2011 IMAS Sample Question
Junior Level Round 1

1. （Geometry）In the following figures, how many of them are not symmetric?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Ans：D.

2. （Number Theory）How many prime number are there among 2, 19, 197, 2009 and 2011?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Ans：E。

$2009 = 7^2 \times 41$；2、19、197 and 2011 are prime, Hence E.

3. （Number Theory）The anicent Chinese use special binary symbols to represent everything, using “⚊” to represent “1”, “⚋” represents “0”. They also use “☶” to represent “1” which is the binary system “001” . And “☱” to represent “6”, which is the binary represntation of “110”. What is the sum of the number in base 10 which are represnted by “☶”,: “ ☷ ”,“ ☰ ”, “ ☵ ” and “ ☲ ”?

(A) 12 (B) 14 (C) 14 (D) 30 (E) 35

Ans：C.

“ ☶ ”、“ ☷ ”、“ ☰ ”、“ ☵ ” and “ ☲ ” are 3, 0, 7, 2, and 5. The sum is 17.

4. （Arithmetic）There are two pools, each with a equal number of fishes. Leong and Hung are competiting to collect as much as fish as they could. When all the fishes in the first pool were collected by them, the ratio of the number of fish that Leong and Hung collected is 3:4. And after all the fish in the second pool were collected, Leong get 33 fishes more than his first catch and the ratio of the number of fishes they collected in the second pool is 5:3. How many fishes originally are in each pool?

(A) 24 (B) 112 (C) 168 (D) 224 (E) 336

Ans 1：C.

The number of fishes in each pool is

$$33 \div \left(\frac{5}{5+3} - \frac{3}{4+3}\right) = 168$$

Ans 2：C.

This could be solved by using ratio, Assume that Leong’ s first catch is 3n, then Hung’ s is 4n. Accordingly, we have $\frac{3n+33}{4n-33} = \frac{5}{3}$，and get n = 24. Hence each pool has $7n$ = 168(fishes).

Ans 3：C.

Let the number of fishes in each pool equal to $56x$. The first catch of Leong is

$24x$, and his second catch is $35x$. Accordingly, $35x-24x=33$ and x =3. Hence the number of fishes in each pool equal to $56x$ =168.

5. （Arithmetic） Two of the fractions are removed from $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$, so that the sum of the remaining three fractions is closest to $\frac{6}{7}$. Which two fractions were removed?

(A) $\frac{1}{2}$、$\frac{1}{5}$ (B) $\frac{1}{2}$、$\frac{1}{6}$ (C) $\frac{1}{3}$、$\frac{1}{5}$ (D) $\frac{1}{3}$、$\frac{1}{4}$ (E) $\frac{1}{4}$、$\frac{1}{5}$

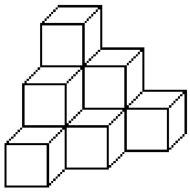
Ans：D.

The fractions are put into equal denominator

$$\frac{1}{2}=\frac{210}{420}、\frac{1}{3}=\frac{140}{420}、\frac{1}{4}=\frac{105}{420}、\frac{1}{5}=\frac{84}{420}、\frac{1}{6}=\frac{70}{420}、\frac{6}{7}=\frac{360}{420}。$$

The sum of the numerator is 609, which is 249 more than 360, and 140+105=245 is closest to 249. So the two fractions removed are $\frac{1}{3}$, $\frac{1}{4}$.

6. （Geometry）As shown in the figure, the three-dimensional figure is composed by 10 cubes, each with side 1 cm. What is the total surface area of the figure?



(A) 24 cm^2 (B) 27 cm^2 (C) 36 cm^2 (D) 48 cm^2 (E) 54 cm^2

Ans：C.

Viewing from top, bottom, front, back, left and right, the surface area are both 6 cm^2, hence the total surface area is 36 cm^2.

7. （combinatoric）1234 is a 4-digit number. The digits of the number are rearrage from the largest to the smallest and from the smallest to the largest to form two numbers. Subtracting the smaller number from the larger number gives 3087 （$4321-1234=3087$）. Then the 4-digit number3087is used to rearrange into a largest and smallest numbers, and subtracting the smaller number from the larger number gives 8352 ($8730-0378=8352$)，working continuously, we have a sequence of numbers:

1234→3087→8352→…

After 12 such operations, what is the sum of the 13 numbers?

