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2014 Taiwan Selection Test for PMWC and EMIC

Preliminary Round Paper I (Time Allowed : 60 Minutes)

- Write down all answers on the answer sheet.
- Each problem is worth 5 points and the total is 100 points.

- $6 \times 72 \times 125 = 6 \times 9 \times 8 \times 125 = 54 \times 1000 = 54000$
- $2 + 7 + 27 + 72 + 272 + 727 + 2727 + 7272 = 9 + 99 + 999 + 9999 = 10 - 1 + 100 - 1 + 1000 - 1 + 1000 - 1 + 10000 = 11110 - 4 = 11106$
- $5 + 605 + 1605 + 21605 + 721605 = 5 \times 5 + 600 \times 4 + 1000 \times 3 + 20000 \times 2 + 700000 = 25 + 2400 + 3000 + 40000 + 700000 = 745425$
- $(9999 + 11118888 + 22227777 + 33336666 + 44445555 + 55554444 + 66663333 + 77772222 + 88881111 + 99990000) \div 9 =$
 $\frac{9999 + 99990000}{9} + \frac{11118888 + 88881111}{9} + \frac{22227777 + 77772222}{9} +$
 $\frac{33336666 + 66663333}{9} + \frac{44445555 + 55554444}{9} = 11111111 \times 5 = 55555555$
- $93339 \times 87655 + 31113 \times 37035 = 93339 \times 87655 + 31113 \times 3 \times 12345 = 93339 \times (87655 + 12345) = 93339 \times 100000 = 9333900000$
- $1.2014 \times 0.3993 + 0.3993 \times 0.7986 + 0.7986 \times 4999 = 0.3993 \times (1.2014 + 0.7986) + 0.7986 \times 4999 = 0.3993 \times 2 + 2 \times 0.3993 \times 4999 = 0.3993 \times (2 + 9998) = 3993$
- $\frac{1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512}{0.01 + 0.04 + 0.09 + 0.18 + 0.32 + 0.42 + 0.49 + 0.64 + 0.81} =$
 $\frac{1024 - 1}{(0.01 + 0.18 + 0.81) + (0.04 + 0.32 + 0.64) + (0.09 + 0.42 + 0.49)} = \frac{1023}{3} = 341$
- $201420142014 \times 201420142017 - 201420142015 \times 201420142015 = (201420142015 - 1) \times (201420142015 + 2) - (201420142015)^2 = (201420142015)^2 + 201420142015 - 2 - (201420142015)^2 = 201420142013$
- $11231 + 12341 + 13451 + 14561 + 15671 + 16781 + 17891 + 18911 + 19121 = 9 \times 10000 + (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) \times 1000 + (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) \times 100 + (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) \times 10 + 9 \times 1 = 90000 + 45000 + 4500 + 450 + 9 = 139959$

$$\begin{aligned}
 10. \quad & \frac{1}{7} + \frac{1}{14} + \frac{1}{28} + \frac{1}{31} + \frac{1}{62} + \frac{1}{124} + \frac{1}{248} + \frac{1}{496} = \frac{1}{7} \left(1 + \frac{1}{2} + \frac{1}{4}\right) + \frac{1}{31} \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right) \\
 & = \frac{1}{7} \times \frac{4+2+1}{4} + \frac{1}{31} \times \frac{16+8+4+2+1}{16} = \frac{1}{7} \times \frac{7}{4} + \frac{1}{31} \times \frac{31}{16} = \frac{1}{4} + \frac{1}{16} = \frac{5}{16}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & \frac{1}{1007 \times 1008} + \frac{1}{1008 \times 1009} + \frac{1}{1009 \times 1010} + \dots + \frac{1}{2013 \times 2014} = \\
 & \left(\frac{1}{1007} - \frac{1}{1008}\right) + \left(\frac{1}{1008} - \frac{1}{1009}\right) + \dots + \left(\frac{1}{2013} - \frac{1}{2014}\right) = \frac{1}{1007} - \frac{1}{2014} = \frac{1}{2014}
 \end{aligned}$$

$$12. \quad \frac{1+20+39+58+\dots+2015}{2+4+6+\dots+2014} = \frac{\frac{1}{2} \times 107 \times (1+2015)}{\frac{1}{2} \times 1007 \times (2+2014)} = \frac{107}{1007}$$

