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MATHEMATICS

SHORT ANSWER PROBLEMS

Name : _____

Index Number : _____

Country : _____



15th International Mathematics and Science Olympiad

Zhejiang Province, China

29 September 2018

Instructions:

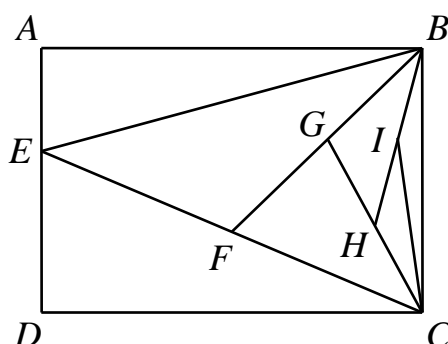
1. Write your name, country and index number on both the Question Booklet and Answer Sheet.
2. Write your Arabic Numerical answers only in the Answer Sheet.
3. There are 25 questions in this paper.
4. For problems involving more than one answer, marks are only awarded when ALL answers are correct.
5. Each question is worth 1 mark. There is no penalty for a wrong answer.
6. You have 60 minutes to complete this paper.
7. Use black pen or blue pen or pencil to write your answer.

Do not turn over this page until you are told to do so.

SHORT ANSWER PROBLEMS

1. A job at Hai Liang Education Park can be done by Alex alone in 6 hours and by Bob alone in 10 hours. Alex works on the job for one hour alone, then Bob continues to work on the job for one hour alone. If they repeat the pattern, in how many hours can the job be done? Express your answer as a common fraction.

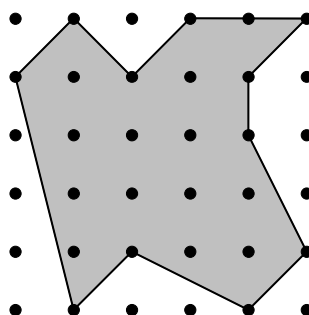
2. In the figure below, E is a point on side AD of rectangle $ABCD$. Points F , G , H and I are midpoints of CE , BF , CG and BH respectively.



If the area of triangle BCI is 1 cm^2 , find the area of rectangle $ABCD$, in cm^2 .

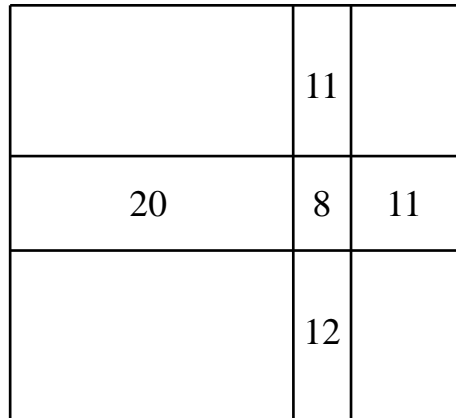
3. In a sequence, the first two terms are 64 and 36. Each subsequent term is the average of the preceding terms. Find the sum of the first 2018 terms.

4. In the figure shown, the distance between adjacent dots in each row and each column is 1 cm. What is the area of the shaded region, in cm^2 ?

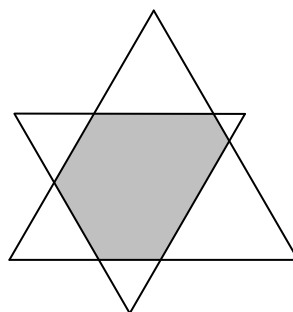


5. For any positive integer n , we define the function $f(n)$ to be the sum of the digits of n and the number of digits of n . For example, $f(218) = 2 + 1 + 8 + 3 = 14$. (Note: The first digit of n , reading from left to right, cannot be 0). What is the sum of maximum and minimum values of n such that $f(n) = 6$?

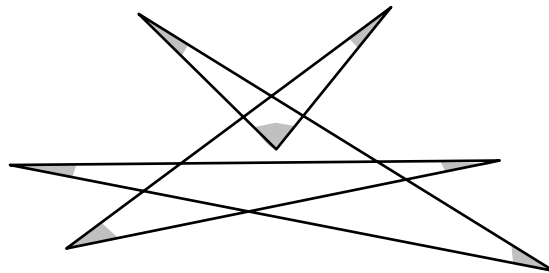
6. A rectangle is divided into 9 smaller portions as shown in the figure below. The perimeter, in cm, of the 5 known portions are also given. Find the perimeter, in cm, of the original rectangle.