Answer：________

Ans：74413。

1234→3087→8352→6174→**6174….**

The sum of the 13 numbers is

$$1234+3087+8352+10\times6174=12673+61740$$
$$=74413$$

Note: The operation is called a Kaprekar operation (after D. R. Kaprekar)

8. (Combinatoric) There are twelve books on the classroom bookshelf. Given that every members of the mathematics groups have borrowed two of the books, and each book on the bookshelf has been borrowed three times. How many members are there in the mathematics group?

Answer：________

Ans：18

As there are 12 books on the shelf, and each one of them were borrowed by 3 members of teh mathematics group, Hence it has been borrowed $12\times3=36$ （times）. Assume there are x members in the mathematics group, each of them borrowed two of the books, hence the total number was $2x$.

$$2x=36$$
$$x=18$$

（By construction, divide 18 people into 6 groups, each group consist of 3 persons. The 12 books is distributed to these 6 groups, and there are 2 books for each person）

2011 IMAS Sample Question
Junior Level Round 2

1. （Combinatoric） The regulations imposed that each member of the national men's basketball league can choose his squad number from 0 to 55. However, if you choose a two-digit number, the numeric value of each digit cannot exceed 5. And the number could not be changes once confirmed. How many different combinations of number available for the team to choose ?

(A) 34 (B) 35 (C) 40 (D) 55 (E) 56

Ans：C.

Number can are available are：0～9、10～15、20～25、30～35、40～45、50～55, 40 in total.

2. （Combinatoric） In figure 1, A large grid is composed by 81 small squares of 1cm^2 each . B and C are two points on the grid. A is a point on the grid so that the area of the triangle △ABC is 3 cm^2. How many such different points A are possible?

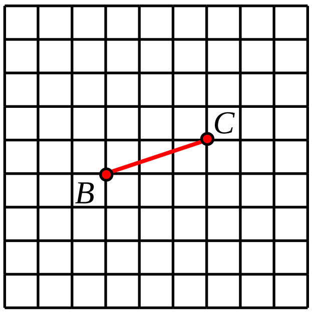


Figure1

(A) 5 (B) 6 (C) 8 (D) 10 (E) 12

Ans 1：C.

Starting from the most upper line, and denote them by 1, 2, …, 10 horizontal lines, and count the points that satisfied the requirement. We got: No point available on line 1, and only one point A_7 on line 2. (as in figure 2). Similarly, we got points A_2, A_1, A_6 that satisfy the requirement. By symmetry, points A_5, A_4, A_3, A_8 also are answer. Hence option C is the answer.

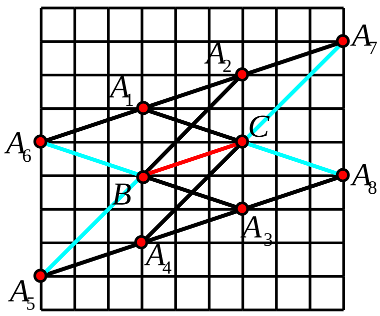


Figure 2

Ans 2：C.

In figure 3, point A_1 obviously is the answer. As the side BC of $\triangle ABC$ is fixed, points above the line BC that satisfy the requirement will pass through point A_1 and parallel to line BC. From the figure, points A_2, A_3, A_4 all satisfy the requirement. By

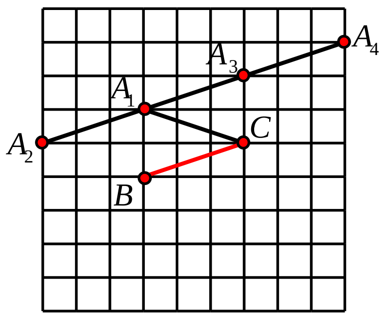


Figure 3

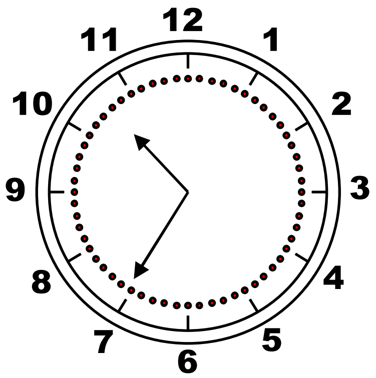
symmetry, there are 4 such points under the line BC. Hence option C is the answer.

