Team Contest

Time limit: 60 minutes 2015/12/11

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on every page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each question is printed on a separate sheet of paper. Each problem is worth 40 points and complete solutions of problem 2, 4, 6, 8 and 10 are required for full credits. Partial credits may be awarded. In case the spaces provided in each problem are not enough, you may continue your work at the back page of the paper. Only answers are required for problem number 1, 3, 5, 7 and 9.

  The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each student must attempt at least one problem. Each will then have 35 minutes to write the solutions of their allotted problem independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together.
- Diagrams are NOT drawn to scale. They are intended only as aids.
- No calculator or calculating device or electronic devices are allowed.
- Answer must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

Team Name：______________________________

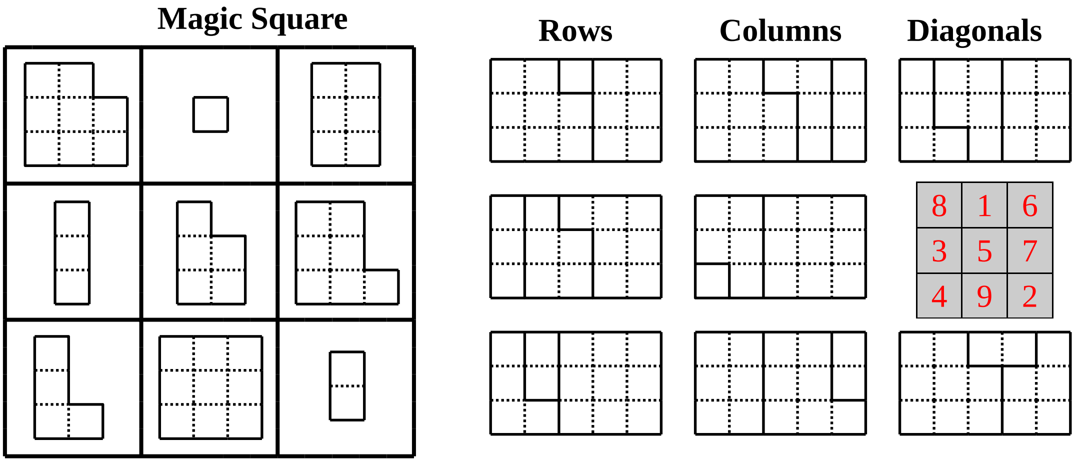
----- Jury use only ---

Problem 1 Score	Problem 2 Score	Problem 3 Score	Problem 4 Score	Problem 5 Score	Total Score
Problem 6 Score	Problem 7 Score	Problem 8 Score	Problem 9 Score	Problem 10 Score	

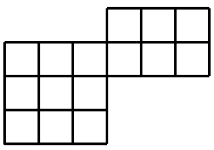
TEAM CONTEST

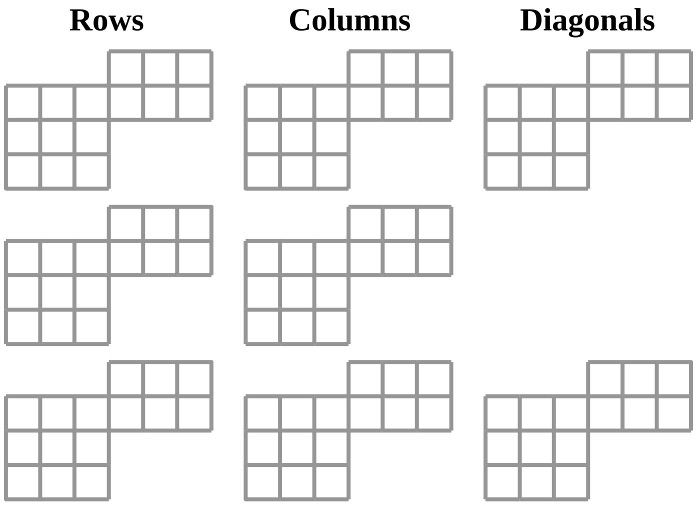
Team : ______________________ *Score* : ____________

1. The geometric magic square in the diagram below on the left is based on the ordinary magic square in the diagram below on the right. The latter is shaded. Of course, the total number of unit squares in the three pieces in each row, column and diagonal is equal to 15, known as magic constant. If that is all, we are not doing anything new. Instead, the magic constant is no longer a number but a figure, which can be formed with the three pieces in each row, column and diagonal. Rotations and reflections of the pieces are allowed. The diagram below on the right shows that the magic constant can be a 3×5 rectangle.



Show that the figure in the diagram below can also be the magic constant for the geometric magic square above.





ANSWER: ______________________

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2. Let a be a positive integer such that $a^2 + b^2 - a$ is a multiple of ab for some positive integer b relatively prime to a. Find the maximum value of a.

ANSWER: ______________________

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3. Let $S(x)$ denotes the sum of the digits in the decimal representation of a positive integer *x*.
Find the largest value of *x* such that $x+S(x)+S(S(x))+S(S(S(x)))=2015$.

