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# ESSAY PROBLEMS

Name: \_\_\_\_\_ Country: \_\_\_\_\_ Index Number: \_\_\_\_\_

1. Five points  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$  are given in the plane such that  $AB = 90$  cm,  $BC = 45$  cm,  $CD = 252$  cm,  $DE = 81$  cm and  $EA = 36$  cm. What is the length, in cm, of  $CE$  ?

*Answer :* \_\_\_\_\_ cm

# ESSAY PROBLEMS

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2. In 2019, Peter's age is four more than the sum of the digits of his birth year.  
What is the possible age of Peter (in 2019)?

*Answer :* \_\_\_\_\_ years old

# ESSAY PROBLEMS

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3. There are 23 students in a room. Their ages are either 10, 11, 12 or 13 years old, and there is at least one student of each age. It is known that their total age is 253, and there are 1.5 times as many 12-year-old students as 13-year-old students. How many 10-year-old students are there?

*Answer :* \_\_\_\_\_ students

# ESSAY PROBLEMS

Name: \_\_\_\_\_ Country: \_\_\_\_\_ Index Number: \_\_\_\_\_

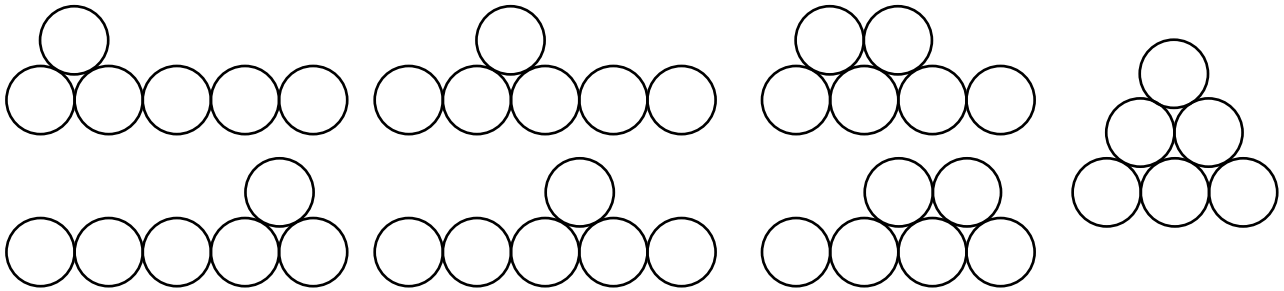
4. Mr. Nam has 10 cuboid prisms: one of size  $2\text{ cm} \times 2\text{ cm} \times 2\text{ cm}$ , three of size  $1\text{ cm} \times 1\text{ cm} \times 2\text{ cm}$ , and six of size  $1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$ . He wants to make a larger cuboid prism by using some of them. What is the biggest possible surface area, in  $\text{cm}^2$ , of the new cuboid prism?