3. （Geometry）An electronic **clock** was installed on the clock tower of a train station. There is a small light on every minute mark along the circumference of the clock. At 9:35:20 p.m., how many lights are on the smaller arc between the hour hand and the minute hand?

(A) 12 (B) 15 (C) 17 (D) 20 (E) 24

Ans：A.

At 9:30:00 pm, the angle subtended by the hour hand and the minute hand is 105º. There are 17 lights within this arc of circle. Five minutes and 20 seconds later, the minute hand pass through 5 lights, and the hour hand did not pass through any light. Hence there are $17-5=12$ lights.



4. （Arithmetic）Jia Xian Triangle（Or Pascal Triangle）：

				1				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1
		…		…		…		

Starting from the third line, the numbers on the two ends of each line are 1. And the value of the number within the interval is the sum of the two numbers just above it at the previous line. What is the sum of all numbers from the first to the tenth line?

Answer：________

Ans：1023.

Observe the sum of each line. The sum of the first line is 1, the sum for the second line is 2, the third is 4, and the fourth is 8. Hence we can conjecture that starting from line 2, sum of number in each line is twice the sum of number of the above line. This can be checked easily, by the following diagram, each number in the upper line is counted twice in order to obtain the line below.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

… … …

Hence the sum of number from line 1 to line 10 is:$1+2+2^2+...+2^9=1023$ 。

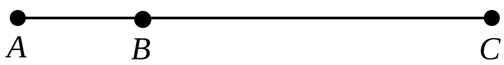
5. （Number Theory）In a game, a number of rectangular bolcks mesaure 2×3×5 is stacked into three piles, each with different number of blocks. The first pile is stacked up with base 3×5. The second pile is stacked with base 2×5. The third pile is stacked with base 2×3. Finally the three piles of blocks are of the same height. How many rectangular blocks are there in total for the three piles.

Answer：________

Ans：31.

The least height of the pile is 30, the LCM of 2, 3 and 5. Then the number of the blocks in each pile is at least $\frac{30}{2}$, $\frac{30}{3}$, $\frac{30}{5}$. The sum is $\frac{30}{2}+\frac{30}{3}+\frac{30}{5}=31$ 。

6. （Arithmetic）As in the figure, Tom and Mary are departing from city A and city B respectively and are going to City C. Tom meets Mary at the distance 2500m away from city C. If Mary starts her journey 10 minutes earlier, then Tom will meet Mary at the distance 1000m away from city C. Given that the speed of Mary is 60m per minute. What is the speed of Tom?



Answer：________m

Ans 1：100.

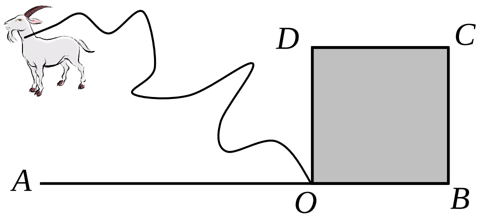
Mary starts 10 minutes earlier, then the difference in distance of them will increase 10×60 = 600 m, and Tom needed to do 2500－1000=1500 m more to meet Mary. That is, when Tom travelled 1500 m, Mary traveled 1500－600 = 900 m. Hence the ratio of speed of Tom and Mary is 1500：900=5：3. Hence the speed of Tom is 60÷3×5= 100 m per minute 。

Ans 2：100.

Assume that the speed of Tom is v m per minute 。

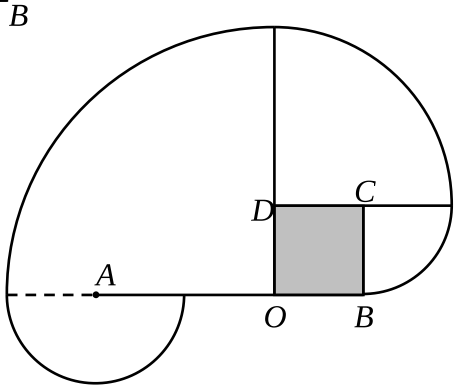
$\frac{600}{v-60}=\frac{1500}{v}$, that is $600v=1500(v-60)$ ，$900v=90000$. Hence $v=100$.

