MATHEMATICS
SHORT ANSWER PROBLEMS

Name : ____________________________ **Index Number :** ___________

Country : ____________________________

14th International Mathematics and Science Olympiad

Singapore

21 November 2017

Instructions:

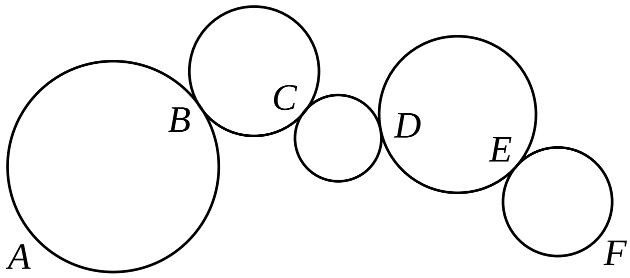
1. Write your name, country and index number on both the Question Booklet and Answer Sheet.
2. Write your Arabic Numerical answers only in the Answer Sheet.
3. Each question is worth 1 mark. There is no penalty for a wrong answer.
4. For problems involving more than one answer, marks are awarded only when ALL answers are correct.
5. There are 25 questions in this paper.
6. You have 60 minutes to complete this paper.
7. Use black or blue pen or pencil to write your answer.

Do not turn over this page until you are told to do so.

SHORT ANSWER PROBLEMS

(1) Find the smallest positive integer n such that the three numbers: $n-96$, n and $n+96$ are positive prime numbers.

(2) The figure below shows five circles touching each other at points B, C, D and E. Points A and B are two different points on the leftmost circle. Points E and F are two different points on the rightmost circle. An ant wants to crawl from A to F along part of the circumference of the circles. If the ant is only allowed to crawl along any parts of the circumference of the circles at most once, how many possible paths can the ant take to crawl from A to F?



(3) Alex spent a total of \$119 in 7 days. Each day, he spent \$4 more than the previous day. How much did Alex spend on the 6th day?

(4) A sequence of numbers is added in the following way:
For example, for the sequence {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, … }, all the numbers in the sequence are first written as decimals as shown below and then added up together.

$$
\begin{array}{l}
0.\mathbf{1} \\
0.0\mathbf{2} \\
0.00\mathbf{3} \\
0.000\mathbf{4} \\
0.0000\mathbf{5} \\
0.00000\mathbf{6} \\
0.000000\mathbf{7} \\
0.0000000\mathbf{8} \\
0.00000000\mathbf{9} \\
0.000000000\mathbf{10} \\
0.0000000000\mathbf{11} \\
0.00000000000\mathbf{12} \\
+ \qquad \vdots \\
\hline
0.123456790123\ldots \\
\hline
\end{array}
$$

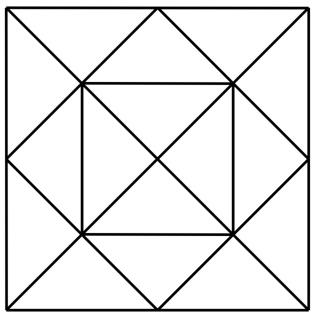
What is the result when the sequence {9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108, 117, … } is added up in the same way?

(5) A positive integer is lucky if each digit in its base-ten representation, starting from the leftmost digit, is not more than the digit on its right. For example, 79, 335 and 679 are lucky numbers but 41, 523 and 786 are not. A super-lucky number is a lucky number with 8 in its ones place and whose square is also lucky. Find the smallest super-lucky number.

(6) Harry, Larry and Parry were each given some marbles on Monday.
On Tuesday, Harry gave Larry and Parry some marbles so that Larry and Parry each ended up with 4 times their original number of marbles.
On Wednesday, Larry gave Harry and Parry some marbles so that Harry and Parry each ended up with 3 times the number of marbles they had on Tuesday.
On Thursday, Parry gave Harry and Larry some marbles so that Harry and Larry each ended up with twice the number of marbles they had on Wednesday.
If Harry, Larry and Parry ended up with 48 marbles each on Thursday, how many marbles did Parry have on Monday?

