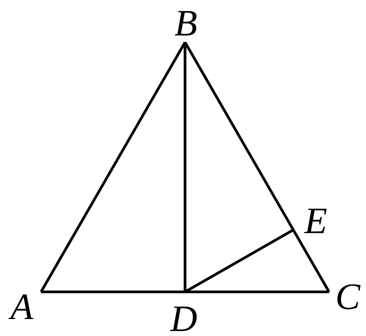
2014 **Taiwan Selection Test for PMWC and EMIC**

Final Round Paper I (Time Allowed : 90 Minutes)

● **Write down all answers on the answer sheet.**
Each problem is worth 10 points and the total is 120 points.

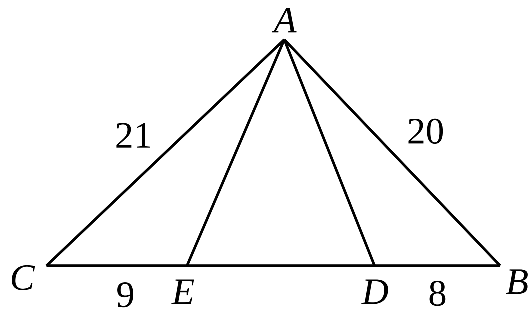
1. Going at the average speed of 40 km per hour, we will be 1 hour late. Going at the average speed of 60 km per hour, we will be 1 hour early. At what average speed, in km per hour, should we go in order to arrive just in time?

2. *ABC* is an equilateral triangle of side length 4 cm. *D* is a point on *AC* such that *BD* is perpendicular to *AC*, and *E* is a point on *CB* such that *DE* is perpendicular to *CB*. What is the area, in cm^2, of a square whose side length is *DE*?



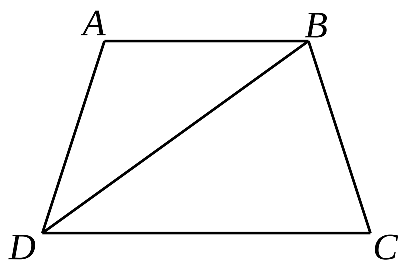
3. Trains go from town A to town B at regular intervals, all travelling at the same constant speed. A train going from town B to town A at the same constant speed along a parallel track meets the trains going in the opposite direction every 10 minutes. How often, in minutes, do the trains go from town A to town B?

4. In triangle *ABC*, *BC* = 29 cm, *CA* = 21 cm and *AB* = 20 cm. *D* and *E* are points on *BC* such that *BD* = 8 cm and *CE* = 9 cm. Determine the measure, in degrees, of $\angle EAD$.



5. What is the largest possible remainder when a two-digit number is divided by the sum of its digits?

6. In the quadrilateral *ABCD*, *AB* is parallel to *DC*, *AB* < *DC* and *AD* = *BC*. The diagonal *BD* divides *ABCD* into two isosceles triangles. Determine the measure, in degrees, of $\angle C$.



7. Every two of A, B and C play one game against each other, scoring 2 points for a win, 1 point for a draw and 0 points for a loss. How many different pairs of numbers are there such that the first is A's total score and the second is B's total score?

8. Find the sum of all positive integers less than 100, each of which has exactly 10 positive divisors.

9. The minute hand of a clock is moving as though it is the hour hand, while the hour hand is moving as though it is the minute hand. At six o'clock in the evening, the clock is showing the correct time. Next day, shortly after seven o'clock in the morning, it shows the correct time again. How many minutes after seven o'clock does that happen?

10. The first three digits of a common multiple of 7, 8 and 9 form the number 523. What is the maximum value of the number formed from its last three digits?

11. Among the positive integers between 1000 and 10000, how many multiples of 9 are there such that the sum of the first two digits is equal to the sum of the last two digits?

12. The numbers 1, 2, . . . , 25 are to be placed in a 5 × 5 table, with one number exactly in each square. Consecutive numbers occupy squares with a common side. Three of the numbers have been placed, as shown in the diagram below. Find the number of different placements of the other 22 numbers.

19		13		
		1		

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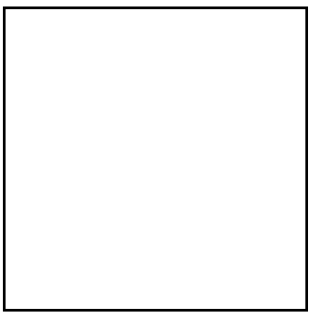
Final Round Paper II (Time Allowed : 60 Minutes)

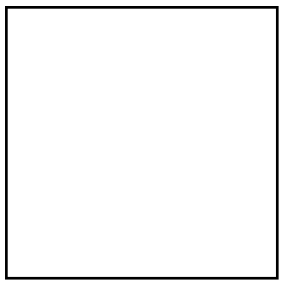
- **Complete solutions of problem 1 and 3 are required for full credits. Partial credits may be awarded. Only answers are required for problem number 2 and 4.**
  Each problem is worth 25 points and the total is 100 points.

1. Insert three plus or minus signs between the digits of 123456789 so that the value of the resulting expression is 100.

Answer：1 2 3 4 5 6 7 8 9 = 100

2. What is the minimum number of obtuse triangles into which a square may be dissected?





Answer：________________

3. There are 10 coins in each of 11 bags. Every coin in 10 of the bags is real, while every coin in the remaining bag looks real but is fake. All real coins weigh the same, and all fake coins also weigh the same. The two weights are unequal. We wish to determine which bag contains the fake coins, but we do not have to find out whether a fake coin is heavier or lighter than a real one. We have a balance which indicates the difference in weight between the contents of its two pans. What is the minimum number of weightings required?

Answer：________________

4. Two copies of each of 1, 2, . . . , 7 are to be placed in a 1×14 table, with exactly one number in each square. The number in the first square is 6, and the number in the last square is 1. There is exactly 1 other number between the two copies of 1, exactly 2 other numbers between the two copies of 2, 3 others between the 3s, 4 others between the 4s, 5 others between the 5s, 6 others between the 6s and 7 others between the 7s. Find all possible placements of the other 12 numbers.

6													1

Answer：

6													1
6													1
6													1
6													1
6													1