7. （Geometry）A shepherd tied a sheep with a 6-meter-long rope at point O on a grassland, where ABCDO is a high fence. AO is 4 meters in length and OBCD is a square of side 2 meters. What is the largest area of grass that the sheep can access in m^2 ?



Ans：16π m².

The largest area of grass that is available is shown in the diagram：



It is formed by a half circle of radius 2 m and three "one-quarter" circles with radii 2 m, 4 m, and 6 m respectively. The area is

$$\frac{1}{2}\times\pi\times2^2+\frac{1}{4}\times\pi\times2^2+\frac{1}{4}\times\pi\times4^2+\frac{1}{4}\times\pi\times6^2$$

$=16\pi$ （m^2）。

8. （Combinatoric）The following is operated on a given non-zero natural number. If the number is even, then the number is divided by 2. If the number is odd, then 1 is added to the number. The above operations will continue until the resulting number 1. How many numbers are there to end with number 1 under exactly 9 operations ?

Ans：34.

1 number go through 1 operation ： 2；
1 number go through 2 operation ： 4；
2 numbers go through 3 operation ： 3，8；
3 numbers go through 4 operation ： 6，7，16；
5 numbers go through 5 operation ： 5，12，14，15，32；
8 numbers go through 6 operation ： 10，11，24，13，28，30，31，64；
13 numbers go through 7 operation ： 9，20，22，23，48，26，27，56，29，60，62，63，128；
21 numbers go through 8 operation ： 18，19，40，21，44，46，47，96，25，52，54，55，112，58，59，120，61，124，126，127，256；
34 numbers go through 9 operation ：17，36，38，39，80，42，43，88，45，92，94，95，192，50，51，104，53，108，110，111，224，57，116，118，119，240，122，123，248，125，252，254，255，512。

Note：The number after n operation is the n-th term of a Fibonacci sequence.

2011 IMAS Sample Question
Secondary Level Round 1

1. （Algebra）Two different liquid A and B are mixed according to the ratio of x:y (in weight) to make a new drink. The cost of liquid A is $5 per 500g and for B is $4 per 500g. Now, the cost of liquid A increases by 10% and the cost of liquid B decreases by 10%, while the cost of the new drink remains unchanged after the changes. What is the value of x: y ?

   (A) 1∶2 (B) 2∶3 (C) 2∶5 (D) 3∶4 (E) 4∶5

Ans：E.

By $5x+4y=5.5x+3.6y$，$5x=4y$，$x:y=4:5$。

2. （Algebra）There are two paper strips. The longer one is 23cm in length, whilst the shorter one is 15 cm. A certain equal length of paper was cut off from both strips, with a result that the remaining length of the longer one is at least two times the shorter one. What is the length of the paper being cut.?

   (A) 6 cm (B) 7 cm (C) 8 cm (D) 9 cm (E) 10 cm

Ans：B.

Let the length of paper being cut is x cm, then

$$23-x\geq 2(15-x)\text{，}$$

And $x\geq 7$. Hence the length of paper cut is at least 7 cm.

3. （Combinatoric）A certain number of ping pong balls were put into 10 boxes, and the numbers of balls in each box are different. However, the number of balls in each box could not be less than 11, could not be equal to 13 and could not be the multiple of 5. What is the minimum number of ping pong balls needed?

   (A) 150 (B) 155 (C) 162 (D) 173 (E) 186

Ans：D.

At lest $11+12+14+16+17+18+19+21+22+23=173$ needed.

4. （Number Theory） Along one side of a 3000-metre new road , a certain number of lamps-posts are placed. According to the original design, the distance between two lamp-posts is 50 metres and the digging work was completed. If the distance between the posts is now changed to 60 metres, how many new holes are needed to be dug?

   (A) 11 (B) 40 (C) 50 (D) 60 (E) 61

Ans：B.

The original number of holes dug is $\dfrac{3000}{50}+1=61$.

The new number of holes is $\dfrac{3000}{60}+1=51$. The LCM of 50 and 60 is 300, hence holes that are 300 m apart are maintained, which equal to $\dfrac{3000}{300}+1=11$. Hence the new work needed is $51-11=40$ holes.

