2017 INTERNATIONAL TEENAGERS MATHEMATICS OLYMPIAD (ITMO)
DAVAO CITY, PHILIPPINES
08-12 NOVEMBER 2017

ORGANIZED BY: MATHEMATICS TRAINERS' GUILD, PHILIPPINES
WWW.MTGPHIL.ORG

KEY STAGE 2 - INDIVIDUAL CONTEST

TIME LIMIT: 90 MINUTES

INFORMATION:

- You are allowed 90 minutes for this paper, consisting of 15 questions to which only numerical answers are required.
- Each question is worth 10 points. No partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found.
- Diagrams shown may not be drawn to scale.

INSTRUCTIONS:

- Write down your name, your contestant number and your team's name on the answer sheet.
- Enter your answers in the space provided on the answer sheet.
- You must use either a pencil or a ball-point pen which is either black or blue.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing the question paper, your answer sheet and all scratch papers.

TEAM:	NAME:	NO.

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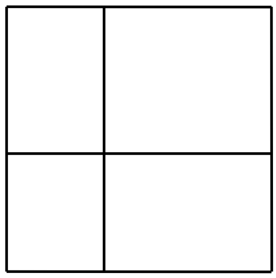
1. A mathematics teacher originally designed a 90-minute exam which contains 75 questions. How many questions should be removed from the exam if the teacher wants to create a 60–minute exam with the same average time for each question as in the original 90–minute exam?

2. What is the remainder when $1+2+2^2+2^3+...+2^{2017}$ is divided by 5?

3. Alex, Bert, Carlos and David are suspects of a bank robbery. It has been found out that:
   (a) If Bert is guilty, then Carlos is a partner in crime;
   (b) If Carlos is guilty, then either Alex is a partner in crime, or Bert is innocent;
   (c) If David is innocent, then Bert is guilty and Alex is innocent;
   (d) If David is guilty, then Bert is also guilty.

   Among the four suspects, how many are innocent?

4. A square is divided by lines parallel to its sides into 4 rectangles as shown in the figure below. The length of a side of the square is known to be an integer, in cm., while the areas of the four rectangles are pairwise distinct positive integers. Find the smallest possible value of the area of the original square, in cm^2.

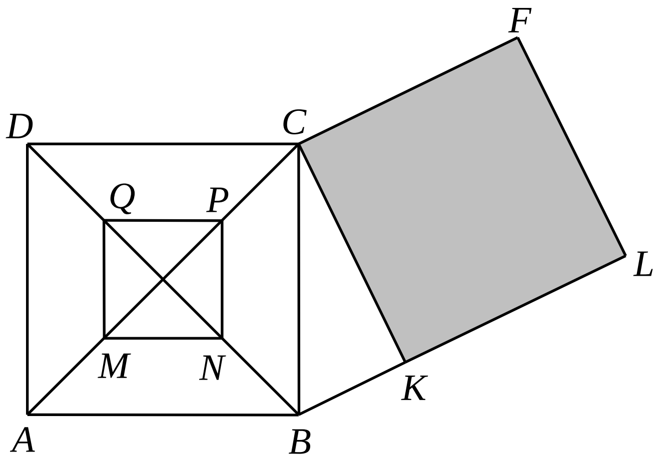


5. Find the greatest positive integer made up of different digits, such that if we remove the leftmost digit, its value would become $\frac{1}{5}$ of the original number.

6. Three brothers have some candies in a pile. On the first day, the eldest brother divided all the candies into three equal parts with a remainder of one candy, and then this eldest brother took one part. On the second day, the youngest brother divided all the remaining candies into five equal parts with one remaining candy, and then this youngest brother took two parts. On the third day, the third brother divided the remaining candies into five equal parts with no extra candy left. After the third brother had taken three parts of candies, what is the minimum number of candies left in all?

