MATHEMATICS
ESSAY PROBLEMS

Name : _____ **Index Number** : _____

Country : _____

15th International Mathematics and Science Olympiad

Zhejiang Province, China

29 September 2018

Instructions:

1. Write your name, team and index number on every page of this booklet.
2. Write your detailed solution in English in the space provided for each question in this booklet. If you need more space for your working solutions, you may use the reverse side of each page, but please write "next side continued" in the last line.
3. There are 13 questions in this paper.
4. Each question is worth 3 marks and partial credits may be awarded.
5. Diagrams are NOT drawn to scale. They are intended only as aids.
6. You have 90 minutes to complete this paper.
7. Use black pen or blue pen or pencil to write your answer.

The following table is for jury use only.

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Score														
Signature														
Score														
Signature														

Do not turn over this page until you are told to do so.

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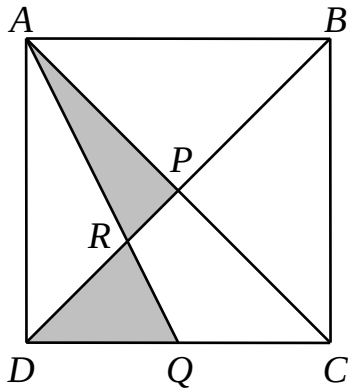
1. In the figure, *ABCD* is a square where $AB = 24$ cm. Let point *Q* be the midpoint of side *CD*, point *P* be the intersection of diagonals *AC* and *BD*, point *R* be the intersection of line segments *AQ* and *BD*. Find the sum of the area, in cm^2, of triangle *ARP* and triangle *DRQ*.



Answer : _____ cm^2

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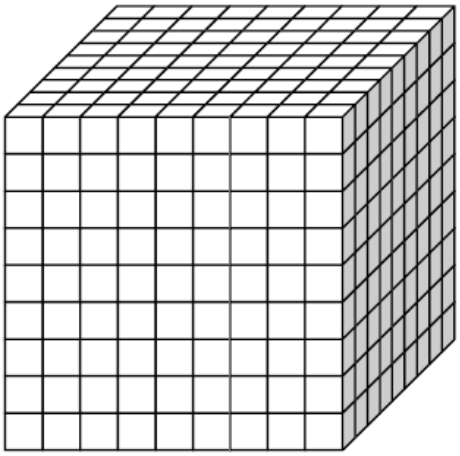
2. A $9\times9\times9$ cube is formed by using 729 unit cubes, each of size $1\times1\times1$. How many different cubes of different sizes can we find in this $9\times9\times9$ cube? (Note: Two cubes having the same size but are located at different positions must be counted as two different cubes)



Answer： _____ cubes

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3. Find all possible positive integer ordered triples (m, n, p) such that $m^2 + n^2 = p^2$, where p is a prime number, $p^2 < 200$ and $m < n$.

Answer : _____

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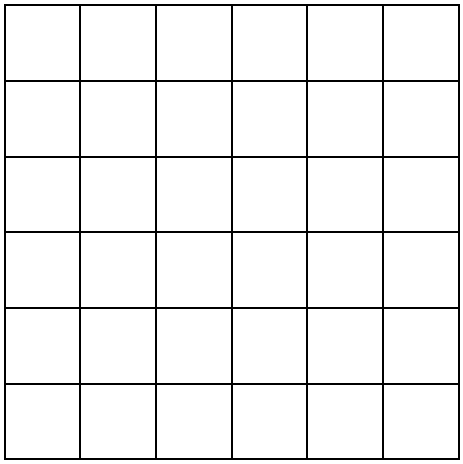
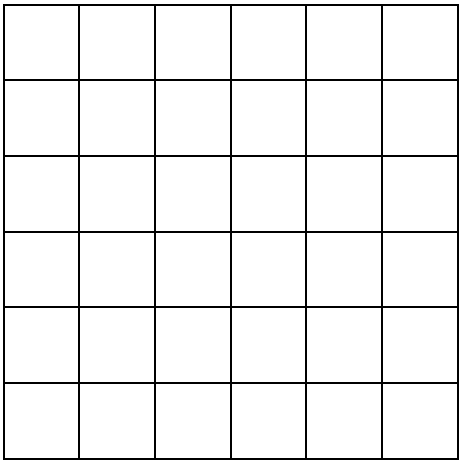
4. Peter is about to shade some cells in the 6×6 grid which is shown below. Any two cells that share a same side must at most have one shaded cell. While on those un-shaded cells, Peter writes a number which is the total number of cells that shares a side with this cell that are shaded.
Given that all the numbers written is at most 3, what is the maximum number of cells that Peter can shade?
Fill the grid with one possible solution of shaded cells and filling number.