5. （Number Theory） In the following five numbers,

$$1234554321、1234554321^2、123454321、$$
$$123454321^2、123454321\times1234554321，$$

How many multiple of 2009 are there?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Ans：B.

We know that

$2009=7^2\times41$，$111111=7\times11\times13\times111$，$11111=41\times271$，

$1234554321=11111\times111111$ is a multiple of 41 and 7, but not 7^2

$123454321=11111\times11111$ is not a multiple of 7, hence only 1234554321^2 is the multiple of 2009.

6. （Geometry）Lights are placed at every coordinates of integral values on a Cartesian plane (x, y are positive). When t = 0, only the light at the origin is on. When t = 1, 2, …., the lights will switch on according to the condition that the distant of the switch-on light is at least 5 unit from another switch-on light. How many lights could never be switched on the Cartesian plane?

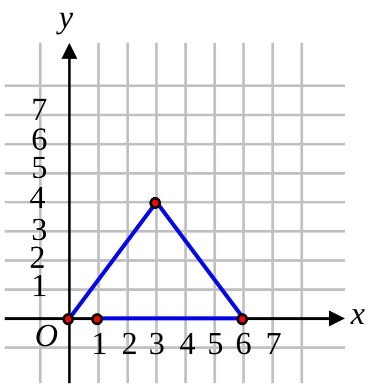
(A) 0 (B) 4 (C) 8 (D) 12 (E) infinite

Ans：A.

All the lights could be switched on.

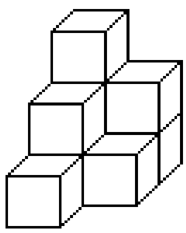
The following is the co-ordinates of sequence of the lights to be switched on （distance of lights being 5）：

$$(0, 0)\to(3, 4)\to(6, 0)\to(1, 0)。$$



This means that lights that have a distance of 1 from the origin could be switched on. Hence all lights could be switched on.

7. （Geometry）The figure shown is composed by 9 cubes, with side 1 cm. What is the total surface area of the figure?



Answer： _________ cm^2

Ans：32.

Viewing from top, bottom, front, back, left and right, the surface areas of the figure are 5 cm^2, 5 cm^2, 5 cm^2, 5 cm^2, 6 cm^2, 6 cm^2，respectively. The total area is 32 cm^2.

8. （Combinatoric）Given the following twelves numbers: 2000, 2001, 2002, … , 2011. How many of them could not be expressed as difference of two perfect squares?

Answer： _________

Ans 1：3.

If a is odd, then

$$a=\left(\frac{a+1}{2}\right)^2-\left(\frac{a-1}{2}\right)^2.$$

If a is a multiple of 4, then

$$a=\left(\frac{a}{4}+1\right)^2-\left(\frac{a}{4}-1\right)^2 。$$

If an even number could be expressed as a difference of two squares, then these two numbers must both be odd or even. The difference of squares of two odd (even) numbers must be a multiple of 4. As 2002, 2006, 2010 are not multiple of 4, they could not be expressed as difference of two squares.

Ans 2：3.

A square number will have reminder 0 or 1 when divided by 4. And when the difference of two squares number divided by 4, the reminder could not be 2. Hence 2002, 2006, 2010could not be expressed as difference of two squares numbers.

1. $a^2-b^2=(a-b)(a+b)=2000$，one of the answer is a=501, b=499。
2. $a^2-b^2=(a-b)(a+b)=2001$，one of the answer is a=335, b=332。
3. $a^2-b^2=(a-b)(a+b)=2003$，one of the answer is a=1002, b=1001。
4. $a^2-b^2=(a-b)(a+b)=2004$，one of the answer is a=502, b=500。
5. $a^2-b^2=(a-b)(a+b)=2005$，one of the answer is a=1003, b=1002。
6. $a^2-b^2=(a-b)(a+b)=2007$，one of the answer is a=1004, b=1003。
7. $a^2-b^2=(a-b)(a+b)=2008$，one of the answer is a=503, b=501。
8. $a^2-b^2=(a-b)(a+b)=2009$，one of the answer is a=1005, b=1004。
9. $a^2-b^2=(a-b)(a+b)=2011$，one of the answer is a=1006, b=1005。

2011 IMAS Sample Question
Secondary Level Round 2

1. （Number Theory）What is the least positive integer that has 20 factors?
   (A) 120 (B) 240 (C) 360 (D) 432 (E) 1536

Ans：B.

