Mathematics Exploration Problems

Name : _____ **Index Number** : _____

Country : _____

14th International Mathematics and Science Olympiad

Singapore

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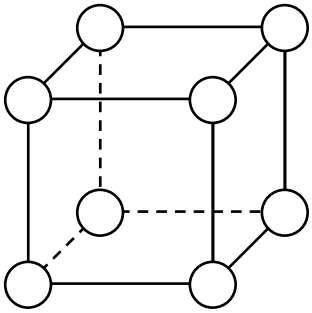
Instructions:

1. Write your name, country and index number on every page of the Answer Sheet.
2. Write your answers only in the Answer Sheet.
3. Answer all questions in Arabic Numerals.
4. Each question is worth 6 marks, and partial credit may be awarded.
5. There are 6 questions in this paper.
6. You have 120 minutes to complete this paper.
7. You are provided with some manipulatives for exploration of some questions.
8. Use black or blue pen or pencil to write your answer.

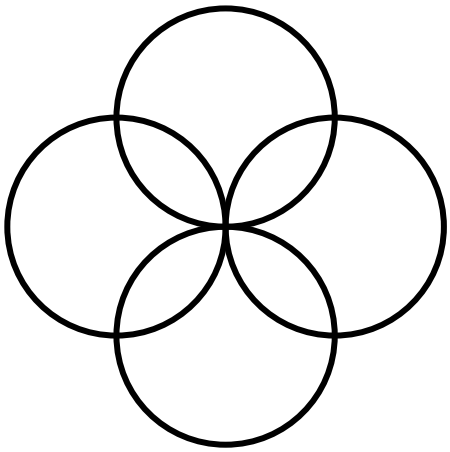
Do not turn over this page until you are told to do so.

EXPLORATION PROBLEMS

(1) Colour four vertices of a cube white and four vertices black so that for any plane passing through three vertices of the same colour, the plane must have a fourth vertex of the other colour.



(2) The figure below shows four intersecting circles enclosing 8 regions. The numbers 1, 2, 3, 4, 5, 6, 7 and 8 are placed in each of the regions with each number being used exactly once. Each region should contain a number such that the sum of the three numbers within each of the four circles is the same. Solutions obtained by reflecting or rotating the figure is considered the same. Find all possible solutions.



(3) The table below shows the travelling times (in hours) between the capital of a country and four other cities A, B, C, and D.

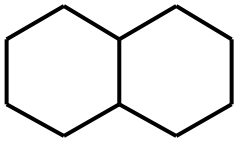
	Capital	A	B	C	D
Capital	0	12	25	8	9
A		0	27	15	12
B			0	20	8
C				0	19
D					0

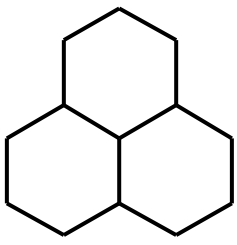
For example, travelling from City A to City B takes 27 hours. Travelling from City B to City A also takes 27 hours.
Mr. Soh who is in the Capital wants to visit each city exactly once before going back to the Capital. What is the shortest possible time for his trip?

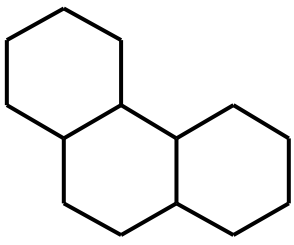
(4) A deck consists of n different cards. A move consists of taking out a stack of consecutive cards from the deck, and then putting the same stack back somewhere else within the deck without flipping any of the cards or changing the order of cards within the stack. You are required to reverse the original order of cards in the deck by such moves. What is the minimum number of moves required if

(i) $n = 5$ (1 point)
(ii) $n = 9$ (2 points)
(iii) $n = 13$ (3 points)

(5) Hexagonal tessellations are formed by joining identical regular hexagons with sides of length 1 cm together by their sides. The figures below are some examples.


Perimeter = 10 cm


Perimeter = 12 cm


Perimeter = 14 cm

If tessellations obtained by reflecting or rotating the figure are considered the same, draw all the different possible hexagonal tessellations with a perimeter of 18 cm in the answer sheet provided.

(6) The Battleship game is played on a 10×10 board, with one 1×4 battleship, two 1×3 cruisers, three 1×2 destroyers and four 1×1 submarines. The ships are placed on the board so that no two ships share common sides or common vertices. However, the ships may touch the edges of the board. On the board provided in the answer sheet, shade a possible position of two 1×3 cruisers, three 1×2 destroyers and four 1×1 submarines on the board such that a 1×4 battleship cannot be fitted onto the board.