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2012 小學數學競賽選拔賽初賽試題

第 一 試：計 算 題 (考試時間 1 小時)

◎請將答案填入答案卷對應題號的空格內，不須計算過程。答案若為分數請化為最簡分數。本題目卷正反面空白處可為作演算草稿紙。每題 5 分，共 100 分

1. $2325+7418+7675-2318=15100$

【解】

$2325+7418+7675-2318=(2325+7675)+(7418-2318)=10000+5100=15100$

2. $14\times25\times28\times50=490000$

【解】

$14\times25\times28\times50=(2\times7\times50)\times(25\times4\times7)=700\times700=490000$

3. $1\frac{1}{2012}\times1\frac{1}{2013}\times1\frac{1}{2014}\times1\frac{1}{2015}\times1\frac{1}{2016}\times1\frac{1}{2017}\times1\frac{1}{2018}\times1\frac{1}{2019}=\frac{505}{503}$

【解】

$1\frac{1}{2012}\times1\frac{1}{2013}\times1\frac{1}{2014}\times1\frac{1}{2015}\times1\frac{1}{2016}\times1\frac{1}{2017}\times1\frac{1}{2018}\times1\frac{1}{2019}$

$=\frac{2013}{2012}\times\frac{2014}{2013}\times\frac{2015}{2014}\times\frac{2016}{2015}\times\frac{2017}{2016}\times\frac{2018}{2017}\times\frac{2019}{2018}\times\frac{2020}{2019}=\frac{2020}{2012}=\frac{505}{503}$

4. $(125+60)\times(16+36)=9620$

【解】

$(125+60)\times(16+36)=125\times52+60\times52=6500+3120=9620$

5. $101\times334+3737\times18=101000$

【解】

$101\times334+3737\times18=101\times(334+37\times18)\ =101\times(334+666)=101\times1000=101000$

6. $2012-2002+1992-1982+1972-1962+\cdots+32-22+12-2=1010$

【解】

$2012-2002+1992-1982+1972-1962+\cdots+32-22+12-2$

$=\underbrace{10+10+\cdots+10}_{101項}=1010$

7. $\frac{12+24+48+96+192}{22+44+88+176+352}=\frac{6}{11}$

【解】

$\frac{12+24+48+96+192}{22+44+88+176+352}=\frac{12(1+2+4+8+16)}{22(1+2+4+8+16)}=\frac{6}{11}$

8. $8+98+998+9998+99998+999998+9999998+99999998+999999998=1111111092$

【解】

$8+98+998+9998+99998+999998+9999998+99999998+999999998$

$=10-2+100-2+1000-2+10000-2+100000-2+1000000-2+10000000$

$-2+100000000-2+1000000000-2$

$=1111111110-2\times9=1111111092$

9. $1111+1212+1313+1414+\cdots+9898+9999=494395$

【解】

$1111+1212+1313+1414+\cdots+9898+9999$

$=101\times(11+12+13+14+\cdots+98+99)$

$=101\times\dfrac{(11+99)\times89}{2}=494395$

10. $20.12\times214\div5.03+2.012\times28900\div50.3=2012$

【解】

$20.12\times214\div5.03+2.012\times28900\div50.3$

$=2012\times214\div503+2012\times289\div503$

$=4\times(214+289)=4\times503=2012$

11. $(2012+2013+2014+\cdots+2209+2210)\div2111=199$

【解】

$(2012+2013+2014+\cdots+2210+2210)\div2111$

$=\dfrac{(2012+2210)\times199}{2}\times\dfrac{1}{2111}=\dfrac{2\times2111\times199}{2}\times\dfrac{1}{2111}=199$

12. $12.34+23.41+34.12+41.23=111.1$

【解】

$12.34+23.41+34.12+41.23$

$=(10+20+30+40)+(2+3+4+1)+(0.3+0.4+0.1+0.2)+(0.04+0.03+0.02+0.01)$

$=(1+2+3+4)\times(10+1+0.1+0.01)$

$=111.1$

13. $2012\times40244024-4024\times20122012=0$

【解】

$2012\times40244024-4024\times20122012=2012\times4024\times(10001-10001)=0$

14. $\dfrac{10101+20202+30303+40404+50505+60606+70707+80808+90909}{10101+20202+40404+80808+161616+323232}=\dfrac{5}{7}$

【解】

$\dfrac{10101+20202+30303+40404+50505+60606+70707+80808+90909}{10101+20202+40404+80808+161616+323232}$

$=\dfrac{10101(1+2+3+4+5+6+7+8+9)}{10101(1+2+4+8+16+32)}=\dfrac{45}{63}=\dfrac{5}{7}$