7. A cylindrical water tank, with diameter 2.8 m and height 4.2 m, is filled in by a pipe of diameter 7 cm, through which water flows at the rate of 4 m/sec. How many minutes will it take for the pipe to completely fill the tank?
(Take $\pi = \frac{22}{7}$)
8. We want to divide a square into obtuse triangles such that every two triangles meet at a common vertex or at a common edge or are disjoint. At least how many triangles can we have?
9. What is the last digit of $12^{2018} + 14^{2018} + 16^{2018} + 18^{2018} + 20^{2018} + \dots + 2014^{2018} + 2016^{2018} + 2018^{2018}$?
10. How many 3-digit positive integers have the property that the product of all of its digits is equal to 18?
11. Two overlapped equilateral triangles are shown in the figure below. The sides of each triangle are parallel to the sides of the other. The perimeter of the two triangles are 744 cm and 930 cm, respectively. What is the perimeter, in cm, of shaded hexagon?



12. Sunny got three boxes from his father, which contained some number of marbles. His father said that the number of marbles inside the first, second and third boxes are three consecutive integers in increasing order and that they are divisible by 5, 7 and 9 respectively. What is the minimum total number of marbles inside the three boxes?
13. Given is the sequence 1, 1, 1, 3, 5, 9, 17, 31, ... , where the n^{th} term after the 3rd term is the sum of three previous terms.
For example, the 4th term is $1+1+1=3$ and the 5th term is $1+1+3=5$.
What is the remainder when the 2018th term is divided by 8?
14. If the eight-digit number $\overline{2018MN28}$ is divided by 7, the remainder is 5. If the same eight-digit number is divided by 11, the remainder is 9. Find the largest possible value of the two-digit number \overline{MN} .
15. Let I, M, S and O represent different digits and the sum of $IMSO, ISMO, OMSI, OSMI, MISO, MOSI, SIMO$ and $SOMI$ is equal to 60012. What is the sum of $I + M + S + O$?
16. Find the sum of all the shaded angles, in degrees, of the figure shown below.



17. In the following 5×5 square grid, the numbers 1, 2, 3, 4 and 5 are filled in, such that each number appears only once in each row and only once in each column. Find the number filled in the shaded square.

1	2			
				1
		4		
2		5		
	5			4

18. We place 2018 distinct points inside a square, then divide the square into triangles, whose vertices are those 2018 points and the 4 vertices of the square. (that is we have 2022 points in total, which are vertices of the triangles). For each triangle, each point is either one of its vertices or lies completely outside the triangle itself. In how many triangles has the square been divided?

19. In the figure shown below, each empty white cell is filled with an integer from 1 to 9. Each number is used at most once. The gray cell is filled with a sign among \times , \div , $+$ or $-$. In each line with 3 numbers, the calculation is from left to right. In the last column, the calculation is from top to bottom. Show one possible solution.

	-		=	
				\times
	\div		=	
				=
			=	

20. In the grid below, each letter represents a different integer from 1 to 7. When comparing the sum of the numbers in any row or column to that of any other rows or columns, the two sums must be of the same parity (either both even or both odd) and differ by at most 2. Find the sum of all possible values of a .

a	b	
c	d	e
	f	g

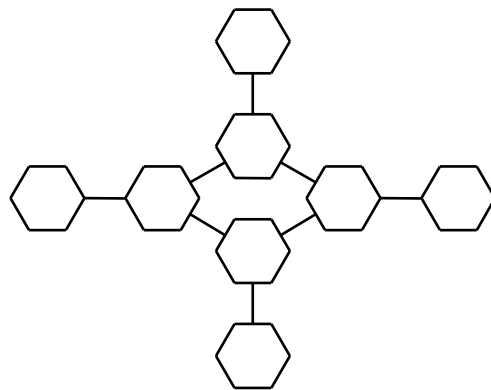
21. Mathrix is a 5×5 puzzle game where we place the digits from 1 to 5 in the board such that each row and column contain the digits from 1 to 5 exactly once. Circles with conditions are placed on some intersections and are meant for the 2 pairs of diagonally adjacent cells. This can be the sum ($+$), difference ($-$) or only odd numbers can be used (*odd*). (For example if we have $+5$ this means that the sum of the two diagonally adjacent numbers is 5)
Find the value of $I + M + S + O$.

	1		M	2
I				
			$7+$	$2-$
	2		3	
$2-$		S		
				<i>odd</i>
2			5	O

22. It is known that \overline{AMMM} and \overline{MMMB} are two 4-digit numbers, where A , B , and M are different digits. If $\frac{\overline{AMMM}}{\overline{MMMB}} = \frac{2}{5}$, find the value of $A + B + M$.

23. What is the units' digit of the expression below:
 $-1 \times 2018 + 2 \times 2017 - 3 \times 2016 + 4 \times 2015 + \dots - 1003 \times 1016 + 1004 \times 1015$?

24. Each hexagon is coloured either red, yellow or blue, such that no two hexagons connected by a line segment have the same colour. In how many different ways can we colour the figure?



25. In the figure shown below, colour each of the equilateral triangles with any one of 4 colours: blue, yellow, green or red so that no two triangles will have the same colour. How many different possible ways of colouring the figure are there? (Two ways of colouring the figure are considered the same if we can obtain one colouring from another by rotating the entire figure, reflecting the figure however counts as a different colouring.)

