Mathematics Essay Problems

Country:____________ Name: ____________________ ID: __________ Score:_______

Instructions:

- Write down your name and country on every page.
- You have 90 minutes to work on this test.
- Write down your detail solutions or working process in English on the space below the question.
- Each problem is worth 3 points, and partial credit may be awarded.
- Use black or blue colour pen or pencil to write your answer.

The following table is for jury use only.

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Score														
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ESSAY PROBLEMS

Country: ________________ Name: __________________________ ID: ____________

1. How many three-digit positive integers $\overline{abc}$ are there such that $a \leq b \leq c$?

ANSWER: ________________ **three-digit positive integers**

ESSAY PROBLEMS

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2. Twenty-four positive numbers are arranged on a circle, each number is equal to the product of its two neighbors. If the two neighboring numbers are 3 and 4, what is the sum of all twenty-four numbers?

ANSWER: ____________________

ESSAY PROBLEMS

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3. Let a, b and c be different positive integers such that $1=\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$.
What is the smallest possible value of $a+b+c$?

ANSWER: ____________________

ESSAY PROBLEMS

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4. Arranged in a circle are 100 plates. Susan places a candy in a plate. Going around the circle, she places a candy on every 15th plate. If she keeps doing so until the candies can no longer be placed in an empty plate, how many plates remain empty?

ANSWER: ______________ **plates**

ESSAY PROBLEMS

Country**:** ________________ Name**:** ________________________ ID**:** ___________

5. What is the largest integer less than or equal to the expression

$$\frac{1}{\frac{1}{1985}+\frac{1}{1986}+\frac{1}{1987}+\cdots+\frac{1}{2015}}\ ?$$

ANSWER:___________________

ESSAY PROBLEMS

Country: ________________ Name: _________________________ ID: ____________

6. There are 81 soldiers lined up in a row with numbers from 1 to 81. In each round onwards, the remaining soldiers call out 1, 2, 3, 1, 2, 3, 1, 2, 3, … . The soldiers who count 1 and 3 are removed from the line. The process continues until only one soldier is left on the line. What is the number of that soldier left on the line?

ANSWER:______________________

ESSAY PROBLEMS

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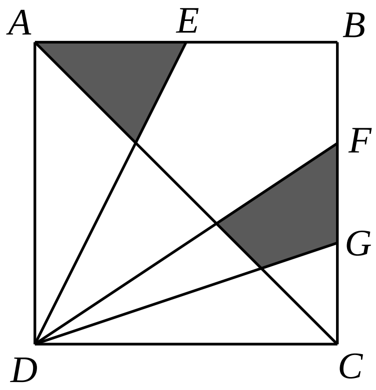
7. The figure below shows a square $ABCD$ of side 6 cm. Given that E is the midpoint of AB, points F and G are on BC so that $BF = FG = GC$. What is the total area of the shaded region in cm^2?



ANSWER: ________________ **cm^2**

ESSAY PROBLEMS

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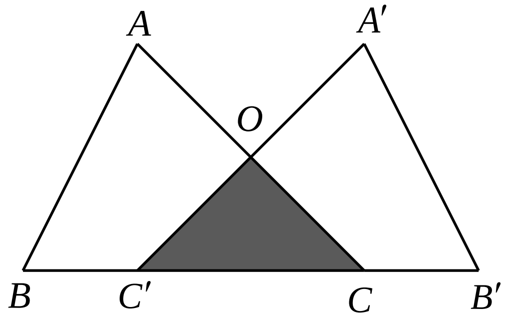
8. In $\Delta\,ABC$, $\angle ACB = 45°$ and $BC = 24$ cm. The length of the altitude from A to BC is 16 cm. Point B', C' are on the line of BC such that $BC = B'C'$. Suppose $AB = A'B'$, $AC = A'C'$, as shown in the figure below. If the area of $\Delta OCC'$ is $\frac{1}{3}$ of the area of ΔABC, what is the length of BB', in cm?



ANSWER: ________________ **cm**

ESSAY PROBLEMS

Country: ________________ Name: _________________________ ID: ___________

9. In a four-digit number, the thousands digit is larger than the units digit, which is not zero, while the hundreds digit is larger than the tens digit. A new four-digit number is obtained from the original number by reversing the order of the digits. How many possible differences of the original and new number are there?

ANSWER: ____________________

ESSAY PROBLEMS

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10. There are three lowest-term fractions, the ratio of their numerator are positive integers in the ratio of 3 : 2 : 4 while the ratio of their denominator are positive integers in the ratio of 5 : 9 : 15. The sum of these three fractions is $\frac{28}{45}$.