ANSWER: ______________________________

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4. A two-player game starts with the number 111 on the blackboard. Anna goes first, followed by Boris, taking turns alternately thereafter. In each move, Anna may reduce the number on the blackboard by 1 or 10, while Boris may reduce it by 1, 2, 8 or 10. The player who reduces the number to 0 wins. Give the winning strategy for Boris.

ANSWER: ______________________________

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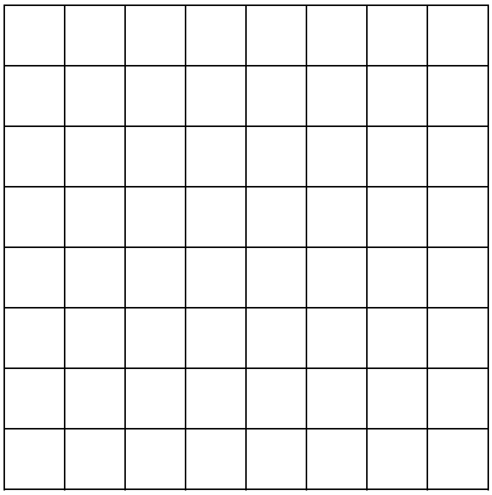
5. In how many different ways can you choose three squares in an 8×8 chessboard so that every two of them share at least one corner?



ANSWER: ______________________ ways

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6. Each square of a 4×4 table is filled with a different one of the positive integers 1, 2, 3, …, 15, 16. For every two squares sharing a side, the numbers in them are added and the largest sum is recorded. What is the minimum value of this largest sum?

ANSWER: ______________________________

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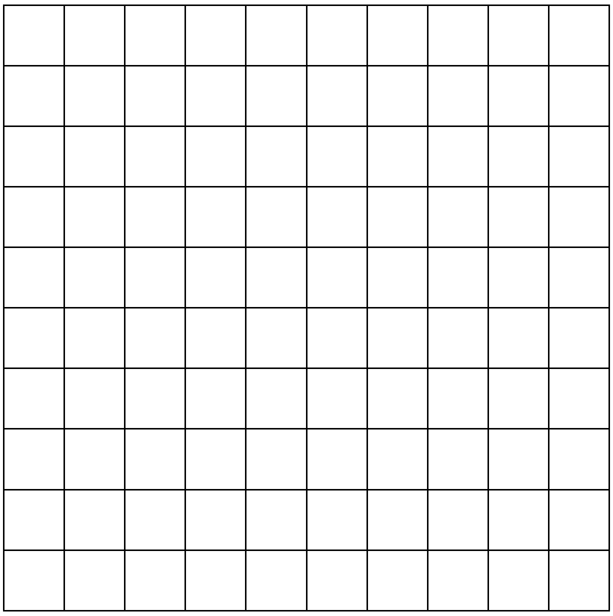
7. A city is divided into 100 squares in a 10×10 configuration. Each police squad occupies two squares which share exactly one corner. What is the minimum number of police squads so that every square not occupied by a police squad must share a side with a square occupied by a police squad?



ANSWER: ____________ police squads

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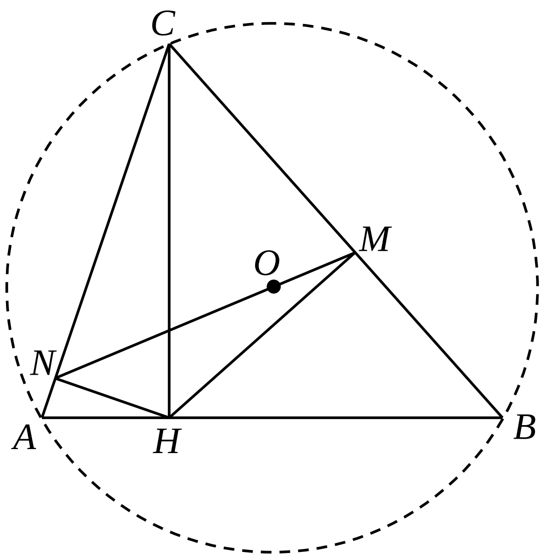
8. *ABC* is an acute triangle. *H* is the foot of the altitude from *C* to *AB*. *M* and *N* are the respective feet of perpendicular from *H* to *BC* and *CA*. The circumcentre of *ABC* lies on *MN*. If the circumradius of *ABC* is $\sqrt{2}$ cm, what is the length of *CH*, in cm?



ANSWER: ______________________ cm

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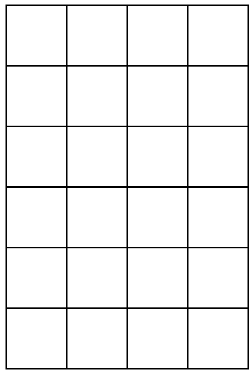
9. Find the number of ways of colouring 12 different squares of a 6×4 chessboard such that there are two coloured squares in each row and three in each column.



ANSWER: ______________________ ways

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10. What is the minimum number of fourth powers of integers, not necessarily distinct, such that their sum is 2015?

ANSWER: ______________________