*Answer :* \_\_\_\_\_  $\text{cm}^2$

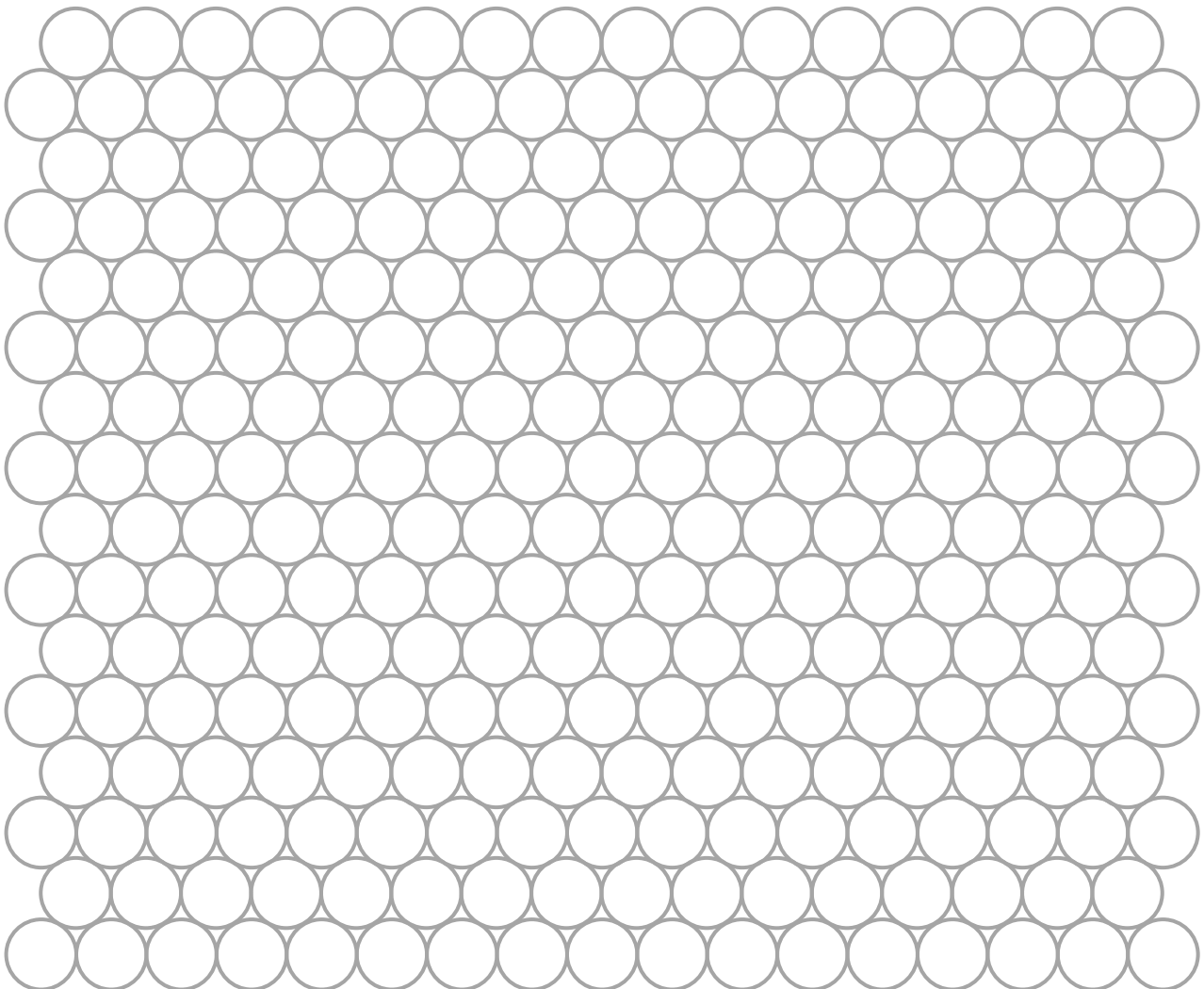
# ESSAY PROBLEMS

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5. Consider the arrangements of  $n$  identical circles, without overlapping, such that there are at least two rows of circles. Each circle should touch the next one in the same row and each circle except those on the bottom row should touch two circles on the row below it. For example, there are 7 possible arrangements for  $n = 6$  as shown in the figure below.



Draw all the possible arrangements for  $n = 8$ .



Answer : \_\_\_\_\_

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6. Each small square in a  $6 \times 6$  table is filled with one of the numbers 1, 2, 3, 4, 5 and 6, so that any row or column has six different numbers. The numbers in some squares are filled as below. How many different completed tables are there?

1	6		5		
5		3	6		
	1	2	3		
3	5	6	2		
6	4				
2		1			

*Answer :* \_\_\_\_\_ tables

# ESSAY PROBLEMS

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7. Find all positive integers  $n$  such that  $n!+2019$  is a perfect square (where  $n!$  denotes the product of all integers from 1 to  $n$ ).

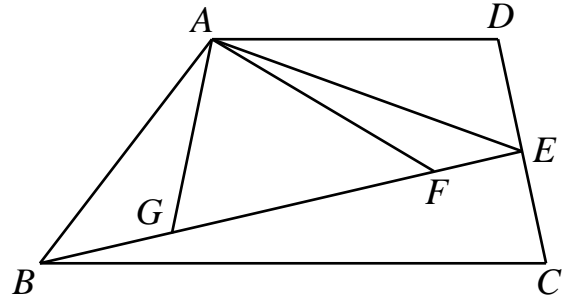
*Answer :* \_\_\_\_\_



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8. In the figure below,  $ABCD$  is a trapezium and  $E$  is the midpoint of  $DC$ . The areas of the triangles  $ABG$ ,  $ABF$  and  $AGE$  are  $15 \text{ cm}^2$ ,  $45 \text{ cm}^2$  and  $60 \text{ cm}^2$ , respectively. What is the area, in  $\text{cm}^2$ , of the trapezium  $ABCD$  ?

