2017 INTERNATIONAL TEENAGERS MATHEMATICS OLYMPIAD (ITMO)
DAVAO CITY, PHILIPPINES

08-12 NOVEMBER 2017

ORGANIZED BY: MATHEMATICS TRAINERS' GUILD, PHILIPPINES
WWW.MTGPHIL.ORG

KEY STAGE 3 - INDIVIDUAL CONTEST

TIME LIMIT: 120 MINUTES

INFORMATION:

- You are allowed 120 minutes for this paper, consisting of 12 questions in Section A to which only numerical answers are required, and 3 questions in Section B to which full solutions are required.
- Each question in Section A is worth 5 points. No partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. Each question in Section B is worth 20 points. Partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

INSTRUCTIONS:

- Write down your name, your contestant number and your team's name in the space provided on the first page of the question paper.
- For Section A, enter your answers in the space provided after the individual questions on the question paper. For Section B, write down your solutions on spaces provided after individual questions.
- You must use either a pencil or a ball-point pen which is either black or blue.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing the question paper and all scratch papers.

TEAM:	NAME:	NO.	SCORE:

FOR JURIES USE ONLY

No.	SECTION A												SECTION B			Total	Sign by Jury
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3		
Score																	
Score																	

Section A.

In this section, there are 12 questions, each correct answer is worth 5 points. Fill in your answer in the space provided at the end of each question.

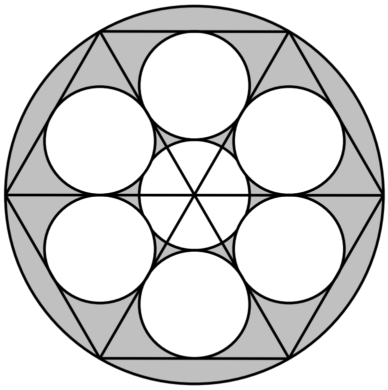
1. If both a and b are positive integers greater than 1, find the smallest possible sum of a and b such that $\sqrt{a\sqrt{a}} = b$.

Answer: ____________

2. Find the largest positive integer d, in which there exists at least one integer n such that d divides both n^2+1 and $(n+1)^2+1$.

Answer: ____________

3. A regular hexagon is inscribed in a circle with radius 12 cm. The hexagon is divided into 6 congruent equilateral triangles and each of them has a small circle inscribed in it. Another small circle is then drawn touching all the inscribed circles. What is the area of the shaded region? Take $\pi = 3.14$.



Answer: ____________ cm^2

4. Find the smallest positive integer n so that $2n$ is a perfect square and $7n$ is a perfect 7^{th} power.

Answer: ____________

5. Find all possible integer values of x, in which $\left(\frac{21}{x}-2\right)^2 - 2\left(\frac{21}{x}-2\right) = x + 42$.

Answer: ____________

6. If the equation $\left(a^2+3b^2\right)x^2 - \left(4a+6b\right)x + 7 = 0$ has a root of 2017, where a and b are real numbers, find the sum of a and b.

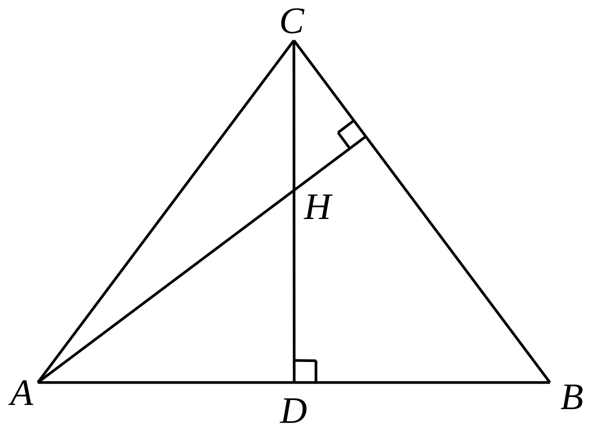
Answer: ____________

7. If $x > 1$, find all possible values of x that satisfies the following equation:

$$\frac{x-2017}{2018} - \frac{x-2018}{2017} = \frac{2017}{x-2018} - \frac{2018}{x-2017}.$$