(7) Form 9-digit numbers using each of the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 exactly once. Let A and B be two such numbers with $B = 8A$. Find the sum of the digits of the number $A + B$.

(8) Straight lines are used to divide a square into identical regions as shown in the figure below. How many triangles are there in the figure?



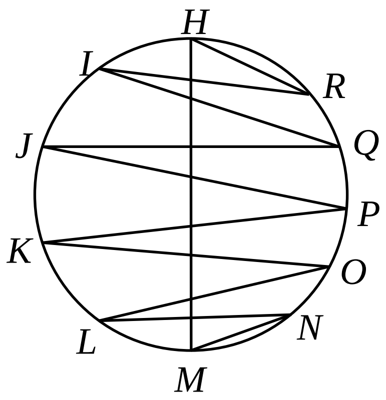
(9) A sculptor wanted to carve a statue from a piece of marble. On the first week, the sculptor carved out 35% of the original piece of marble. On the second week, he carved out 20% of what was left, and on the third week, he carved out 25% of the remaining block to complete the statue. The weight of the completed statue is 48.75 kg. What was the weight, in kg, of the original piece of marble?

(10) Find the greatest 10-digit positive integer that is divisible by 36 and in which each of the digits 0 to 9 appears exactly once.

(11) Jack wants to climb up a 26-step staircase. He is only allowed to take 2 or 4 steps for each move. In how many different ways can he climb up to the top of the staircase with exactly 8 moves?

(12) What number should be subtracted from the numerator of the fraction $\frac{537}{463}$ and added to the denominator so that the resulting fraction is equal to $\frac{1}{9}$?

(13) In the figure below, *H, I, J, ... , Q* and *R* are 11 points on the circle such that $HI = IJ = JK = KL = LM$ and $NO = OP = PQ = QR$. If *HM* is the diameter of the circle, find the total measure, in degrees, of $\angle HRI$, $\angle IQJ$, $\angle JPK$, $\angle KOL$ and $\angle LNM$.

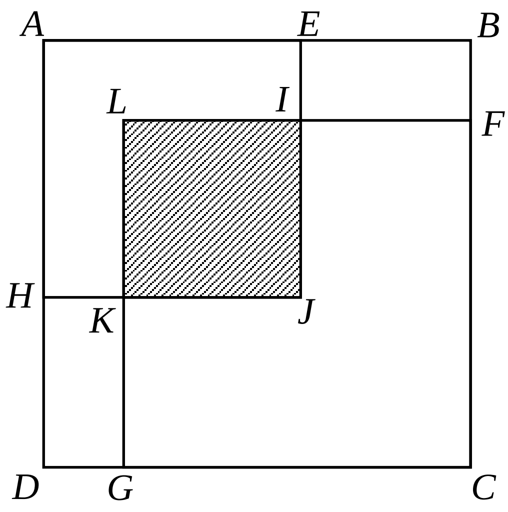


(14) A palindrome is a number which remains the same when its digits are written in the reverse order. For example, 131 is a palindrome. A car's odometer reads 16961 km. How much further should the car travel in kilometers before the odometer reads the next palindrome?

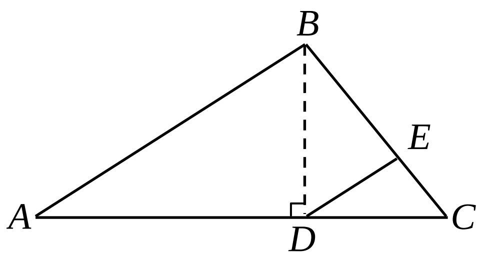
(15) If $\overline{abcdef}$ represents a 6-digit number, where *a, b, c, d, e* and *f* denote different digits, what is the largest possible value of $\overline{abc} + \overline{bcd} + \overline{cde} + \overline{def}$?

(16) Alvin and Charles both walk from Town *Q* to Town *M* at a speed of 4.8 km/h and 5.4 km/h respectively. Bob cycles from Town *M* to Town *Q* at a speed of 10.8 km/h. If all three of them start at the same time, Alvin will meet Bob 5 minutes after Charles meets Bob. How many minutes will Bob take to travel from Town *M* to Town *Q*?