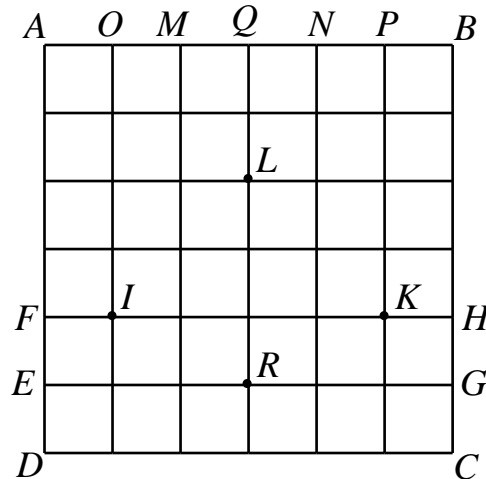


Answer : \_\_\_\_\_  $\text{cm}^2$

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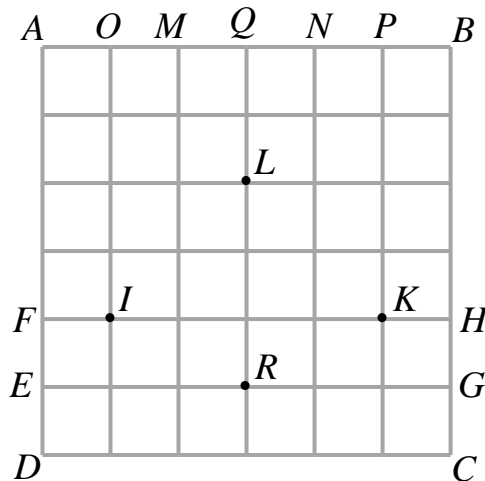
9. Mr. Nobu used a square grid paper  $ABCD$ , as shown in the figure, to fold an envelope.



Then, he folded it as follows:

- 1) Fold  $D$  to  $F$ ;
- 2) Fold  $E$  to  $I$  and  $G$  to  $K$ ;
- 3) Fold  $A$  to  $M$  and  $B$  to  $N$ ;
- 4) Fold  $O$  to  $L$  and  $P$  to  $L$ ;
- 5) Fold  $Q$  to  $R$ .

Given that  $AB = 12\text{cm}$ , what is the area, in  $\text{cm}^2$ , of the envelope he obtained?  
Draw the envelope he obtained.



Answer :

$\text{cm}^2$

# ESSAY PROBLEMS

Name: \_\_\_\_\_ Country: \_\_\_\_\_ Index Number: \_\_\_\_\_

**10.** What is the maximum number of digits that can be removed from the 1000-digit number 20192019...2019 so that the sum of the remaining digits is 2019? What is the biggest resulting number?

Remove digits,

*Answer :* the biggest number is

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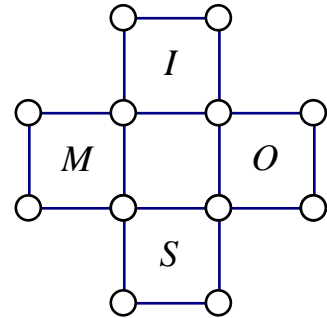
**11.** A football tournament has 8 participating teams, each team playing against each other once. A win is worth 3 points, a draw 1 point, and a loss 0 points. After each team has played exactly 5 matches, all 8 teams have different numbers of points. Given that there are 6 draws among the matches played, and that the current third team has 10 points. What is the greatest possible number of points of the current sixth team?

*Answer :* \_\_\_\_\_ **points**

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- 12.** There are four squares labelled as  $I$ ,  $M$ ,  $S$  and  $O$ , as shown in the figure. Each vertex of these squares is painted in one of the three colours: blue, red, yellow. A colouring is called “*even*” if in each of the four squares, the 4 vertices are painted in exactly two colours so that 2 vertices are of the same colour and the remaining 2 vertices are of the other colour. How many different *even* colourings are there?



*Answer* : \_\_\_\_\_ *even* colourings

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**13.** Laila, Mary and Nelly took turns to pick beads from a box according to the following increasing pattern: Laila picked 1 bead, Mary picked 2 beads, Nelly picked 3 beads, then Laila picked 4 beads, Mary picked 5 beads, Nelly picked 6 beads, and again, Laila picked 7 beads, and so on.

When the number of remaining beads was smaller than the number of beads required for the next turn, all those beads would go to the girl who was in the turn before. In the end, Laila got a total of 2019 beads. How many beads were there in the box at the beginning?

*Answer :* \_\_\_\_\_ beads