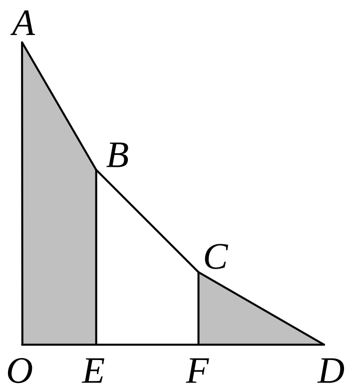
7. How many numbers from all 2-digit, 3-digit and 4-digit numbers are neither divisible by 12 nor by 30?

8. If $ABCD$ and $KLFC$ are two squares so that B, K and L are collinear. Points M and P are on AC, points N and Q are on BD so that $MNPQ$ is also a square, as shown in the figure below. If $MN = BK$ and area of quadrilateral $BCPN$ is 2017 cm^2, find the area of square $KLFC$, in cm^2.



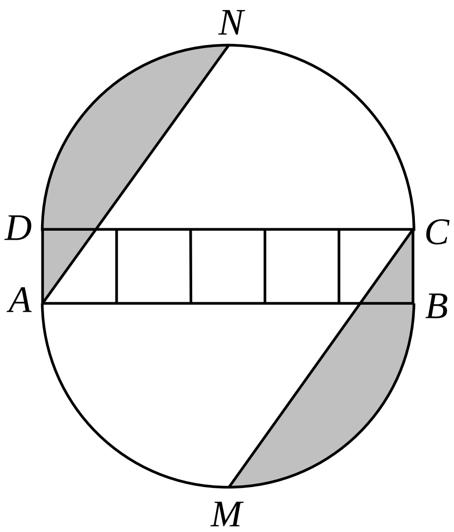
9. There are 70 boys and 30 girls that participated in a school chess tournament. In each round, all participants are grouped into 50 pairs and each pair plays one game. After three rounds, 21 games were played between girls. How many games were played between boys?

10. If OA and OD are perpendicular segments each with length of 20 cm. AB, BC and CD are segments of equal length such that $\angle OAB = 30° = \angle ODC$. E and F are points on OD such that BE and CF are perpendicular to OD. Let $[P]$ denote the area of the polygon P. Determine $\dfrac{[OABE]+[CDF]}{BC}$, in cm.

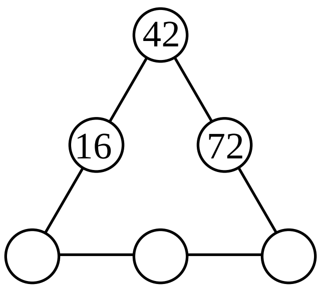


11. In today's school fair, donuts were given out for snacks. Each student got 2 donuts, while each adult got 3 donuts. All of the girl students ate just one of their donuts, and gave away their second one to the boy students. The adults ate all of their share of donuts. Finally, there are 23 boy students that ate 4 donuts and all of the remaining boy students ate 3 donuts. We know that a total of 237 people joined the fair and ate a total of 508 donuts. How many boy students attended the school fair?

12. In the figure below, $AB = 5AD = 70$ cm. *DNC* and *AMB* are two semicircles and points *M* and *N* are the midpoints of arcs *DC* and *AB*, respectively. Find the area of the shaded region, in cm^2. Take $\pi = \frac{22}{7}$.



13. The diagram shows six circles lying in 3 different lines. Three of the circles are numbered 42, 16 and 72 while the remaining three circles should contain positive integers, such that product of the numbers written in each of the three lines are the same. How many different ways can this be done?



14. Suppose one of the symbols ○, △ , ▽ is inserted into each of the unit squares of the 2×6 chessboard. How many different ways can these symbols be inserted into the unit squares of the chessboard such that the same symbols are not placed between any two unit squares that share a common side? (Note: Rotation or flipping should be considered different.)

15. How many ways can we choose 3 different numbers from the set {1, 2, 3, …, 24, 25} so that they have no common divisors greater than 1?