15. $\dfrac{1}{5+\dfrac{1}{4+\dfrac{1}{3+\dfrac{1}{2}}}}=\dfrac{30}{157}$

【解】

$$\frac{1}{5+\dfrac{1}{4+\dfrac{1}{3+\dfrac{1}{2}}}}=\frac{1}{5+\dfrac{1}{4+\dfrac{1}{\dfrac{7}{2}}}}=\frac{1}{5+\dfrac{1}{4+\dfrac{2}{7}}}=\frac{1}{5+\dfrac{1}{\dfrac{30}{7}}}=\frac{1}{5+\dfrac{7}{30}}=\frac{1}{\dfrac{157}{30}}=\frac{30}{157}$$

16. $\dfrac{1}{24}+\dfrac{1}{40}+\dfrac{1}{60}+\dfrac{1}{84}+\dfrac{1}{112}+\dfrac{1}{144}+\dfrac{1}{180}+\dfrac{1}{220}+\dfrac{1}{264}=\dfrac{1}{8}$

【解】

$$\begin{aligned}
&\frac{1}{24}+\frac{1}{40}+\frac{1}{60}+\frac{1}{84}+\frac{1}{112}+\frac{1}{144}+\frac{1}{180}+\frac{1}{220}+\frac{1}{264}\\
=&\frac{1}{8}(\frac{1}{3}+\frac{1}{5})+\frac{1}{60}+\frac{1}{84}+\frac{1}{112}+\frac{1}{144}+\frac{1}{180}+\frac{1}{220}+\frac{1}{264}\\
=&(\frac{1}{15}+\frac{1}{60})+(\frac{1}{84}+\frac{1}{112})+(\frac{1}{144}+\frac{1}{180})+(\frac{1}{220}+\frac{1}{264})\\
=&\frac{1}{15}(1+\frac{1}{4})+\frac{1}{28}(\frac{1}{3}+\frac{1}{4})+\frac{1}{36}(\frac{1}{4}+\frac{1}{5})+\frac{1}{44}(\frac{1}{5}+\frac{1}{6})\\
=&\frac{1}{15}\times\frac{5}{4}+\frac{1}{28}\times\frac{7}{12}+\frac{1}{36}\times\frac{9}{20}+\frac{1}{44}\times\frac{11}{30}\\
=&\frac{1}{3}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{12}+\frac{1}{4}\times\frac{1}{20}+\frac{1}{4}\times\frac{1}{30}\\
=&\frac{1}{4}(\frac{1}{3}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30})\\
=&\frac{1}{4}\times\frac{20+5+3+2}{60}=\frac{1}{4}\times\frac{30}{60}=\frac{1}{8}
\end{aligned}$$

17. $\left(55+\dfrac{9}{7}-\dfrac{2}{3}\right)\times\left(\dfrac{1}{8}+\dfrac{7}{9}+\dfrac{1}{13}+\dfrac{13}{7}\right)-\left(\dfrac{1}{8}+\dfrac{7}{9}-\dfrac{1}{13}+\dfrac{13}{7}\right)\times 52$
$+\left(\dfrac{1}{8}+\dfrac{7}{9}+\dfrac{1}{13}+\dfrac{13}{7}\right)\times\left(\dfrac{2}{3}-\dfrac{9}{7}-3\right)=8$

【解一】

$$\begin{aligned}
&\left(55+\frac{9}{7}-\frac{2}{3}\right)\times\left(\frac{1}{8}+\frac{7}{9}+\frac{1}{13}+\frac{13}{7}\right)-\left(\frac{1}{8}+\frac{7}{9}-\frac{1}{13}+\frac{13}{7}\right)\times 52+\left(\frac{1}{8}+\frac{7}{9}+\frac{1}{13}+\frac{13}{7}\right)\times\left(\frac{2}{3}-\frac{9}{7}-3\right)\\
=&\left(55+\frac{9}{7}-\frac{2}{3}+\frac{2}{3}-\frac{9}{7}-3\right)\times\left(\frac{1}{8}+\frac{7}{9}+\frac{1}{13}+\frac{13}{7}\right)-\left(\frac{1}{8}+\frac{7}{9}-\frac{1}{13}+\frac{13}{7}\right)\times 52\\
=&52\times\left(\frac{1}{8}+\frac{7}{9}+\frac{1}{13}+\frac{13}{7}\right)-\left(\frac{1}{8}+\frac{7}{9}-\frac{1}{13}+\frac{13}{7}\right)\times 52\\
=&52\times\left(\frac{1}{8}+\frac{7}{9}+\frac{1}{13}+\frac{13}{7}-\frac{1}{8}-\frac{7}{9}+\frac{1}{13}-\frac{13}{7}\right)\\
=&52\times\frac{2}{13}=8
\end{aligned}$$