    What is the sum of their denominator?

ANSWER: ____________________

ESSAY PROBLEMS

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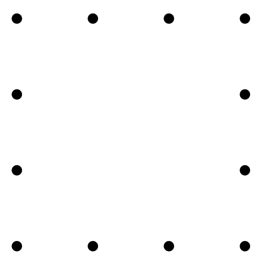
11. Sixteen points are on the sides of a 4×4 grid so that the center portion of 2×2 are removed. How many triangles are there in total that have vertices chosen from those remaining points and at least 1 interior angle equal to $45°$?



ANSWER: **______________ triangles**

ESSAY PROBLEMS

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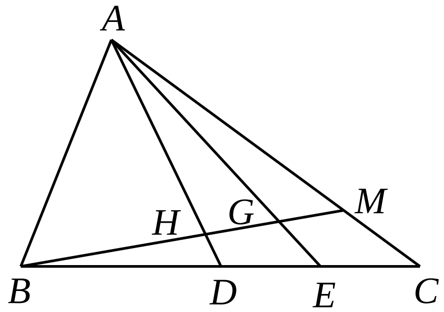
12. In ΔABC, points D and E are on BC such that $BD : DE : EC = 2 : 1 : 1$. The point M is on AC such that $\frac{CM}{MA} = \frac{1}{3}$. BM intersects AD, AE at point H, G respectively. Find $BH : HG : GM$.

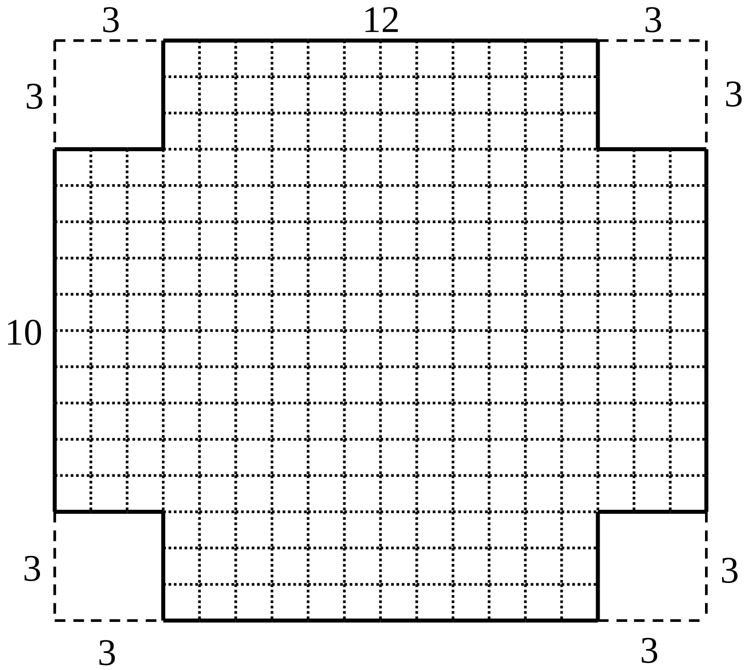


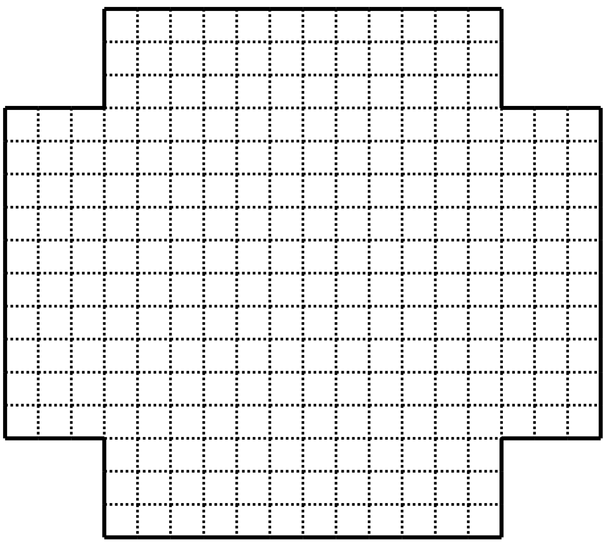
ANSWER: ***BH*** : ***HG*** : ***GM*** **=** ______ : ______ : ______

ESSAY PROBLEMS

Country**:** ________________ Name**:** _________________________ ID**:** ____________

13. From a 16 cm by 18 cm piece of paper, a 3 cm by 3 cm square is cut off from each corner. At most how many 3 cm by 4 cm rectangles can be cut off from the remaining part of this piece of paper?





ANSWER: ________ **rectangles,**