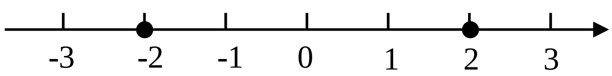
As 20=2×10=4×5=2×2×5, the positive integer that has 20 factors is either the multiple of 3 and 2(9 times), which is 3×2×2×2×2×2×2×2×2×2=1536, or a multiple of 3 (3times) and 2 (4 times), which is 3×3×3×2×2×2×2=432, or a multiple of 3, 5 and 2 (4 times), which is 3×5×2×2×2×2=240. Hence the least positive integer is 240.

2. （Number Theory）Given that m and n are positive integers, and only one of the following statement is incorrect. Which one of the following statement is incorrect?
   (A) $m+n$ is divisible by 3(B) $m+1$ is divisible by n (C) $m+7n$ is prime
   (D) $m=2n+5$ (E) mn is even

Ans：A.

If (A) is correct, then $m+7n=(m+n)+6n$ is larger than 3 and is not divisible by3,and is not prime. Hence (C) is incorrect. Option (D) is incorrect, otherwise, by $m=2n+5$ and the correctness of (A), we have：
$m+n=(2n+5)+n=3n+5$ being divisible by 3. As $3n+5$ is not divisible by 3, option (D) is incorrect. The above disagree with “one of the options (A), (B), (C), (D), (E) is incorrect”. Hence option (A) is incorrect. In fact, it could be checked with taking $m=17$，$n=6$.

3. (Algebra) Given that the points -2 and 2 divides the number line into three sections. If for arbitrary n different points on the number line, at least three of the points lie in one of the sections. What is the least value of n?



   (A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Ans：D.

The 3 sections of the number line include all corresponding points, and by Pigeon-Holes theorem, option (D) is correct.

4. （Algebra）A watch is always 3 minutes slow in one hour than a normal watch. If the watch is accurate at the time 4:30 a.m., and later that morning, the hands of the watch show 10:50 a.m. What is the accurate time shown on a normal watch?

Answer：_________

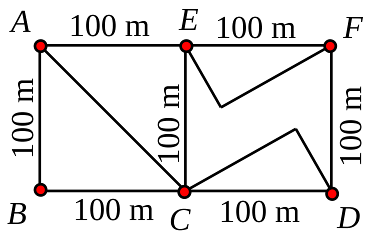
Ans：11:10am.

Let the accurate time be x hour, accordingly

$$\frac{1}{20}\left(x-4\frac{1}{2}\right)=\left(x-10\frac{5}{6}\right)$$

And we have $x=11\frac{1}{6}$ （hours） =11:10am.

5. （Combinatoric） As shown in the figure, A represents the post office whilst B,C,D,E and F are 5 households. The distance between each household is shown in the figure. A postman starts from the post office to deliver letters to each household (every household gets a letter). The last stop for the postman is D and he is not required to return to the post office. What is the shortest route in metres covered by the postman?



Answer：__________

Ans：500 m.

$$A\xrightarrow{100}B\xrightarrow{100}C\xrightarrow{100}F\xrightarrow{100}E\xrightarrow{100}D$$ 。

6. （Geometry）As shown in figure 1, the length of road along the lake-side are AB = 3km, CD = 12 km, and AD = 13km respectively. AB is perpendicular to BC. The shaded area in the figure represent the grassland, the rest is the water. If we take a yacht and start from C, with a speed of 10 km per hour, how much time in hours is needed to reach the other side AD? 。

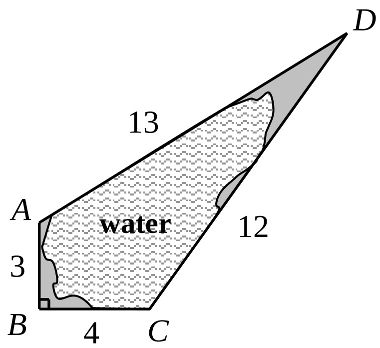


Figure 1

Answer：__________

Ans：$\frac{6}{13}$.