Answer： _____ cells,

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5. Find all possible three-digit numbers $\overline{abc}$ such that $\overline{abc} = a! + b! + c!$.
(Note: $n!$ (read as n factorial) means $n! = n \times (n-1) \times (n-2) \times ..... \times 3 \times 2 \times 1$.
For example : $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$).

Answer : _____

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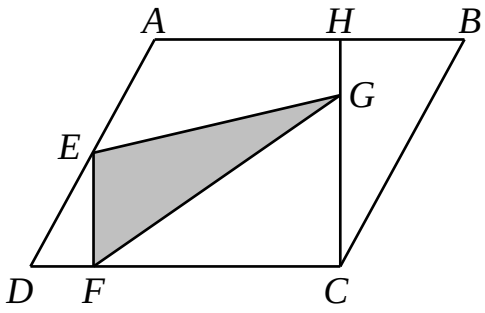
6. In parallelogram *ABCD*, point *E* is on *AD* such that $AE = DE$, point *F* is on *CD* and point *H* is on *AB* such that $EF \perp DC$ and $CH \perp AB$.
Let *AH : HB* = 3 : 2 and point *G* is on line *CH*. If the area of triangle *EFG* is 296 cm^2, find the area, in cm^2, of parallelogram *ABCD*.



Answer : _____ cm^2

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7. One may express a number in base-8. For example, 37 could be written as $(45)_8$ in base-8 because $(45)_8 = 4\times 8 + 5 = 37$. On the other hand, $(49)_8$ does not exist because $4\times 8 + 9 = 41$ should be expressed as $(51)_8$. Let $\overline{AB}$ be a two-digit prime number in base-10, where A, $B < 8$. If the value of $(\overline{AB})_8$ is also a prime number in base-10, how many possible values of $(\overline{AB})_8$ can we obtain?

