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ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

1. Positive integers are arranged in the following pattern.

1	2	3	4	...	100
2	3	4	5	...	101
3	4	5	6	...	102
4	5	6	7	...	103
⋮	⋮	⋮	⋮	⋮	⋮
100	101	102	103	...	199

Find the sum of all the integers in the table above.

Answer : _____

ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

2. In 2017, Alex's age is equal to the sum of all the digits of his year of birth and his mother's age is also equal to the sum of all the digits of her year of birth. How old was Alex's mother when she gave birth to Alex?

Answer : _____ years old

ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

3. A 4035-digit number is a multiple of 13. Its first 2017 digits are all 5s, and its last 2017 digits are all 6s. What is the middle digit?

$$\underbrace{555 \cdots 55}_{2017 \text{ digits}} ? \underbrace{666 \cdots 66}_{2017 \text{ digits}}$$

Answer : _____

ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

4. All positive integers from 1 to 10,000 are arranged according to the following rules:

- The first row contains all the integers whose sum of their digits is 1.
- The second row contains all the integers whose sum of their digits is 2.
- The third row contains all the integers whose sum of their digits is 3.
-
- Integers in each row are arranged in increasing order.
- Each of these rows are then rearranged in a line such that the first number in the second row comes after the last number of the first row; the first number in the third row comes after the last number of the second row;... and so on.

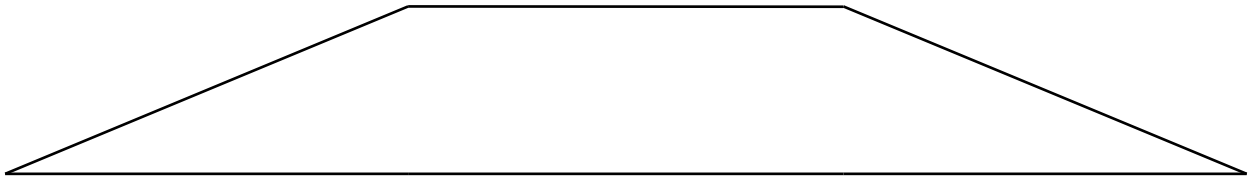
Which term is the number 9799 in the final arrangement?

Answer : _____ *term*

ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

5. A trapezium has three equal sides, and the base is 2 cm less than the sum of these three sides. If the distance between the parallel sides is 5 cm, find the area, in cm^2 , of the trapezium.



Answer : _____ cm^2

ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

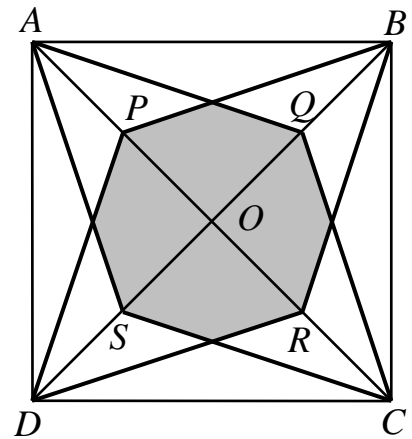
6. A box contained some red and blue balls. At least 90% of the balls were red. Peter first took the balls out of the box one at a time. Out of the first 50 balls that he took out, only one was blue. Subsequently, he took out 8 balls each time where one was blue and 7 were red. He continued in this manner until fewer than 8 balls were left in the box. Find the maximum number of balls the box could have contained at the beginning.

Answer : _____ balls

ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

7. Square $ABCD$ has sides of length 12 cm. The centre of the square is O , and P , Q , R and S are the midpoints of OA , OB , OC and OD respectively. Find the area, in cm^2 , of the shaded region.



Answer : _____ cm^2

ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

8. Cindy used red and green pens to write all 3-digit positive integers in increasing order. All the even numbers were written in red and all the odd numbers were written in green. How many among all the digits '6' were written in red?