(17) In the figure below, square *AEJH* and square *FCGL* overlap to form the shaded square *KJIL*. The area of *KJIL* is $\frac{1}{4}$ the area of *FCGL* and $\frac{4}{9}$ the area of *AEJH*. What fraction of the square *ABCD* is shaded?



(18) In the figure below, *D* is a point on the line *AC* such that *BD* is perpendicular to *AC*. Point *E* is on the line *BC* such that *AB* is parallel to *DE*. The area of $\triangle CDE$ is 9 cm^2. Given that $BD = 8$ cm and $DC = 6$ cm, find the length, in cm, of *AD*.



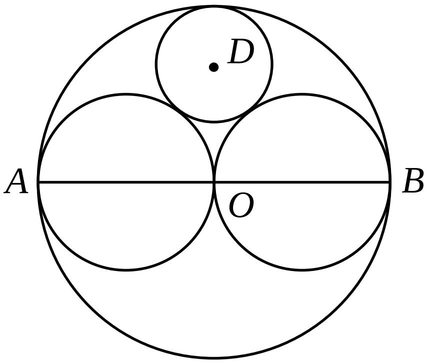
(19) *A*, *B* and *C* are positive integers. The sum of 160 and the square of *A* is equal to the sum of 5 and the square of *B*. The sum of 320 and the square of *A* is equal to the sum of 5 and the square of *C*. Find the positive integer *A*.

(20) Eight positive integers are arranged in a row as shown below.

$$2017,\ a,\ b,\ c,\ d,\ e,\ f,\ g$$

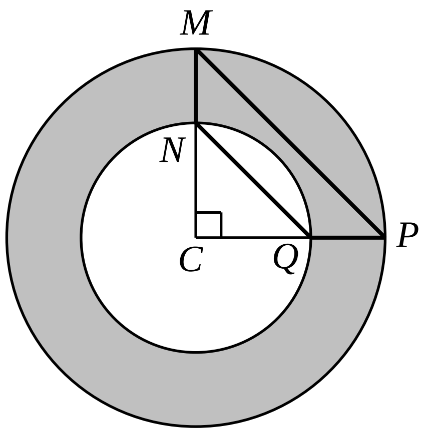
Starting from *b*, each number is the average of the previous two numbers. If the difference between *f* and *g* is 1, what is the smallest possible value of *a*?

(21) In the figure below, *AB* is the diameter of the circle with centre *O*. Two circles are drawn with *AO* and *OB* as diameters. In the region between the circumferences, a circle with centre *D* is inscribed to touch the other three circles. If the length of the radius of the circle *D* is 8 cm, find the length, in cm, of *AB*.



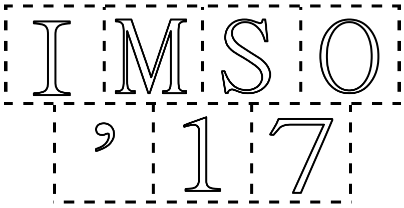
(22) A total of 555 balls are distributed into 11 boxes such that all boxes contain different numbers of balls. If the number of balls in each box must contain the digit ‘5’, what is the greatest possible number of balls in a box?

(23) In the figure below, the area of trapezium *MNQP* is 21 cm^2. *C* is the centre of both the big and small circles and *MC* is perpendicular to *CP*. Find the area, in cm^2, of the shaded region. (Take $\pi = \frac{22}{7}$)



(24) Every unit square on a 8×8 grid table is painted in one of eight different colours. Each colour is used at least once and any two unit squares sharing a common side cannot be of the same colour. Two colours are friendly if there are two unit squares sharing a common side which are painted in these two colours. What is the minimum number of friendly pairs of colours ?

(25) The organizer wants to paint the figure below which comprises of 7 dotted grid squares with characters written on it.



Each grid square should not be of the same color as any of its adjacent grid squares. For example, the grid square with S cannot have the same color as those with M, O, 1 and 7. If there are four different colors to choose from, in how many different ways can the organizer paint the figure?