Answer： _____ values

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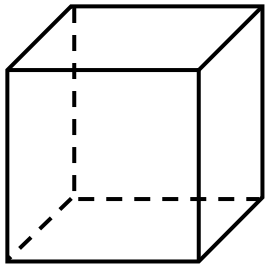
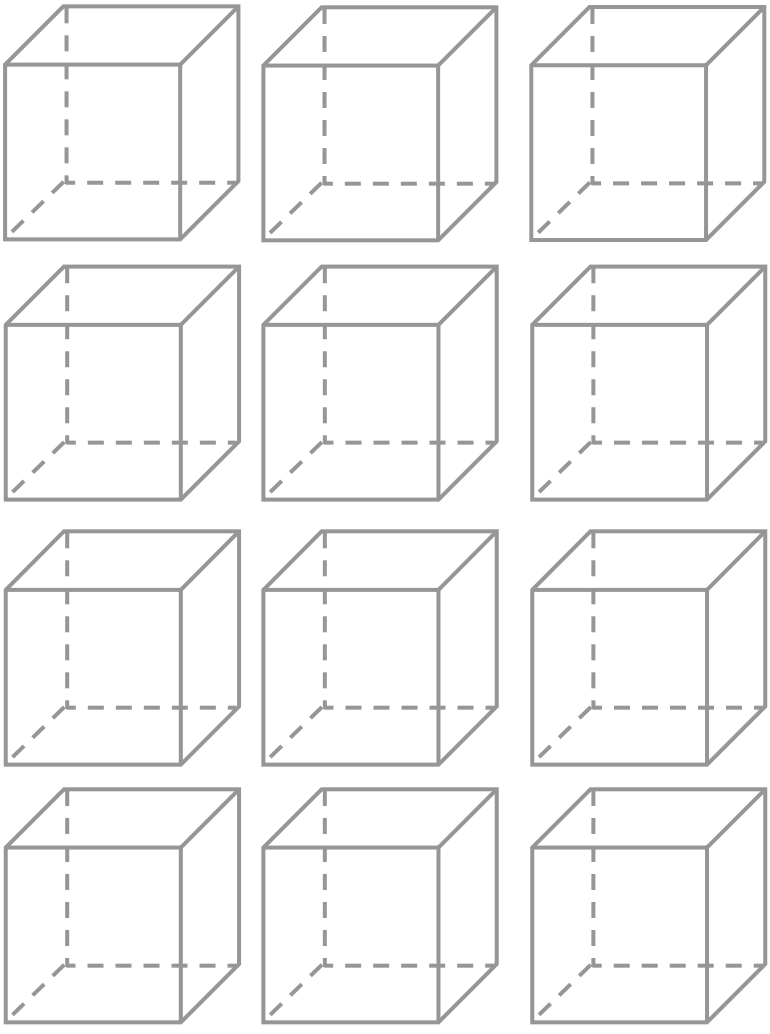
8. Colour each side of the cube below by one of the 3 colours: blue, red or yellow so any 2 sides that share the same vertex should have different colours. How many different ways can we color the cube? (Note: Coloring is considered the same if when rotating or reflecting, we see it the same way.)





Answer： _____ ways,

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9. Using digits 1, 2, 3, 4, 5 and 6 to form a six-digit number $\overline{abcdef}$, where different letters represent different digits. If $2|\overline{ab}$, $3|\overline{abc}$, $4|\overline{abcd}$, $5|\overline{abcde}$ and $6|\overline{abcdef}$, find all possible six-digit numbers $\overline{abcdef}$.

Answer：_____

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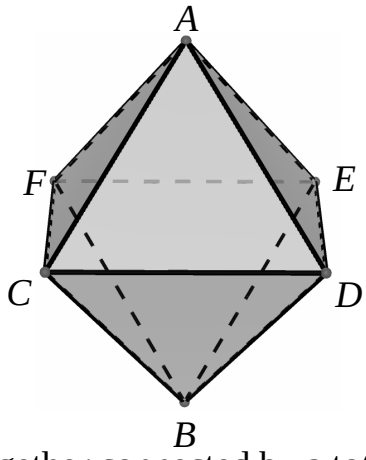
10. A regular octahedron is shown in the diagram below.



There are 6 vertices altogether connected by a total of 12 edges. An ant starts from vertex *A* and moves along any edge but cannot go through any edge more than once.

If the ant visits each vertex exactly once before it returns to vertex *A*, how many different routes are there?

Answer : _____ routes

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11. Let A be a positive integer consisting of different digits, and B be a positive integer whose digits are a rearrangement of the digits of A. Suppose that $A - B = \underbrace{11\cdots1}_{m}$ (m copies of 1's). Find the maximum value of m that satisfy the given requirements.

Answer：_____

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12. Let n be a positive integer such that all of its digits are distinct and that its digit sum is 17. If $n < 2018$, how many different possible values of n are there?

Answer： _____

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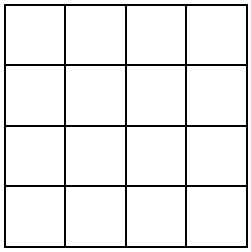
13. Using 16 black pawns and 16 white pawns, how many ways can we place 16 pawns into the 4×4 chessboard, where each pawn is occupying 1 cell, such that in each row and in each column the number of white pawns is even?



Answer : _____ ways