2017 INTERNATIONAL TEENAGERS MATHEMATICS OLYMPIAD (ITMO) DAVAO CITY, PHILIPPINES

08-12 NOVEMBER 2017

MTG PHILIPPINES

ORGANIZED BY: MATHEMATICS TRAINERS' GUILD, PHILIPPINES
WWW.MTGPHIL.ORG

KEY STAGE 3 - TEAM CONTEST

TIME LIMIT: 60 MINUTES

INFORMATION:

- You are allowed 60 minutes for this paper, consisting of 10 questions printed on separate sheets. For questions 1, 3, 5, 7 and 9, only numerical answers are required. For questions 2, 4, 6, 8 and 10, full solutions are required.
- Each question is worth 40 points. For odd-numbered questions, no partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. For even-numbered questions, partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

INSTRUCTIONS:

- Write down your team's name in the space provided on every question sheet.
- Enter your answers in the space provided after individual questions on the question paper.
- During the first 10 minutes, the four team members examine the first 8 questions together, and then altogether discuss them. Then they distribute the questions among themselves, with each team member is allotted at least 1 question.
- During the next 35 minutes, the four team members write down the solutions of their allotted problems on the respective question sheets, with no further communication /discussion among themselves.
- During the last 25 minutes, the four team members work together to write down the solutions of the last 2 questions on the respective questions sheets.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing all question sheets and all scratch papers.

TEAM:	SCORE:

FOR JURIES USE ONLY

No.	1	2	3	4	5	6	7	8	9	10	Total	Sign by Jury
Score												
Score												

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10th November, 2017, Davao City, Philippines

Team：_____ Score：_____

1. Let $f(x)=\frac{9^x}{9^x+3}$. Calculate $f\left(\frac{1}{2017}\right)+f\left(\frac{2}{2017}\right)+...+f\left(\frac{2015}{2017}\right)+f\left(\frac{2016}{2017}\right)$.

Answer: _____

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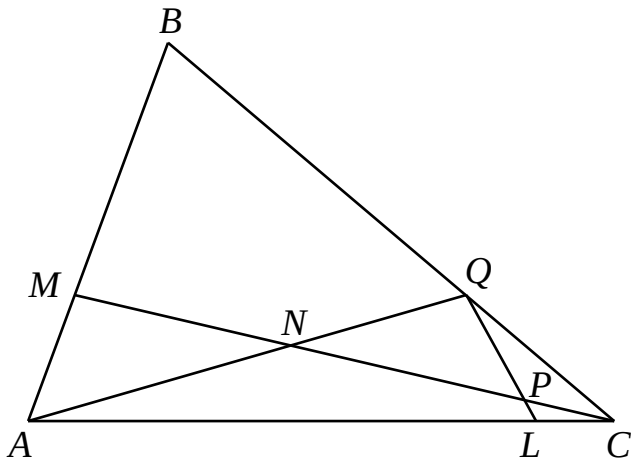
2. In $\triangle ABC$, point M is between A and B such that $AM : MB = 1 : 2$. Points N and P are between C and M such that $CN : NM = 3 : 2$, $CP : PM = 1 : 5$. Segments AN and BC intersect at point Q. Segments PQ and AC intersect at point L. Find the ratio $CL : LA$.



Answer: $CL : LA =$ _____ : _____

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3. Find the largest integer p such that $14^{2017}+2^{2017}$ is divisible by 2^p.

Answer: _____

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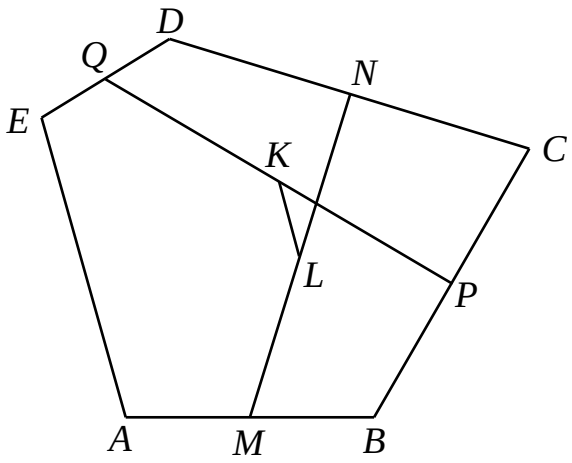
4. In pentagon *ABCDE*, points *M*, *P*, *N* and *Q* are midpoints of *AB*, *BC*, *CD* and *DE* respectively. While points *K* and *L* are midpoints of *QP* and *MN*, respectively, as shown in the figure below. If $KL = 25$ cm, find the length of *EA*, in cm.