$$\begin{aligned}
 13. \quad & 1 - \frac{1}{5} - \frac{1}{50} - \frac{1}{500} - \frac{1}{5000} - \frac{1}{50000} - \frac{1}{500000} = \\
 & 1 - \left(\frac{100000+10000+1000+100+10+1}{500000}\right) = 1 - \frac{111111}{500000} = \frac{500000-111111}{500000} = \\
 & \frac{388889}{500000} = 1 - 0.2 - 0.02 - 0.002 - 0.0002 - 0.00002 - 0.000002 = 0.777778
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \frac{2014^2 - 1995^2 + 1976^2 - 1957^2 + 1938^2 - 1919^2}{361} = \\
 & \frac{(2014-1995)(2014+1995) + (1976-1957)(1976+1957) + (1938-1919)(1938+1957)}{19^2} \\
 & = \frac{4009+3933+3857}{19} = 621
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{(1+2012)\left(1+\frac{2012}{2}\right)\left(1+\frac{2012}{3}\right)\dots\left(1+\frac{2012}{2014}\right)}{(1+2014)\left(1+\frac{2014}{2}\right)\left(1+\frac{2014}{3}\right)\dots\left(1+\frac{2014}{2014}\right)} = \\
 & \frac{(1+2012) \times \frac{(2+2012)}{2} \times \frac{(3+2012)}{3} \times \dots \times \frac{(2014+2012)}{2014}}{(1+2014) \times \frac{(2+2014)}{2} \times \frac{(3+2014)}{3} \times \dots \times \frac{(2014+2014)}{2014}} = \\
 & \frac{2013 \times 2014 \times 2015 \times \dots \times 4026}{2015 \times 2016 \times 2017 \times \dots \times 4028} = \frac{2013 \times 2014}{4027 \times 4028} = \frac{2013}{8054}
 \end{aligned}$$

$$\begin{aligned}
16. & (10000 + 2014 \times 2014 + 0.2014)(2014 \times 2014 + 0.2014 + 0.2014 \times 2014) - \\
& (10000 + 2014 \times 2014 + 0.2014 + 0.2014 \times 2014)(2014 \times 2014 + 0.2014) = \\
& (10000 + 2014 \times 2014 + 0.2014) \times (2014 \times 2014 + 0.2014) \\
& + (10000 + 2014 \times 2014 + 0.2014) \times 0.2014 \times 2014 \\
& - (10000 + 2014 \times 2014 + 0.2014) \times (2014 \times 2014 + 0.2014) \\
& - 0.2014 \times 2014 \times (2014 \times 2014 + 0.2014) = 0.2014 \times 2014 \times 10000 = 2014.2014
\end{aligned}$$

$$\begin{aligned}
17. & \frac{60}{10^2 - 1} + \frac{60}{11^2 - 1} + \frac{60}{12^2 - 1} + \dots + \frac{60}{30^2 - 1} = \\
& \frac{2 \times 30}{(10+1)(10-1)} + \frac{2 \times 30}{(11+1)(11-1)} + \frac{2 \times 30}{(12+1)(12-1)} + \dots + \frac{2 \times 30}{(30+1)(30-1)} = \\
& 30 \times \left[\left(\frac{1}{9} - \frac{1}{11} \right) + \left(\frac{1}{10} - \frac{1}{12} \right) + \left(\frac{1}{11} - \frac{1}{13} \right) + \dots + \left(\frac{1}{29} - \frac{1}{31} \right) \right] = 30 \times \left(\frac{1}{9} + \frac{1}{10} - \frac{1}{30} - \frac{1}{31} \right) = \\
& 30 \left(\frac{1}{10} - \frac{1}{30} \right) + 30 \left(\frac{1}{9} - \frac{1}{31} \right) = 2 + \frac{220}{93} = \frac{406}{93} = 4 \frac{34}{93}
\end{aligned}$$