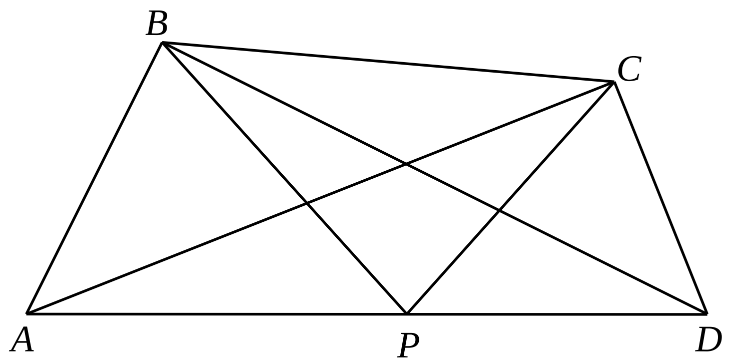
Answer: ____________

8. In the figure below, *ABC* is an isosceles triangle with base *AB*. Its orthocentre *H* divides its altitude *CD* into two segments *CH* and *HD*, where $CH = 7$ cm and $HD = 9$ cm. Find the perimeter of triangle *ABC*, in cm.



Answer : ________cm

9. In quadrilateral *ABCD*, $\angle ABD = \angle ACD = 90°$ and a point *P* is on *AD* so that $\angle APB = \angle CPD$, as shown in the figure below. If $AP = 24$ cm and $DP = 19$ cm, find the value of $PB \times PC$.



Answer : ______________

10. A girl tosses a fair coin 100 times and a boy tosses a fair coin 101 times. The boy wins if he has more heads than the girl has, otherwise, he loses. Find the probability that the boy win this game.

Answer : ____________%

11. A 6×6 chessboard is formed by 36 unit squares. How many different combinations of 4 unit squares can be selected from the chessboard so that no two unit squares are in the same row or column?

Answer : ______________

12. If $\overline{abcd}$ is a 4-digit number, where each different letter represents a different digit such that $a < b$, $c < b$ and $c < d$. How many such 4-digit numbers are there?

Answer : ______________

Section B.

Answer the following 3 questions, each question is worth 20 points. Partial credits may be awarded. Show your detailed solution in the space provided.

1. Let α, β and γ be the three roots of the polynomial x^3-5x+1. If p and q are relatively prime positive integers such that

$$-\frac{p}{q}=\frac{\alpha^3}{(6\beta+1)(6\gamma+1)}+\frac{\beta^3}{(6\alpha+1)(6\gamma+1)}+\frac{\gamma^3}{(6\beta+1)(6\alpha+1)}.$$

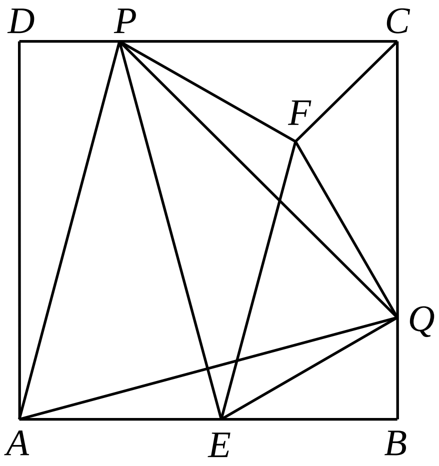
Find the sum of p and q.

Answer : _____________

2. A 21×21 table contains 21 copies of each of the numbers 1, 2, 3, . . . , 20 and 21. The sum of all the numbers above the main diagonal (diagonal from the top-left cell to the bottom-right cell) is equal to three times the sum of all the numbers below the main diagonal. Find the sum of all the numbers on the main diagonal of the table.

Answer : _____________

3. In the figure below, $ABCD$ is a square. Points E, Q and P are on sides AB, BC and CD, respectively, such that $PE \perp AQ$ and $\triangle AQP$ is an equilateral triangle. Point F is inside $\triangle PQC$ such that $\triangle PFQ$ and $\triangle AEQ$ are congruent. If $EF = 2\,\text{cm}$, find the length of FC, in cm.



Answer: ___________ cm