【解二】

令$\left(\frac{1}{8}+\frac{7}{9}+\frac{13}{7}\right)=a$，原式成為

$\left(55+\frac{9}{7}-\frac{2}{3}\right)\times\left(a+\frac{1}{13}\right)-\left(a-\frac{1}{13}\right)\times 52+\left(a+\frac{1}{13}\right)\times\left(\frac{2}{3}-\frac{9}{7}-3\right)$

$=\left(55+\frac{9}{7}-\frac{2}{3}+\frac{2}{3}-\frac{9}{7}-3\right)\times\left(a+\frac{1}{13}\right)-\left(a-\frac{1}{13}\right)\times 52$

$=52\times\left(a+\frac{1}{13}\right)-\left(a-\frac{1}{13}\right)\times 52$

$=52\times\frac{2}{13}$

$=8$

18. $\dfrac{(\frac{10}{7}+\frac{10}{11}+\frac{10}{13})\div(7\times 11+11\times 13+13\times 7)}{12\times 12-7\times 7+5\times 5}=\dfrac{1}{12012}$

【解】

$\dfrac{(\frac{10}{7}+\frac{10}{11}+\frac{10}{13})\div(7\times 11+11\times 13+13\times 7)}{12\times 12-7\times 7+5\times 5}$

$=\dfrac{10\times(\frac{7\times 11+11\times 13+13\times 7}{7\times 11\times 13})\times\frac{1}{7\times 11+11\times 13+13\times 7}}{120}$

$=\frac{1}{12012}$

19. $(1+\frac{2012}{45})+(22+\frac{2012}{45}\times 2)+(333+\frac{2012}{45}\times 3)+\cdots\cdots+(999999999+\frac{2012}{45}\times 9)$

$=1097395697$

【解】

$(1+\frac{2012}{45})+(22+\frac{2012}{45}\times 2)+(333+\frac{2012}{45}\times 3)+\cdots\cdots+(999999999+\frac{2012}{45}\times 9)$

$=1+22+333+\cdots+999999999+\frac{2012}{45}\times(1+2+3+\cdots\cdots+9)$

$=(1+2+3+\cdots+9)+(20+30+40+\cdots+90)+(300+400+500+\cdots+900)+\cdots$

$+(80000000+90000000)+900000000+2012$

$=45+440+4200+39000+350000+3000000+24000000+170000000+900000000+2012$

$=1097393685+2012$

$=1097395697$

20. $\frac{2011}{2012}+\frac{20119}{20120}+\frac{201199}{201200}+\cdots+\frac{20119\overbrace{9\cdots 9}^{2011個9}}{2012\underbrace{0\cdots 0}_{2011個0}}$ 的整數部分是多少？

【解一】

$$\frac{2011}{2012}+\frac{20119}{20120}+\frac{201199}{201200}+\cdots+\frac{20119\overbrace{9\cdots9}^{2011個9}}{2012\underbrace{0\cdots0}_{2011個0}}>\underbrace{\frac{2011}{2012}+\frac{2011}{2012}+\frac{2011}{2012}+\cdots+\frac{2011}{2012}}_{2012項}>2011$$

且$\frac{2011}{2012}+\frac{20119}{20120}+\frac{201199}{201200}+\cdots+\frac{20119\overbrace{9\cdots9}^{2011個9}}{2012\underbrace{0\cdots0}_{2011個0}}<\underbrace{1+1+1+\cdots+1}_{2012項}=2012$，故所求為 2011

【解二】

$$\frac{2011}{2012}+\frac{20119}{20120}+\frac{201199}{201200}+\cdots+\frac{20119\overbrace{9\cdots9}^{2011個9}}{2012\underbrace{0\cdots0}_{2011個0}}$$

$$=(1-\frac{1}{2012})+(1-\frac{1}{20120})+(1-\frac{1}{201200})+\cdots+(1-\frac{1}{2012\underbrace{0\cdots0}_{2011個0}})$$

$$=2012-\frac{\overbrace{1\cdots1}^{2012個1}}{2012\underbrace{0\cdots0}_{2011個0}}$$

因$0<\frac{\overbrace{1\cdots1}^{2012個1}}{2012\underbrace{0\cdots0}_{2011個0}}<1$，故所求為 2011。

答：2011