$$\begin{aligned}
18. & \frac{2000^2}{1999 \times 2001} + \frac{2002^2}{2001 \times 2003} + \frac{2004^2}{2003 \times 2005} + \frac{2006^2}{2005 \times 2007} + \frac{2008^2}{2007 \times 2009} + \\
& \frac{2010^2}{2009 \times 2011} + \frac{2012^2}{2011 \times 2013} + \frac{2014^2}{2013 \times 2015} = \\
& \frac{2000^2}{2000^2 - 1} + \frac{2002^2}{2002^2 - 1} + \frac{2004^2}{2004^2 - 1} + \frac{2006^2}{2006^2 - 1} + \frac{2008^2}{2008^2 - 1} + \frac{2010^2}{2010^2 - 1} + \\
& \frac{2012^2}{2012^2 - 1} + \frac{2014^2}{2014^2 - 1} = \\
& 8 + \frac{1}{2000^2 - 1} + \frac{1}{2002^2 - 1} + \frac{1}{2004^2 - 1} + \frac{1}{2006^2 - 1} + \frac{1}{2008^2 - 1} + \frac{1}{2010^2 - 1} + \\
& \frac{1}{2012^2 - 1} + \frac{1}{2014^2 - 1} = \\
& 8 + \frac{1}{2} \left(\frac{1}{1999} - \frac{1}{2001} + \frac{1}{2001} - \frac{1}{2003} + \dots + \frac{1}{2013} - \frac{1}{2015} \right) = 8 + \frac{1}{2} \left(\frac{1}{1999} - \frac{1}{2015} \right) = \\
& 8 + \frac{1}{2} \left(\frac{1}{1999} - \frac{1}{2015} \right) = 8 + \frac{1}{2} \times \frac{16}{2015 \times (2000 - 1)} = 8 \frac{8}{4027985} = \frac{32223888}{4027985}
\end{aligned}$$

$$\begin{aligned}
19. & \frac{1}{150 \times 151 \times 152} + \frac{1}{151 \times 152 \times 153} + \dots + \frac{1}{199 \times 200 \times 201} = \\
& \text{Since } \frac{1}{a(a+1)(a+2)} = \frac{1}{a+1} \left(\frac{1}{a(a+2)} \right) = \frac{1}{2} \times \frac{1}{a+1} \left(\frac{1}{a} - \frac{1}{a+2} \right) = \\
& \frac{1}{2} \left(\frac{1}{a(a+1)} - \frac{1}{(a+1)(a+2)} \right) = \frac{1}{2} \left(\frac{1}{a} - \frac{1}{a+1} - \frac{1}{a+1} + \frac{1}{a+2} \right) = \frac{1}{2} \left(\frac{1}{a} - \frac{2}{a+1} + \frac{1}{a+2} \right)
\end{aligned}$$

$$\begin{aligned}
& \text{故 } \frac{1}{150 \times 151 \times 152} + \frac{1}{151 \times 152 \times 153} + \cdots + \frac{1}{199 \times 200 \times 201} = \\
& \frac{1}{2} \left[\left(\frac{1}{150} - \frac{2}{151} + \frac{1}{152} \right) + \left(\frac{1}{151} - \frac{2}{152} + \frac{1}{153} \right) + \left(\frac{1}{152} - \frac{2}{153} + \frac{1}{154} \right) + \cdots + \left(\frac{1}{199} - \frac{2}{200} + \frac{1}{201} \right) \right] = \\
& \frac{1}{2} \left(\frac{1}{150} - \frac{1}{151} - \frac{1}{200} + \frac{1}{201} \right) = \frac{1}{2} \times \frac{121404 - 120600 - 91053 + 90600}{18210600} \\
& = \frac{351}{36421200} = \frac{39}{4046800}
\end{aligned}$$

20. $1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{999}}}}}}}}}} = ?$

Since $1 - \frac{999}{1234} = \frac{235}{1234}$ 、 $1 - \frac{1}{1 - \frac{999}{1234}} = 1 - \frac{1234}{235} = -\frac{999}{235}$ 、

$$1 - \frac{1}{1 - \frac{1}{1 - \frac{999}{1234}}} = 1 - \left(-\frac{235}{999} \right) = \frac{1234}{999}$$
、 $1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{999}{1234}}}} = 1 - \frac{999}{1234}$ 、

So we have a cycle of length 3, since $7=3 \times 2+1$, we can get

$$1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{999}}}}}}}} = 1 - \frac{999}{1234} = \frac{235}{1234}。$$