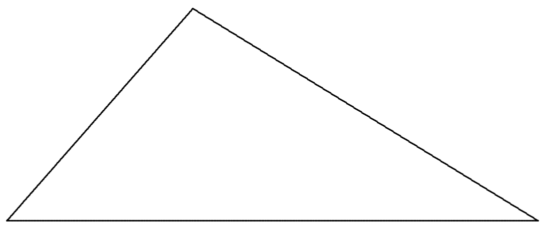
15. The length of each side of a triangle like the one below is a different prime number and its perimeter is also a prime number.



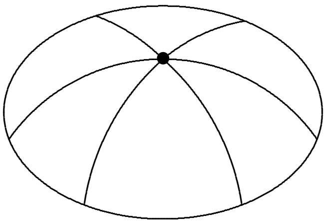
What is the smallest possible perimeter of such a triangle?

(A) 11 (B) 17 (C) 19 (D) 23 (E) 29

16. The 5-digit number $a986b$, where a is the first digit and b is the units digit, is divisible by 72. What is the value of $a+b$?

(A) 9 (B) 10 (C) 12 (D) 13 (E) 15

17. A cap consists of six pieces, all the same size and shape.



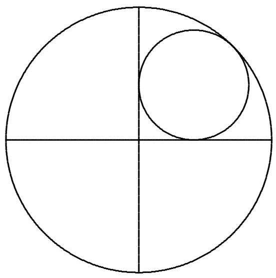
If each piece can be either gold or brown, how many different caps can be made?

(A) 12 (B) 14 (C) 16 (D) 18 (E) 20

18. For all positive integers n, $\text{Snap}(n) = 2n$ if n is even and $\text{Snap}(n) = 3n$ if n is odd. If p is a prime number greater than 2, what is the value of $\text{Snap}\big(\text{Snap}(p-1) - p\big)$?

(A) $p-2$ (B) $2p-2$ (C) $2(p-2)$ (D) $3p-2$ (E) $3(p-2)$

19. A circle is inscribed in a quadrant of a larger circle. The ratio of the area of the inner circle to that of the quadrant is



(A) $2:3$ (B) $4:5$ (C) $3:(2+\sqrt{3})$ (D) $\sqrt{2}:\sqrt{3}$ (E) $4:(3+2\sqrt{2})$

20. The operation $\otimes$ means $a \otimes b = a + b^2$. If $a > 0$ and $(a \otimes a) \otimes a = a \otimes (a \otimes a)$, then a equals

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{2}-1$ (D) $\sqrt{2}+1$ (E) 2

Questions 21 to 25, 5 marks each

21. The *super factorial* number $1! \times 2! \times 3! \times \cdots \times 12!$ can be written as a factorial times a perfect square, that is, in the form $m! \times n^2$. What is the value of m?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

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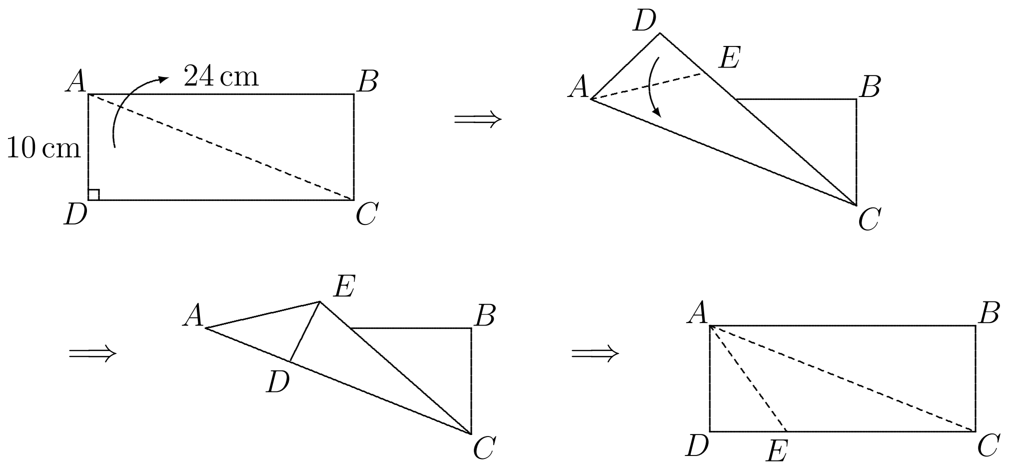
22. The rectangular piece of paper pictured has length $AB = 24\,\text{cm}$ and width $AD = 10\,\text{cm}$. It is folded along the diagonal AC and then triangle ACD is folded along the line AE so that AD is aligned with AC.



How long, in centimetres, is DE?

(A) $\frac{13}{2}$ (B) $\frac{10}{\sqrt{3}}$ (C) $\frac{20}{3}$ (D) 8 (E) 12

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23. There are sixteen different ways of writing four-digit strings using 1s and 0s. Three of these strings are 1010, 0100 and 1001. These three can be found as substrings of 101001. There is a string of nineteen 1s and 0s which contains all sixteen strings of length 4 exactly once. If this string starts with 1111, the last four digits are

(A) 1110 (B) 0000 (C) 0110 (D) 1010 (E) 0111

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24. What is the smallest n such that no matter how n points are placed inside or on the surface of a cube of side length 16 units, there are at least two of these points which are closer than 14 units to each other?

(A) 8 (B) 9 (C) 11 (D) 12 (E) 13

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25. Loki stands at the centre of a forest which has trees with trunks of identical radii at every integer coordinate point except the origin, where he is standing. From where he is, he cannot see beyond the second tree in any direction. That is, he cannot see any tree with either coordinate of magnitude greater than 2. What is the smallest possible radius of the tree trunks?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{\sqrt{10}}$ (D) $\frac{1}{\sqrt{13}}$ (E) $\frac{1}{2(\sqrt{13}-3)}$

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

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26. If $m + n = 11$ and $m^2 + n^2 = 99$, what is the value of $m^3 + n^3$?

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27. A 3-digit number is subtracted from a 4-digit number and the *result* is a 3-digit number.

□□□□ − □□□ = □□□

The 10 digits are all different.
What is the smallest possible result?

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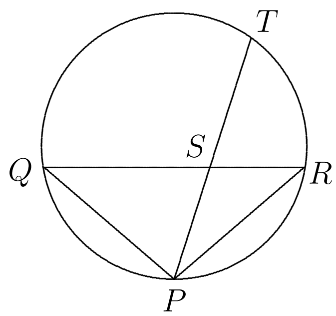
28. In the triangle PQR, $PQ = PR = 40\,\text{cm}$ and S is a point on QR such that $PS = 25\,\text{cm}$. The extension of PS meets the circle through PQR at T.



What is the length, in centimetres, of PT?

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29. A polynomial f is given. All we know about it is that all its coefficients are non-negative integers, $f(1) = 6$ and $f(7) = 3438$. What is the value of $f(3)$?

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30. There are many towns on the island of Tetra, all connected by roads. Each town has three roads leading to three other different towns: one red road, one yellow road and one blue road, where no two roads meet other than at towns. If you start from any town and travel along red and yellow roads alternately (RYRY...) you will get back to your starting town after having travelled over six different roads. In fact RYRYRY will always get you back to where you started. In the same way, going along yellow and blue roads alternately will always get you back to the starting point after travelling along six different roads (YBYBYB). On the other hand, going along red and blue roads alternately will always get you back to the starting point after travelling along four different roads (RBRB). How many towns are there on Tetra?