Answer: _____ cm

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5. Let x and y be positive integers, where $0 < x < y < 2018$. How many ordered pairs (x, y) are there such that $x^2 + 2018^2 = y^2 + 2017^2$?

Answer: _____ pairs

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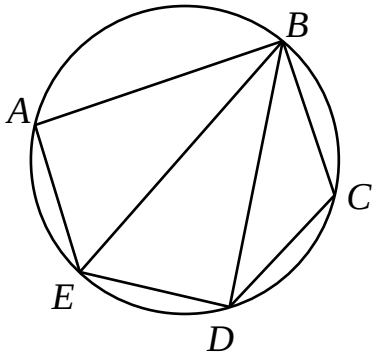
6. Points *A*, *B*, *C*, *D* and *E* are on the circumference. Chord *AC* is a diameter of the circle, as shown in the figure below. If $\angle ABE = \angle EBD = \angle DBC$, $BE = 16$ cm and $BD = 12\sqrt{3}$ cm, find the area of pentagon *ABCDE*.



Answer: _____ cm

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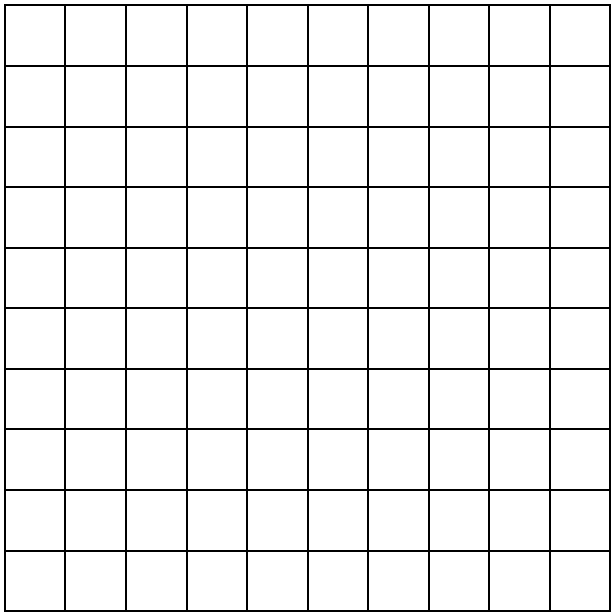
7. A 10×10 chessboard is dissected into thirty-three 1×3 or 3×1 rectangles and one unit square. In how many different positions can this unit square be, if the chessboard may not be reflected or rotated?



Answer: _____

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8. Prove the inequality: $\sqrt{99\times101}+\sqrt{98\times102}+...+\sqrt{1\times199}<\frac{100^2\pi}{4}$.

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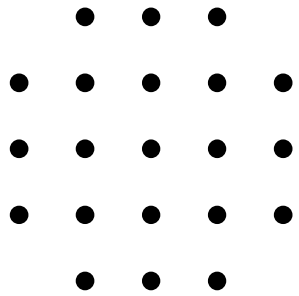
9. A computer randomly chooses three different points on the given grid below (all points have the equal chance of being chosen).

Let $\frac{p}{q}$ be the probability to form a triangle with these points (this fraction is written in its irreducible form). Find the sum of p and q.



Answer: _____

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10. Jane has 12 marbles, where in one is fake. She are not certain if the fake marble is heavier or lighter than the real marble. What is the minimum number of weightings needed to find the fake marble and determine whether the fake marble is heavier or lighter than the real marble? Explain your answer.

Answer: _____ ways