Answer : _____

ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

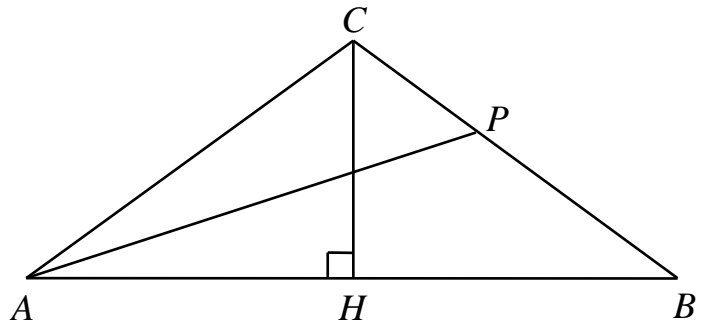
9. Each face of a cube is divided into four equal small squares. Each small square is painted with one of the three different available colours. Small squares with common sides or edges are painted with different colours. What is the maximum number of small squares that can be painted with the same colour?

Answer : _____ quarters

ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

10. Triangle ABC is an isosceles triangle with $AC = BC$. Point H is on AB such that CH is perpendicular to AB and the angle bisector of $\angle CAB$ meets BC at point P . If $AP = 2CH$, find the measure, in degrees, of $\angle ACB$.



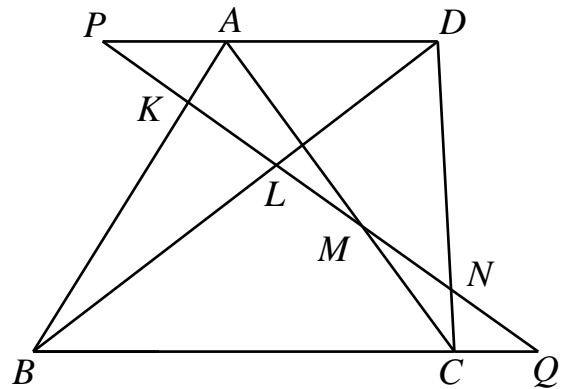
○

Answer : _____

ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

11. In the figure below, $ABCD$ is a trapezium where AD is parallel to BC . Points P , A and D lie on a straight line. Points B , C and Q also lie on a straight line. Line PQ intersects AB , BD , AC and CD at K , L , M and N respectively, such that $PK = KL = LM = MN = NQ$. If $AD = 20$ cm, what is the length, in cm, of BC ?

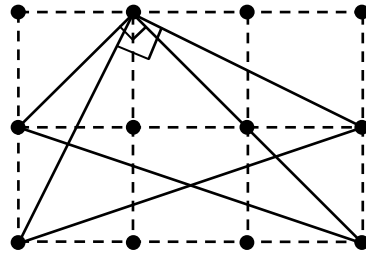
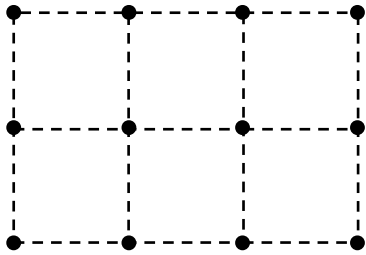


Answer : _____ cm

ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

12. The figure on the left shows a 3×2 table with 12 grid points. Three grid points are chosen to form a right triangle. The figure on the right shows two such right triangles. How many right triangles can be formed?



Answer : _____ right triangles

ESSAY PROBLEMS

Name: _____ Team: _____ Index Number: _____

- 13.** A square is drawn on a plane. Mary chooses a point P on this plane but keeps its position secret. You may draw any straight line on the plane and Mary will tell you whether P lies on one side or on the other side of the line, or on the line itself. The answer is given to you immediately after each line has been drawn. Regardless of the position of the point Mary chooses, what is the minimum number of lines you have to draw to find out whether P lies inside or outside the square or on the sides of the square? Explain your answer.

Answer : _____ *lines*