Connect AC (as in figure 2), and by Pythagorean theorem, AC=5 km. As $5^2+12^2=13^2$, ΔACD is an right angle triangle, $\angle ACD$=90°. To start from C with a speed of 10 km per hour and arriving AD with the shortest time, a shortest distance is required, which is the distance from C to AD. This is the height of the right angle triangle ACD.

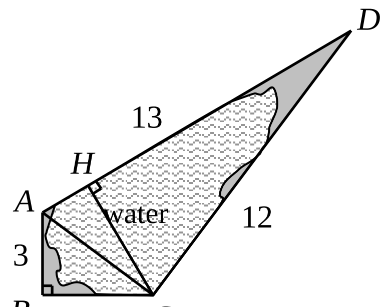


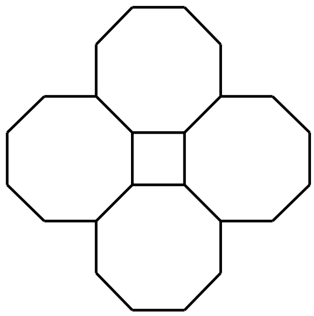
Figure 2

$$\text{Height CH } = \frac{AC \times CD}{AD} = \frac{5 \times 12}{13} = \frac{60}{13} \text{km}。$$

Hence the shortest time required is

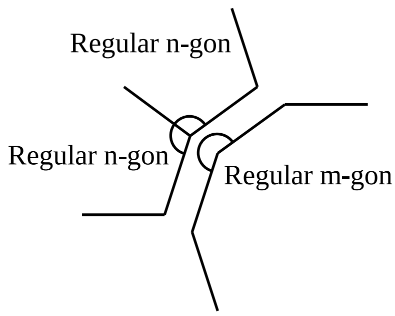
$$\frac{60}{13} \div 10 = \frac{6}{13} \text{ hours.}$$

7. （Geometry）A regular m-sided polygon is tessellated by m n-sided polygon (The diagram show the case when m=4 and n=8). When m = 10, what is the value of n?



Ans：

An interior angle of a regular *m-sided* polygon is $\frac{(m-2)\times 180^\circ}{m}$, hence an interior angle of a 10-sided polygon（m=10）is $\frac{(10-2)\times 180^\circ}{10} = 144^\circ$.

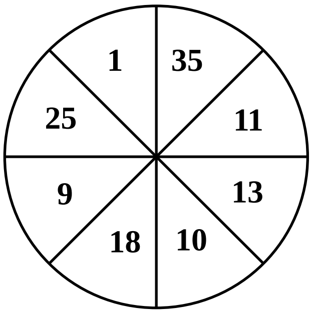


There are two n-sided regular polygon tessellated on every vertex of a regular m-side polygon. Hence the angle sum of the interior angle of m-side polygon and two times the interior angle of a n-sided polygon is equal to 360°. And

$$144^0 + 2 \times \frac{n-2}{n} \times 180^\circ = 360^\circ,$$

hence $n = 5$。

8. （Combinatoric）A group of people darts at the board shown (8 sectors). The number shown on sectors of the board represents the number of scores. Everyone darts four times and everyone scores a total of 62. Any two of the players will have at least one score sector different from the other. At most how many people are involved in the game?



Ans：

To solve the problem, we need to know how many of the darts did not hit the target, and the different combinations of at most four of the following numbers (1, 9, 10, 11, 13, 18, 25 and35) to represent the sum 62. The numbers appeared could be repeated. There are only two even numbers here, the scores could be 18 and 18, 10 and 18, 10 and 10, or all numbers are odd. This helps to limit the search area. Hence we have 8 possible outcomes for score 62.

$$\begin{aligned}\mathbf{62} &= \mathbf{35+25+1+1} \\ &= \mathbf{35+13+13+1} \\ &= \mathbf{35+9+9+9} \\ &= \mathbf{25+25+11+1} \\ &= \mathbf{25+18+18+1} \\ &= \mathbf{25+18+10+9} \\ &= \mathbf{25+13+13+11} \\ &= \mathbf{18+18+13+13}\end{aligned}$$

There is only one possibility for using 3 darts to strike 62, which is {35, 18, 9}. Since the sum of two largest number appeared on the board is 25+35=60, it is not possible to score 62 by two darts.

Hence at most 9 persons is involved in the game.