Mathematics Exploration Problems

Country: _____ Name: _____ No.: _____ Score: _____

Instructions:

* Write down your name and country on the answer sheet.

* Write your answer on the answer sheet.

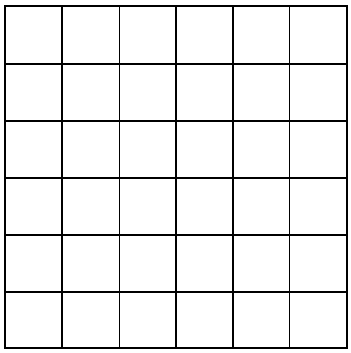
* You have 120 minutes to work on this test.

* Each problem is worth 6 points, and partial credit may be awarded.

* Use black or blue pen or pencil to write your answer.

EXPLORATION PROBLEMS

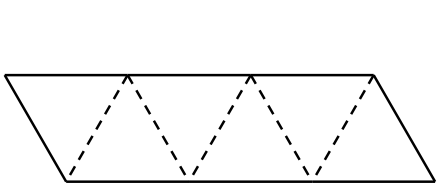
1. Each of the 36 cells in a 6 × 6 table is to be filled with numbers 0, 1 or 2. The sum of the six numbers in each row and each column is then computed. How do you fill the cells to get the maximum number of different values among the 12 sums?

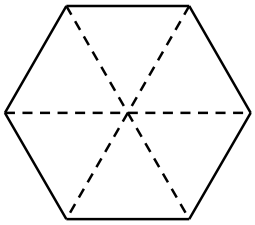


2. In the Indonesian village Sevenia, each bill is worth a number of rupiahs equal to a power of 7, namely, $7^0 = 1$, $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$, $7^5 = 16807$, $7^6 = 117649$, $7^7 = 823543$, and so on. A millionaire has some bills worth a total of exactly 1000000 rupiahs, but at most 8 bills of any kind. No two millionaires have exactly the same combinations of bills. What is the maximum number of millionaires in Sevenia?

3. The diagram below shows two figures connected and constructed from six unit equilateral triangles satisfying the following two conditions.
   (1) When two equilateral triangles are joined together, an entire edge of one must match up with an entire edge of the other.
   (2) The figure is in one connected piece.

   Draw all other such figures.

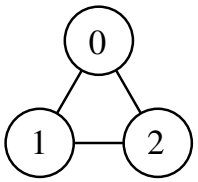




EXPLORATION PROBLEMS

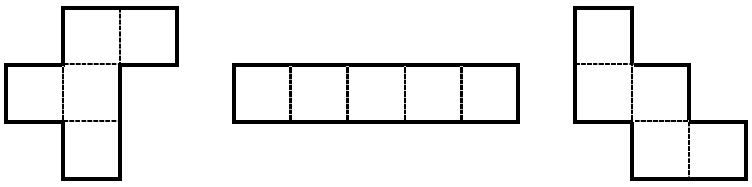
4. Write down all positive integers $n \geq 9$ which are divisible by every positive integer not greater than $\frac{n}{9}$.

5. A magic triangle consists of several circles in a triangular configuration, and filled with consecutive integers starting from 0. The integer in a circle supported by two other circles below is the remainder when the sum of the integers in the supporting circles is divided by the number of circles. Moreover, the integer in the bottom leftmost circle is less than the integer in the bottom rightmost circle. The diagram below shows an example of a triangle with three circles such that the only division $(1 + 2) \div 3$ leaves a remainder of 0.



   (a) Write down all magic triangles with six circles. (2 marks)
   (b) Write down all magic triangles with ten circles. (4 marks)

6. The three shapes in the diagram below are called F, I and W respectively.



   (a) Use two copies of F to make a figure, and then two copies of W to make the same figure. No overlap is allowed. (1 mark)
   (b) Use several copies of F to make a figure, and then the same number of copies of I to make the same figure. No overlap is allowed. (2 marks)
   (c) Use several copies of W to make a figure, and then the same number of copies of I to make the same figure. No overlap is allowed. (3 marks)